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A Model of Peaked Density Profile and Inward Pinch in Tokamaks

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A model theory of peaked density profile and inward flux of ohmic discharges in tokamak is presented. Anomalous particle fluxes in the presence of radial electric field, E_r, and radial current are consistently solved in stationary state. Viscous damping of differential flow due to E_r balances with particle diffusion, which determines density and E_r structures. In viscous plasma, peaked profile is expected. Reduction of edge neutrals induces further inward flux; Change of E_r with ion viscous damping propagates into center, causing the ion heating as well.

key words: anomalous pinch, peaked density profile, improved confinement, tokamak transport, ion viscosity, radial electric field

Transport study has been a key issue in magnetic confinement fusion research. In a development of tokamak devices, there remain many uncertainities in particle confinement. From the early stage of tokamak experiments, inward pinch and the density peaking are well known facts, however, they are still unresolved problems. Without a knowledge of particle transport, it is difficult to control the partilcle balance. A few theories of inward pinch has been reported such as Ware pinch[1], ion mixing mode[2], and anomalous pinch due to drift waves.[3] Recently several new improved confinements other than H-mode[4] have been found, such as supershot[5], pellet mode[6-8], the counter injection of neutral beam[9], the improved divertor confinement[10] the improved L-mode[11] and improved Ohmic confinement(IOC)[12,13]. They have attracted attensions because of better confinement, which is compatible with particle exhaust. These discharges have a common feature of the peaked density profile associated with inward particle pinch. Peaking is triggered by the reduction of gas-puffing/degassing and closely correlate with recycling of particles. The density peaking propagates into the center with an inward flux. The typical time scale is order of energy confinement time. The central density rise is hardly explained without a pinch against the density gradient. Simultaneously, the ion temperature also becomes peaked, while the electron temperature remains similar.

Theoretically, the temperature gradient driven drift mode had been nominated[2] as a mechanism to cause an inward pinch. In this mechanism, work done by the temperature gradient and the heat flux causes the inward drift of particles. However, in improved confinements, the electron temperature gradient, which is the origin of the inward particle flux in ref.[2], does not decrease and both the inward pinch of density and the steepening of ion temperature profile have been observed during the transition. Furthermore, no significant enhancement of fluctuation level has been obeserved, the fact indicates that models based on the fluctuation[2,3] may not be applicable. Thermodynamically, an inward pinch must be compensated by an increase of other dissipations, when no external power is supplied.

In this letter we present a new model to explain the inward particle drift (density peaking), which can also explain the IOC. This model theory shows that differential part of the plasma rotation due to radial electric field, E_r, induces the inward flow of particles. The balance of particle diffusion and the viscous damping of the velocity shear determines the E_r structure. The reduction of neutral particles causes the additional peaking of the profile. Due to the viscous damping, ion heating is also expected associated with the pinch. The ratio of diffusion coefficient and the viscosity is found to dictate the density peaking.

The model plasma is bounded by separatrix or limiter surface at $r = a$. We divide the region into two; the inner region I ($r<a-\Delta$, $\Delta \ll a$), where the effect of neutral particles is small and is neglected, and the outer thin region II ($a-\Delta<r<a$), where neutrals play a role to determine edge conditions of n and E_r. Neutral density, $n_0(r,t)$, is assumed to be given as a parameter.

In region I, we use cylindrical coordinates and we treat the region II as a slab.

To obtain the structure of radial electric field, $E_r(r)$, consistent with the density and temperature profiles, n(r) and T(r), we consider particle fluxes as follows. Electron and ion fluxes are assumed to be anomalous driven by drift-type micro-turbulence. In the framework of quasilinear theory, the flux for the mode frequency, ω,and the poloidal wave number, m/r, is written as [14),]

$$\Gamma_{s,a} = -D_s n_s [\frac{n_s'}{n_s}(1+\alpha_s \eta_s) - \frac{eE_r}{T_s} + \frac{er\omega B_t}{mcT_s}] \qquad (1)$$

where B_t is the toroidal magnetic field, η and α denote the ratio of temperature gradient to density gradient and the numerical coefficient of order unity. Prime denotes the radial derivative and the suffix shows particle species, s. In the presence of static radial electric field, the dispersion relation of the drift wave gives the doppler shifted frequency of the mode, which exactly cancels the rigid rotation term due to E_r.[14)] Only the shear term of E_r/r in Eq.(1), which corresponds to the differential rotaion, affects the particle flux.

