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K. Urata, T. Hatori and T. Amano

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Current Bubble Formation by Nonlinear Coupling of Resistive Tearing Modes

Kazuhiro Urata,
Tadatsugu Hatori, and Tsuneo Amano

Abstract

The current bubble means a helical duct with low current density invading the inner high current region of current carrying toroidal plasmas. It is a non-turbulent analogue of the MHD clump. The numerical simulations show that it appears right after the absorbtion of a small magnetic island into the chaotic layer in the separatrix region of a large magnetic island.

Key words ;
tearing mode, mode coupling, chaotic magnetic field, and clump.

1 Introduction

We know that the current disruption is the most dangerous phenomenon in tokamaks. The breakdown of the magnetic surface caused by magnetic perturbations with different helicities is thought to be a central ingredient of the mechanism for the disruption. In this letter, we study numerically the mode-coupling process of resistive tearing modes with different helicities based on a reduced set of magnetohydrodynamics (MHD) in the tokamak ordering[1,2,3,4]. Dominant two tearing modes are $m = 2/n = 1$ (2/1 in shorthand notaion) and 3/2. Here m and n represent the poloidal and the toroidal mode numbers, respectively. B.V.Waddell et al. [5] and B.Carreras et al. [6] studied the nonlinear destabilization of the 3/2 mode and the nonlinear coupling of the 3/2 mode and the 2/1 mode resulting in the chaotic magnetic field. J.W.Eastwood and W.Arter[7] demonstrated that the final phase of rapid growth observed in the previous simulations can be accounted for a nonphysical feedback mechanism.

The motivation of the present study is to understand the fundamental mechanisms of selfconsistent chaotic evolution of MHD system toward the turbulent state. We find out an unexpected new phenomenon which has been overlooked in Refs. [5,6], the formation of a current bubble, which occurs right after the disappearance of the 3/2 magnetic islands. The current bubble is a duct with nearly zero toroidal current density J_ζ , which invades the central region of the high J_ζ around the magnetic axis in the poloidal cross-section. It deforms into an irregular pattern after a finite time and is at last followed by a turbulent state. At the stage of bubble formation, there is no numerical problem of finite radial mesh size which was pointed out in Ref.[7]. After describing in Sec.2 the observation of the current bubble, its significances are discussed in Sec.3 .

2 Bubble formation

The basic equations of the present simulation are the reduced set of equations[1,2,3,4];

$$\frac{\partial}{\partial t}U = [\phi, U] - [\psi, J_\zeta] - \frac{\partial J_\zeta}{\partial \zeta} + N\nabla_\perp^2 U \tag{1}$$

$$\frac{\partial}{\partial t}\psi = [\phi, \psi] - \frac{\partial}{\partial \zeta}\phi + \frac{1}{S}(\eta J_\zeta + E_\zeta^w) \tag{2}$$

where $[\alpha, \beta] \equiv \vec{\zeta} \cdot \nabla\alpha \times \nabla\beta$, magnetic field $\mathbf{B} = -\epsilon\nabla_\perp\psi \times \vec{\zeta} + \vec{\zeta}$ (ϵ is the inverse aspect ratio), velocity $\mathbf{v} = \nabla_\perp\phi \times \vec{\zeta}$, toroidal current density $J_\zeta = \nabla_\perp^2\psi$, vorticity $U = \nabla_\perp^2\phi$, (The functions ψ and ϕ are the poloidal flux and velocity stream function, and $\vec{\zeta}$ is the unit vector of toroida direction). The time is now expressed in units of $\tau_A = a/v_A$ (a is the minor radius, v_A the Alfvén velocity), so all results presented here will be expressed in the Alfvén times. Conventional notations $S = \tau_r/\tau_A$ and η are the ratio of the resistive skin time to the Alfvén time and the resistivity, respectively. N is the ratio of the Alfvén time τ_A to the viscous diffusion time a^2/ν(ν is the kinematic viscosity).

