Three Dimensional Extended Boundary Node Method to Potential Problem

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The potential problems often appear in the various engineering fields such as the nuclear fusion. The boundary-element method (BEM) has been applied to the problems and has made a lot of wonderful results; however, a boundary surface has to be divided into a set of boundary elements before applying the BEM to the problems.

Alternatively, Chati et al. proposed the boundary-node method (BNM) [1] as a numerical method for solving the three-dimensional (3D) potential problems. In contrast to the BEM, the BNM requires only the node on a boundary surface. In other words, elements of a geometrical structure are no longer necessary. However, the surface must be divided into a set of integration cells to evaluate surface integrals such as influence coefficients. In this sense, the BNM still has a concept of boundary elements partly.

The purpose of the present study is to formulate the BNM without using any integration cells. To this end, a surface boundary is represented in terms of an implicit function and the 3D local coordinate is used for evaluating surface integrals. Throughout the present study, this method is called the eXtended Boundary-Node Method (X-BNM).

In the X-BNM, a boundary surface is assumed as an implicit surface $f(\boldsymbol{x}) = 0$ and the shape function is assumed to have a support of radius R. Under the above assumption, influence coefficients can be written in the form,

$$I = \int_S F \, \mathrm{d}S \ . \tag{1}$$

Here, S denotes a part of the implicit surface that is contained in a sphere of center $\boldsymbol{y}$ and radius R. Different coordinates are used for the numerical integration of (1), depending on whether S contains a singularity $\boldsymbol{z}$ of $F(\boldsymbol{x})$ or not. For the case where S does not contain any singularity of $F(\boldsymbol{x})$, we use the 3D polar coordinate (ρ, θ, φ) whose center coincides with the sphere center $\boldsymbol{y}$. For this case, the vector equation of S is given by $\boldsymbol{x} = \boldsymbol{g}(\rho, \theta(\rho, \varphi), \varphi)$ $(0 \le \rho \le R, 0 \le \varphi < 2\pi)$ and, by means of the equation, the integration can be evaluated. Note that $\theta(\rho, \varphi)$ is determined by solving the nonlinear equation, $f(\boldsymbol{g}(\rho, \theta, \varphi)) = 0$.

On the other hand, a slightly different coordinate is employed for the case where S contains $\boldsymbol{z}$. For this case, we use the 3D polar coordinate $(\rho^*, \theta^*, \varphi^*)$ whose center is $\boldsymbol{z}$. In addition, the vector equation is given by $\boldsymbol{x} = \boldsymbol{g}(\rho^*, \theta^*(\rho^*, \varphi^*), \varphi^*)$ $(0 \le \rho^* \le R^*(\varphi^*), 0 \le \varphi^* < 2\pi)$. Incidentally, the equation $R^*(\varphi^*)$ for the edge of S is determined by solving the following nonlinear systems: $\sigma_1^*(\rho^*, \theta^*, \varphi^*) \equiv f(\boldsymbol{g}(\rho^*, \theta^*(\rho^*, \varphi^*), \varphi^*)) = 0$, $\sigma_2^*(\rho^*, \theta^*, \varphi^*) \equiv |\boldsymbol{g}(\rho^*, \theta^*(\rho^*, \varphi^*), \varphi^*) - \boldsymbol{y}|^2 - R^2 = 0$.

The Laplace problem is chosen as a typical potential problem, and the performance of the X-BNM is investigated numerically.

[1] M. K. Chati and S. Mukherjee, Int. J. Numer. Methods Eng. **47** (2000), 1523.