The ion flux also comes from the viscosity flow and the momentum loss by charge exchange in the presence of toroidal rotation, U_ϕ, and the polarization drift. They are given by [15,16)], $\Gamma_{i,v}=(qR/erB)n_i m_i \nabla\mu_\perp \nabla U_\phi$, $\Gamma_{i,cx} = (qR/erB)n_i m_i U_\phi n_0 \langle\sigma_{cx} v\rangle$, $\Gamma_{i,p} = (\varepsilon_0 c^2/ev_A{}^2)\partial E_r/\partial t$, where $U_\phi = (qT_i R/erB)[n_i'/n_i(1+c_i\eta_i)-$

$eE_r/T_i]$, R and a are major and minor radii, $\mu_\perp$ is the viscosity, $\langle\sigma_{cx}v\rangle$ is the charge exchange crosssection, and c_i is a neoclassical coefficient[15], respectively. We adopt the basic equations as the Poisson's equation,

$$(\varepsilon_0\varepsilon_\perp/e)\partial E_r/\partial t = \Gamma_{e,tot} - (\Gamma_{i,a}+\Gamma_{i,v}+\Gamma_{i,cx}) \qquad (2)$$

and the continuity as

$$\partial n_{e,i}/\partial t = -\nabla\cdot\Gamma_{e,i,tot} + S_{e,i} \, . \qquad (3)$$

Particle sources $S_{e,i}$ are localized near the edge and are given by the ionization of neutrals. We assume that $S_e = S_i$ ($\equiv n_0 n_e\langle\sigma_i v\rangle$; where $\langle\sigma_i v\rangle$ is the ionization crosssection). The condition, $\varepsilon_\perp \gg 1$, holds inside the plasma,[17] the electron density profile of Eq.(3), where $\Gamma_{e,tot}$ equals to $\Gamma_{e,a}$, is considered in the following.

We introduce the normalized electric filed, E ($= a^2 eE_r/rT_i$), and its shear part {E} ($\equiv$E-const.). For simplicity, we consider the homogeneous tempearture case ($\alpha=0$) with $T_e = T_i$ and the transport coefficients are set to be numerical factors. In the coordinate y ($=r^2/a^2$) with N=ln n, Eq.(3) is written

$$\frac{\partial n}{\partial t} = \frac{2D_e}{a^2}\frac{d}{dy} ny(\ \{E\} + 2N'\ ) + S_e \qquad (4)$$

In a stationary state, the integration of Eq.(4) over y gives,

$-2N' = \{E\} + a^2\langle S\rangle/2D_e \equiv F$, where $\langle S\rangle = \int n_e n_0 \langle \sigma_i v\rangle dy/n_e y$. Particle source is limited to the thin layer near edge of the order of the penetration length of neutrals, Δ. $\Delta \gg \lambda_D$ holds ($n_e = n_i$), where λ_D is the Debye length. For simplicity, the averaged source term $\langle S\rangle$ is approximated to be constant in space. Substituting N' into Eq.(2) we have,

$$\frac{\partial E}{\partial t} = -\nu_a F + \nu_\mu \frac{1}{y}\frac{d}{dy} y \frac{d}{dy} F + \nu_{cx} \frac{1}{y} (F+C) + \frac{a^2}{\rho_i^2} \langle S\rangle \qquad (5)$$

where $\nu_\mu = q^2R^2\mu_\perp/a^4$, $\nu_a = D_i/\rho_i{}^2$, $\nu_{cx} = q^2R^2n_0\langle\sigma_{cx}v\rangle/a^2$, ρ_i is toroidal gyroradius and C, $\{E\} = E-C$, is constant of integrand which is determined by the boundary condition.