The resistivity η is taken to be constant in time, and its radial dependence is obtained by assuming a constant loop voltage in the equiliblium and not allowing the magnetic flux to evolve in the absence of tearing mode activity[6]. The quantities ψ and ϕ are expanded in Fourier series in the toroidal and poloidal angles to give

$$\psi = \sum_{m,n} \psi_{mn}(r) \cos(m\theta + n\zeta) \tag{3}$$

and

$$\phi = \sum_{m,n} \phi_{mn}(r) \sin(m\theta + n\zeta) \tag{4}$$

and a finite difference scheme is employed in the radial variable. The results presented here use 29 Fourier components (maximum poloidal number is 14 and maximum toroidal number is 7) and radial grid has 200 points.

The q profiles are fixed by the formulae;

$$q(r) = 1.4(1 + (\frac{r}{0.6})^8)^{\frac{1}{4}}. \tag{5}$$

As initial perturbations, we set in eigen functions of 2/1 and 3/2 modes decided by a linear shooting code. The value of S ranges up to 1.0×10^6. At any value of S, current bubbles are formed. We describe here the case where S is 2×10^5. The viscosity is totally neglected; $N = 0$, in the subsections 2.1 through 2.3 .

2.1 Linear stage

Firstly, 2/1 and 3/2 modes grow exponentially. On account of the difference in the growth rate, the 2/1 mode dominates at the end of the linear stage.

2.2 Stage of developed 2/1 island

The 2/1 mode comes to grow algebraically, and the 3/2 magnetic islands become visible. The Poincaré intersections of magnetic field lines with a poloidal plane ($\zeta = 0$) show that a pair of large 2/1 magnetic islands have appeared, meanwhile nested regular magnetic surfaces remain around the magnetic axis, although their shapes of concentric circles turn into ellipses gradually. Cross-sections of those regular magnetic surfaces with the $\zeta = 0$ plane always agree with contour lines of equal J_ζ , that is $\nabla_{\parallel} J_\zeta = 0$.

At the end of this stage, there is a high 'mountain' of J_ζ in the central elliptic region and two moderate 'hills' of J_ζ lie in the 2/1 island as shown in Fig.1 . Between the 'mountain' and 'hills', there exist the 'valley' region where $J_\zeta \simeq 0$. At the edge of the 'mountain', the 3/2 islands become considerably large, then a layer of chaotic magnetic field begins to develop along the 'valley'.

2.3 Stage of bubble formation

The 3/2 mode is abruptly destabilized by nonlinear effects. The 3/2 islands are shown in Fig.2(b). It should be noted that the stream function becomes large in the area where O-points of 2/1 and 3/2 modes face each other. Fig.5(a) shows that there appears a strong inflow toward the magnetic axis. But the flow is blocked at first because regular magnetic surfaces survive in the central region as shown in Fig.3(b). The 3/2 island plays a role of a 'bank' against the flow.

Once the 3/2 magnetic islands are absorbed into the chaotic layer and disappear, a dramatic change in J_ζ takes place. The gradient of J_ζ in the direction of magnetic field becomes finite ($\nabla_{\parallel} J_\zeta \neq 0$), and induces a finite inertia term dU/dt , because the equation of motion (1) may be rewritten as $dU/dt = -\nabla_{\parallel} J_\zeta + N\nabla_{\perp}^2$. So the low J_ζ region in the chaotic layer invades the central ellipse with high J_ζ . If we look at the point S in Fig.5(b), there exists a stagnation point of the flow, which is a remnant of the O-point of 3/2 mode. The value of J_ζ remains high around the stagnation point as shown at a point S in Fig.4(a). A series of 7/4 magnetic islands which lie in the ellipse, limit the size of the bubble in the radial direction as shown in Fig.4(b) . The time scale of the formation, the persistence and the decay of the current bubble is of the order of $10\tau_A$. After the bubble has deformed, a turbulent stage sets in, where J_ζ is roughly uniform.

The current bubble has the same helical structure as the magnetic island of the 3/2 mode. It connects with itself after the three rotations in the toroidal direction and the two rotations in the poloidal direction, because the bubble appears when the 3/2 mode is strongly influenced by the inflow of the 2/1 mode around the O-point of that mode. Thus the most regular cross-section of the current bubble is formed around the point A in Fig.4(a) where the O-points of the 2/1 mode and the 3/2 mode face each other in a shortest distance. If we start from the point A in Fig.4(a) and go along the current bubble in the toroidal direction, the duct structure of the bubble gradually changes to the gap structure. After just one rotation in the toroidal direction, we arrive at the point B in the same figure. If we go in the opposite direction, we arrive at the point C. At the points B and C, where the duct of the bubble and the 2/1 mode are separete in 60 degrees of the poloidal angle, the cross-section of the bubble is smaller than the regular one and is deformed. If they are separate in more than 70 degrees of the poloidal angle, no bubble structure can be identified.

