Application of Algebraic Approximation to Three Dimensional Multibody Coulomb Problem

Koichiro HIGASHI, Shun-ichi OIKAWA, Yuhei SUTO, and Yutaka MATSUMOTO

Graduate School of Engineering, Hokkaido University, Sapporo 060-8628 Japan

infinite@fusion.qe.eng.hokudai.ac.jp

Since it is difficult to rigorously deal with multibody Coulomb and gravitational collisions, the current classical theory considers them as a series of temporally-isolated binary Coulomb and gravitational collisions within the Debye sphere.
We have proposed the algebraic analysis approximation (ALG) for Coulomb multibody interactions [1-3]. When all the field particles are moving, the ALG has relatively poor prediction ability [2,3] on the two-dimensional motion of the test particle initially at rest. Nonetheless, the ALG approximation gives excellent results for the statistical quantities [3], such as variance of velocity changes or the scattering cross section, for a sufficiently large number of Monte Carlo trials.
Figure 1 shows the variance of change in velocity of the test particle for three-dimensional Coulomb multibody problem. The CPU time taken for one Monte Carlo trial of the direct integration method, DIM, is 31 ms and that of the ALG is 0.13 ms on a computer with an Intel® Celeron® D 346@3.06 GHz. That is, the ALG achieves 263 times speed up for $N = 28$ body problems.
Figure 2 shows the CPU time for the ALG and the DIM. The number of particles dependence of the CPU time for the ALG approximation is in proportion to $N^{3.02}$, while $N^{2.34}$ for the DIM. The ALG is suited for multibody problems for moderate number of particles of $N \lesssim 3 \times 10^5$.

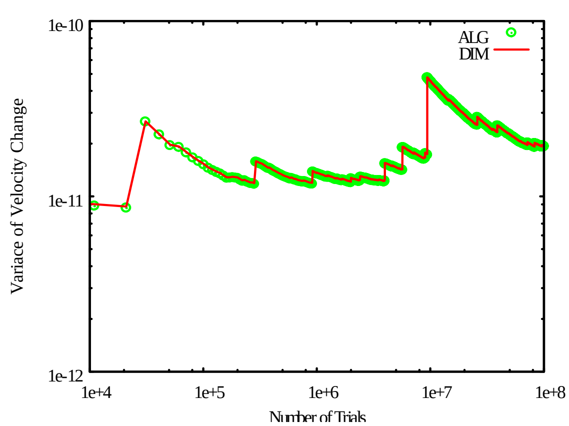

Figure 1: Variance of velocity changes for $N = 28$ for the ALG in green and the DIM in red.

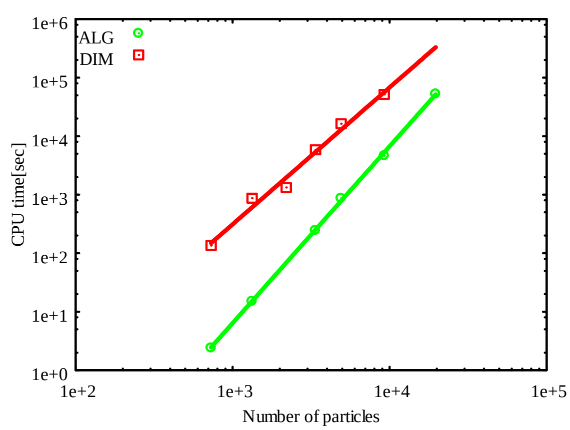

Figure 2: CPU time vs the number of particles N with fitting lines for the ALG in green and the DIM in red.

[1] S. Oikawa and H. Funasaka, Plasma Fusion Res. **3** S1073 (2008).
[2] K. Higashi et al., in *Proc. 18th Int. Toki Conf.* (ITC18), P2-58, Toki (2008).
[3] S. Oikawa, K. Higashi, H. Funasaka, and Y. Kitagawa, to be published in Plasma Fusion Res. **4** (2009).