In the region I, where $n_0 \simeq 0$, we get solutions of Eqs.(4) and (5), in terms of the 0-th order modified Bessel's function as,

$$\{E\} = A_1 I_0(\sqrt{\nu_a/\nu_\mu}\, y) \ , \qquad (6)$$

and $\{E\} = -2N'$ (i.e., $\Gamma = 0$). In the outer slab region II, we obtain, $\{E\} + a^2\langle S\rangle/D_e = A_2e^{\alpha y} + A_3e^{-\alpha y} + A_4$, where $\alpha^2 = (\nu_a - \nu_{cx})/\nu_\mu$. Coefficients A_1 to A_4 are to be determined from continuity and boundary conditions. The continuity of $\{E\}$ and $\{E\}'$ are imposed between regions I and II at $r = a - \Delta$. As the boundary condition at r=a, we give the density and velocity fall-off lengths as, $-dn/dr/n\big|_{r=a} = 1/\lambda_n$, $-dU_\phi/dr/U_\phi\big|_{r=a} = 1/\lambda_v$, which are determined by scrape-off layer (SOL) transport.[18] If there is no momentum loss via viscosity, the value of λ_v becomes infinity.

Imposing these conditions, we obtain A_1 and the solution in the core as

$$\{E\} = \{ (\frac{a}{\lambda_n} - \frac{a^2}{2D_e}\langle S\rangle) \frac{I_0(\sqrt{\nu_a/\nu_\mu}\, r^2/a^2)}{I_0(\sqrt{\nu_a/\nu_\mu}\,(1-2\Delta/a))} \}$$

$$\times \{ 1 - \frac{2\Delta}{a}\sqrt{\nu_a/\nu_\mu}\frac{I_1(\sqrt{\nu_a/\nu_\mu})}{I_0(\sqrt{\nu_a/\nu_\mu})} - \frac{2\Delta}{a}\frac{2a}{\lambda_V}\frac{1}{I_0(\sqrt{\nu_a/\nu_\mu})} \} \qquad (7)$$

and $-(dn/dr/n)/r = \{E\}$. To understand the essential effects on profiles of the electric field and the density, let us consider the 0-th order contribution of Δ/a. The solution (7) shows that the balance between particle diffusion and viscous damping of rotation dictates the shape the density profile. The peaked profile can be sustained even in the absence of particle source. This is due to the differential rotation in the presence of E_r. To show the effect of viscosity, we plot the density profile for various values of $\sqrt{\nu_a/\nu_\mu}$ in Fig. 1(a). For fixed line averaged density, this ratio is changed from 0.1 to 10. Numerical coefficient at the edge, $A_0 = (a/\lambda_n - a^2\langle S\rangle/D_e)$, is chosen to be 10. We see that the ratio of ν_a and ν_μ strongly affects the internal structure. When the viscosity is large, we obtain the peaked density profile. In the less viscous plasma, the shear flow only exists near the edge, similar to ordinary fluid. The actual value of ν_a/ν_μ is not known, however, if we evaluate D_i to be order of $1 m^2/sec$ and ν_μ to be the neoclassical value, then the value of ν_a/ν_μ roughly becomes order of 10 in ASDEX discharges

[12,13]. If a quasilinear theory is applied to drift wave fluctuations this ratio becomes order of unity[19].

Neutral source is expected to become small, when the gas-puff is reduced, and the value A_0 rises. In Fig.1(b) we show the density profiles for various values of A_0 for the fixed values of ν_a/ν_μ and the line averaged density. We see that the reduction of the source causes the density peaking. The relation between the density peaking and the particle confinement time, τ_p is seen from Eq.(7). From the definition of τ_p^{-1} as $\int n_e n_0 \langle \sigma_i v \rangle dV / \int n_e dV$, we observe the τ_p dependence upon the peakedness (i.e., the averaged value of $-\{E\}$ for the given profile and the transport coefficients). Associated with the density peaking, τ_p increases when the neutrals are reduced. The SOC/IOC transition is attributed to this mechanism. In the case of less viscous plasma, when ν_a/ν_μ is large, the sensitivity of the internal density profile to the change of the edge source is found to be small.