At the point D where O-point of 2/1 mode and X-point of 3/2 mode face each other, no current bubble is formed. Comparing Figs.4(a) and (b), we can recognize that the bubble structure exists well inside the chaotic region.

Some results are obtained in the case of finite viscosity N, the value of which ranges upto 0.75×10^5. As N increases, two characteristic features get remarkable. First, the growth rates of principal modes tend to be smaller, so that the time sequence of bubble formation and the onset of turbulence is delayed. Secondly, the fine structure of J_ζ is averaged out to be hard to be seen even in the turbulent state. It is confirmed that finite viscosity does not prevent the bubble from forming at least when N is less than 0.75×10^5.

3 Significance of the Current Bubble

We show in this section a theory in which the current bubble is involved. D.J.Tetreault[9, 10] developed a MHD clump theory in current-carrying plasma. He applied the original clump theory, which had been first developed to study the Vlasov turbulence, to the MHD turbulence and predicted clumps, that are the correlated structures of magnetic field lines. The clump is caused by mixing of the mean current density and decays as a result of magnetic stochasticity.

The present current bubble is created right after the absorption of 3/2− islands into the chaotic magnetic layer. Note that the rotation of the Lorentz force is the gradient of the toroidal current density along the field line ;

$$\nabla \times (J \times B) = B \cdot \nabla J_\zeta \equiv \nabla_{\|} J_\zeta. \tag{6}$$

In the magnetic island, the quasi-equiliblium state with $\nabla_{\|} J_\zeta \simeq 0$ is retained. After the onset of magnetic chaos, however, it is replaced by a state with finite value of $\nabla_{\|} J_\zeta$, at least at a transient stage when the magnetic field is stochastic but J_ζ remains coherent. The flow associated with the 3/2− island is enhanced by the Lorentz force, according to the vorticity equation;

$$\frac{dU}{dt} = -\nabla_{\|} J_\zeta + N \nabla_\perp^2 U. \tag{7}$$

Thus both the current bubble and the MHD clump are created by the strong flows which mix the inhomogeneous mean current density. The difference is that the bubble results from a strong inward coherent flow, not from the turbulent mixing flow. Except for the difference in flow patterns, the current bubble is the first numerical observation of the MHD clump as predicted by D.J.Tetreault.

Formation of a current bubble means the local interchange of the high and low current density regions, therefore it gives one of the mechanisms for the rapid radial transport of the toroidal current density. It is well known that the force-free equilibrium states are frequently realized in the laboratory and astrophysical plasmas through the rapid magnetic relaxation process. The associated violent transport of the current density is explicable by the current bubble.

4 Conclusions and discussion

The current bubble formation in the coupling process of resistive tearing modes is found by numerical simulations based on the reduced set of MHD equations. It is formed right after the disappearance of 3/2 island. This indicates that it is a structure induced by the magnetic chaos. It persists within a time interval of the order of $10\tau_A$. It is to be noted there exists a spatical structure which we call the current bubble even in the chaotic magnetic field. Current bubbles are formed anywhere O-points of different modes face each other.

Investigation of decay process of the bubble will be published elsewhere, in which the aliasing errors in radial mesh as pointed by J.W.Eastwood and W.Arter[7] must be avoided.

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Figure captions

Fig.1

The 3-D representation of the toroidal current density J_ζ on a poloidal plane; $\zeta = 0$ at $t = 640\tau_A$.

Fig.2

(a) Contour lines of the equal toroidal current density J_ζ on a poloidal plane; $\zeta = 0$ at $t = 660\tau_A$. The central high J_ζ region has deformed to be elliptic.

(b) Poincaré plots of some magnetic field lines on a poloidal plane; $\zeta = 0$ at the same time as (a), initial points of which are marked by triangles. The 3/2 magnetic islands appear in addition to the large 2/1 islands.

The value of S is 2.0×10^5 .

Fig.3

A state immediately before the onset of the current bubble formation. The value of S is 2.0×10^5.