In summary, our analysis shows that an inward particle pinch is due to the viscous damping of sheared flow. The corresponding differential rotation is caused by the radial electric field. In order to show this effect, the radial profile of the potential corresponding to the shear part $\{E\}$ is plotted in Fig.1(c) for various values of $\sqrt{\nu_a/\nu_\mu}$. We clearly see that the viscous plasma has stronger pinch effect into the core region.

Associated with the inward drift, the dissipation due to ion viscous damping takes place. So that our theory also predicts the anomalous ion heating during the particle pinch. According to this heating, ion temperature rise and its peaked profile are expected. The input rate, ΔQ_i, is roughly estimated as $\Delta Q_i = nTD_a\Delta(\nabla n/n)^2$. In the case of SOC/IOC transition the change of $\Delta(\nabla n/n)^2 \sim (1/a)^2$ has been observed.[13] Therefore $\Delta Q_i \sim nT/\tau_E$ is expected from our model. Electron drug can cause the damping of the velocity shear, so that the electron heating may occur under a certain condition. The analysis is left for future.

There have been experimental observations of radial electric field in relation to confinements.[20,21,22] The different radial structures in potential have been reported from the linear regime to SOC regime in Ohmic discharges.[22] Though the experimentally observed electric potential contains the contribution from the rigid rotation, the comparison study may explore the underlying physics.

We mainly consider the density peaking in ohmic discharges. In particular, transition from the SOC to the IOC is studied. The obtained results reveal the common feature of the improved confinement of peaked density profile and can be applied to the other discharges. Detailed analyses of this model and applications to the other improved confinement will be made in separate papers.

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Fugure Caption

Fig. 1 Density profiles for various values of $\sqrt{\nu_a/\nu_\mu}$.(a). This ratio is changed from 0.1 to 10. Numerical coefficient at the edge, $A_0=(a/\lambda_n-a^2\langle S\rangle/D_e)$, is 10. Density profiles for various values of A_0 (3, 5, 10, 20)for the fixed values of ν_a/ν_μ(= 1).(b).
(c) Radial profile of electric potential corresponding to the shear part {E} is plotted in for various values of $\sqrt{\nu_a/\nu_\mu}$ (1, 2, 3, 5, 10). A_0 = 10.
The line averaged density is kept constant.

Fig. 1(a)

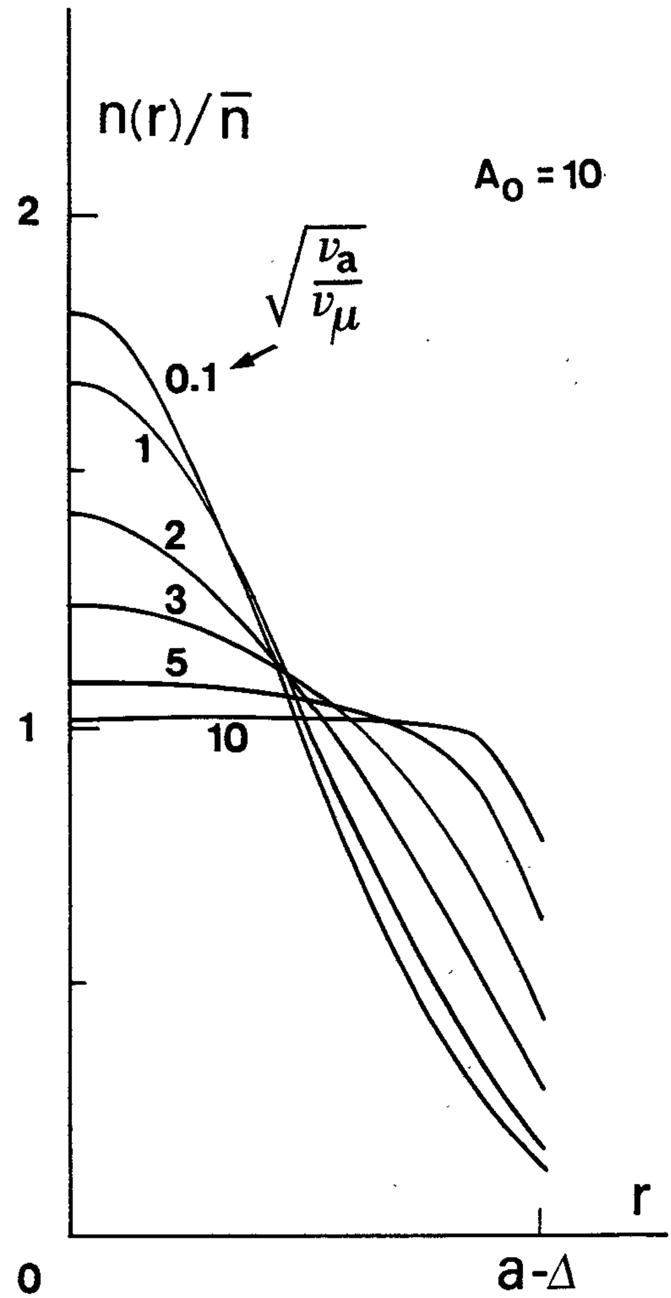


Fig.1 (b)

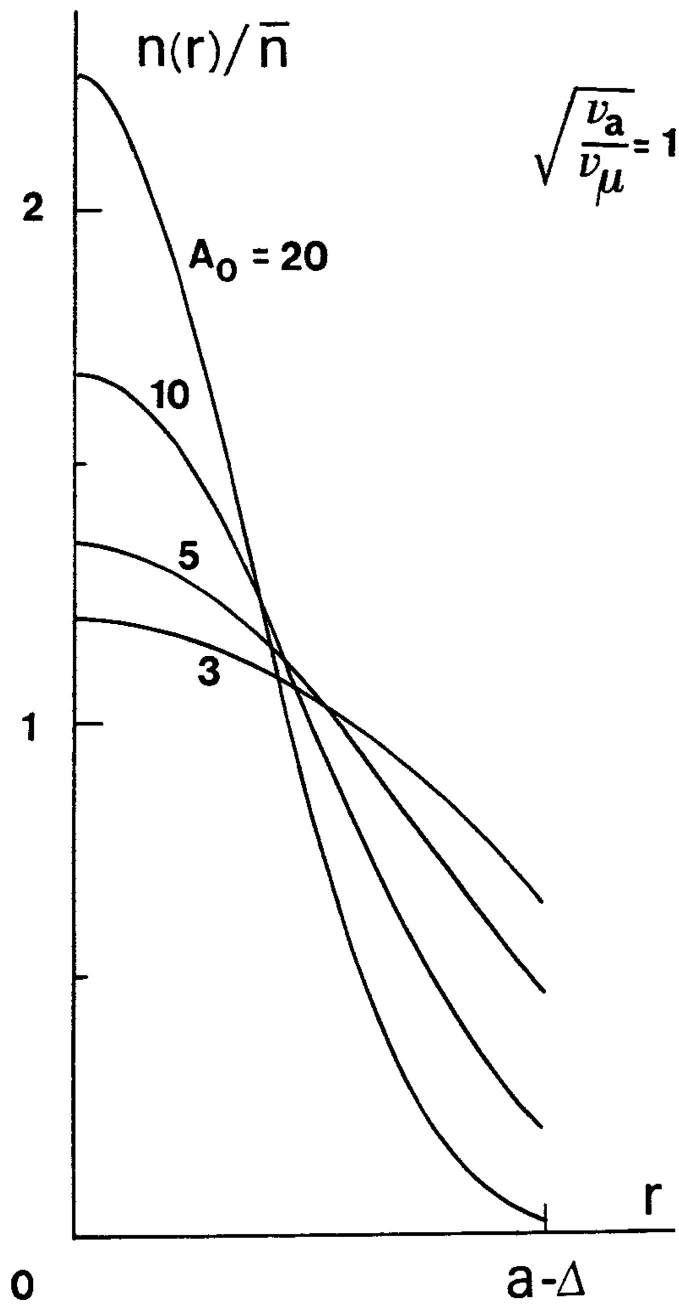
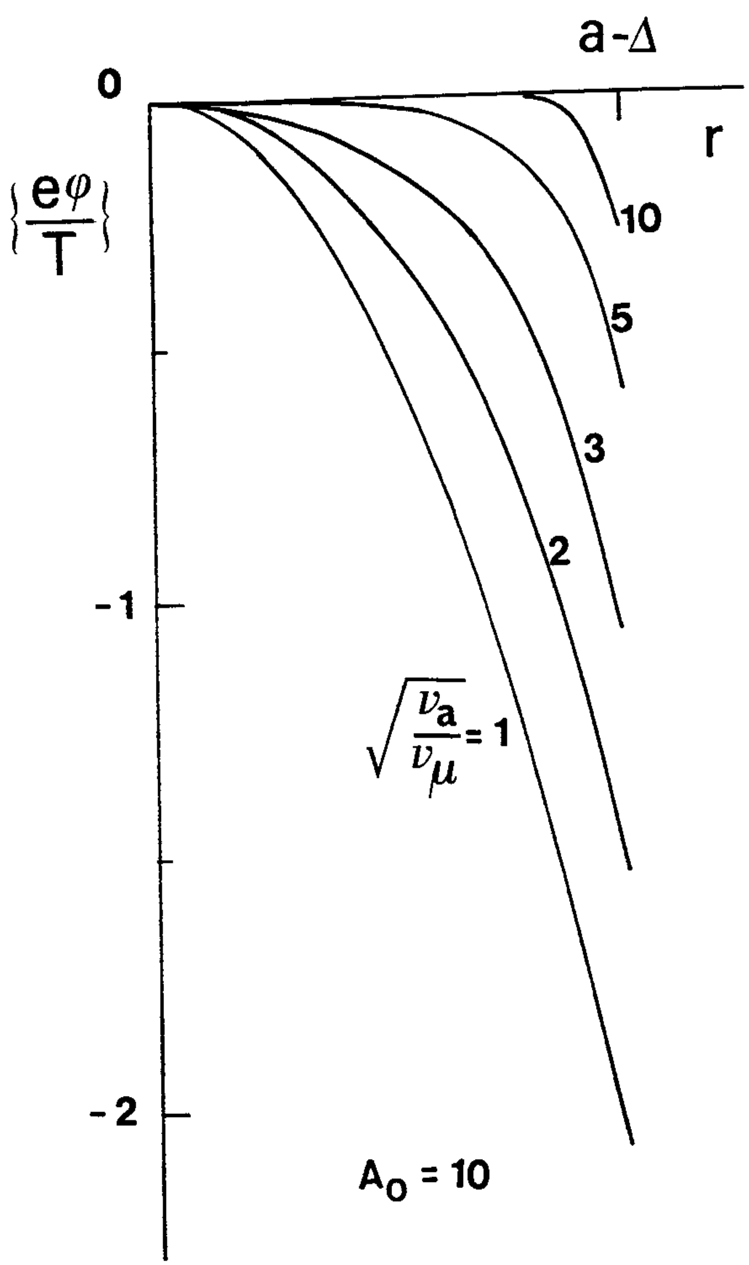


Fig.1(c)


a-Δ
0
r
$\left\{\frac{e\varphi}{T}\right\}$
10
5
3
2
-1
$\sqrt{\frac{\nu_a}{\nu_\mu}} = 1$
-2
$A_0 = 10$