(a) Contour lines of the equal toroidal current density J_ζ on a poloidal plane; $\zeta = 0$ at $t = 680\tau_A$.

(b) Poincaré plots of some magnetic field lines on a poloidal plane; $\zeta = 0$ at the same time as (a), initial points of which are marked by triangles. The bubble formation is prevented by the 3/2 island.

Fig.4

The formation of the current bubble. The value of S is 2.0×10^5.

(a) Contour lines of the equal toroidal current density J_ζ on a poloidal plane; $\zeta = 0$ at $t = 720\tau_A$. The regular cross-section is shown around the point A and the deformed ones are at the points B and C. The point S is the stagnation point of the flow (see Fig.5(b)), where J_ζ is kept high.

(b) Poincaré plots of some magnetic field lines on a poloidal plane; $\zeta = 0$ at the same time as (a), initial points of which are marked by triangles.

Fig.5

The flow velocity on a plane; $\zeta = 0$. The length of the arrows represents the intensity of the flow. The value of S is 2.0×10^5.

(a) At $t = 680\tau_A$, the flow is brocked by the 3/2 magnetic island. (see Fig.3)

(b) At $t = 720\tau_A$, the strong flow to the central elliptic region takes place. The point S is the stagnation point of flow. (see Fig.4)

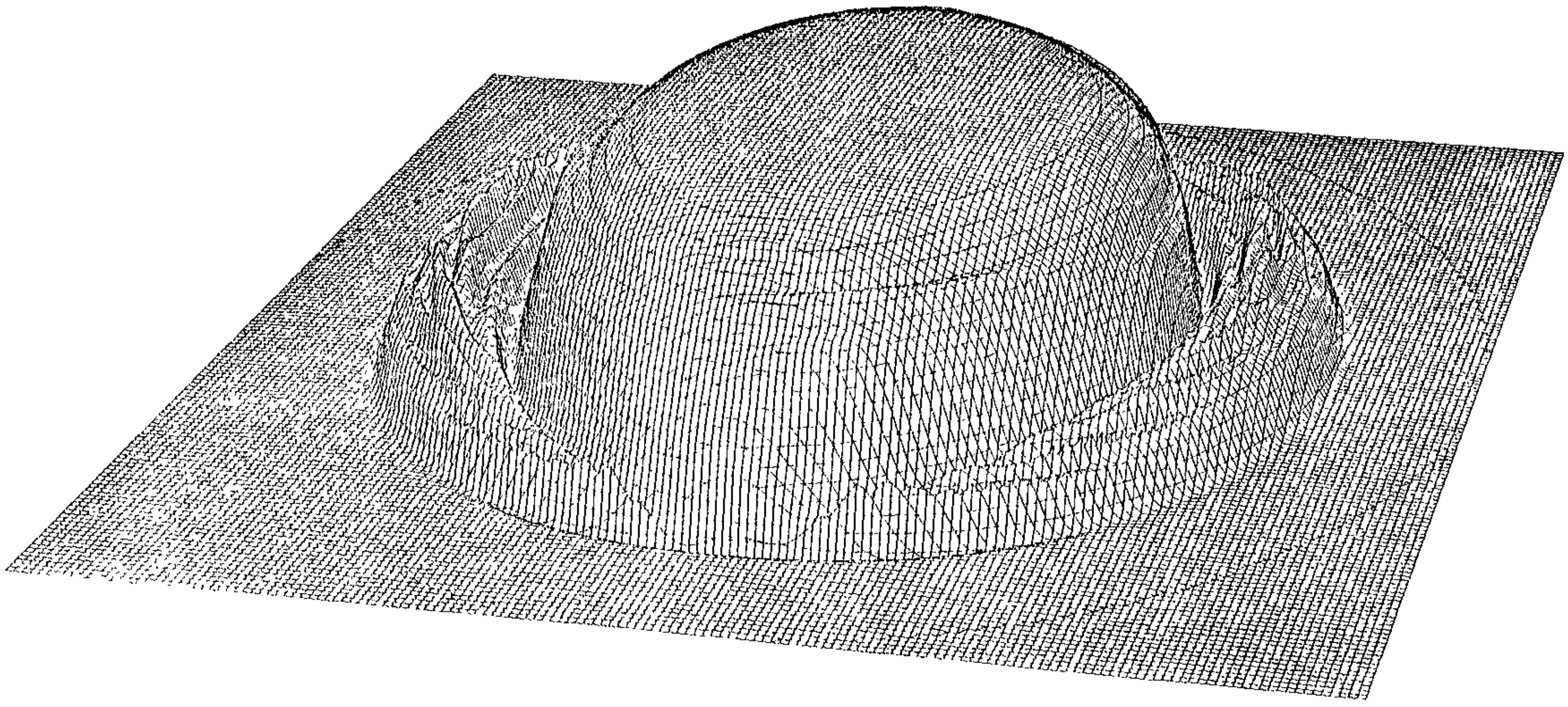


Fig.1

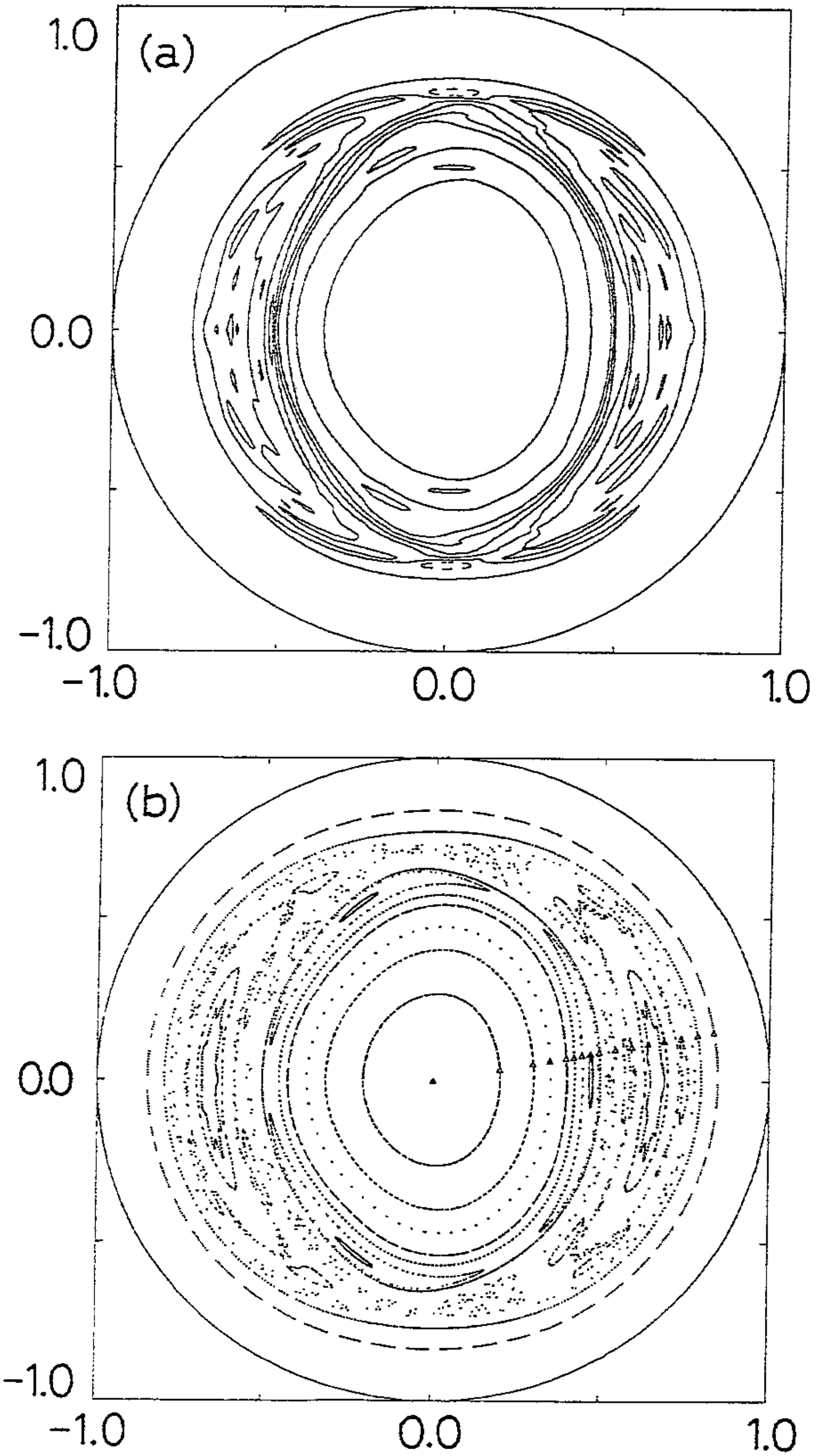


Fig.2

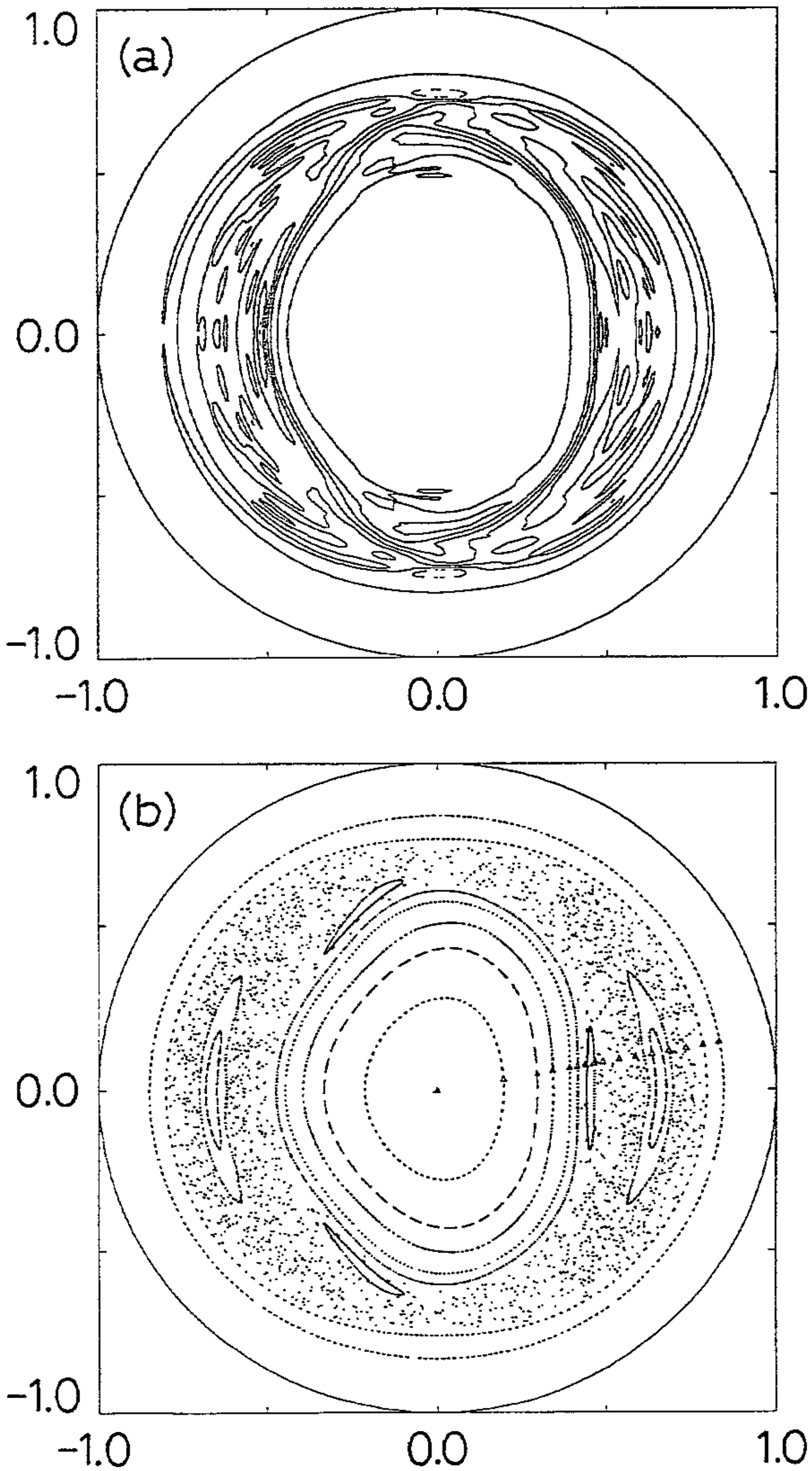


Fig.3

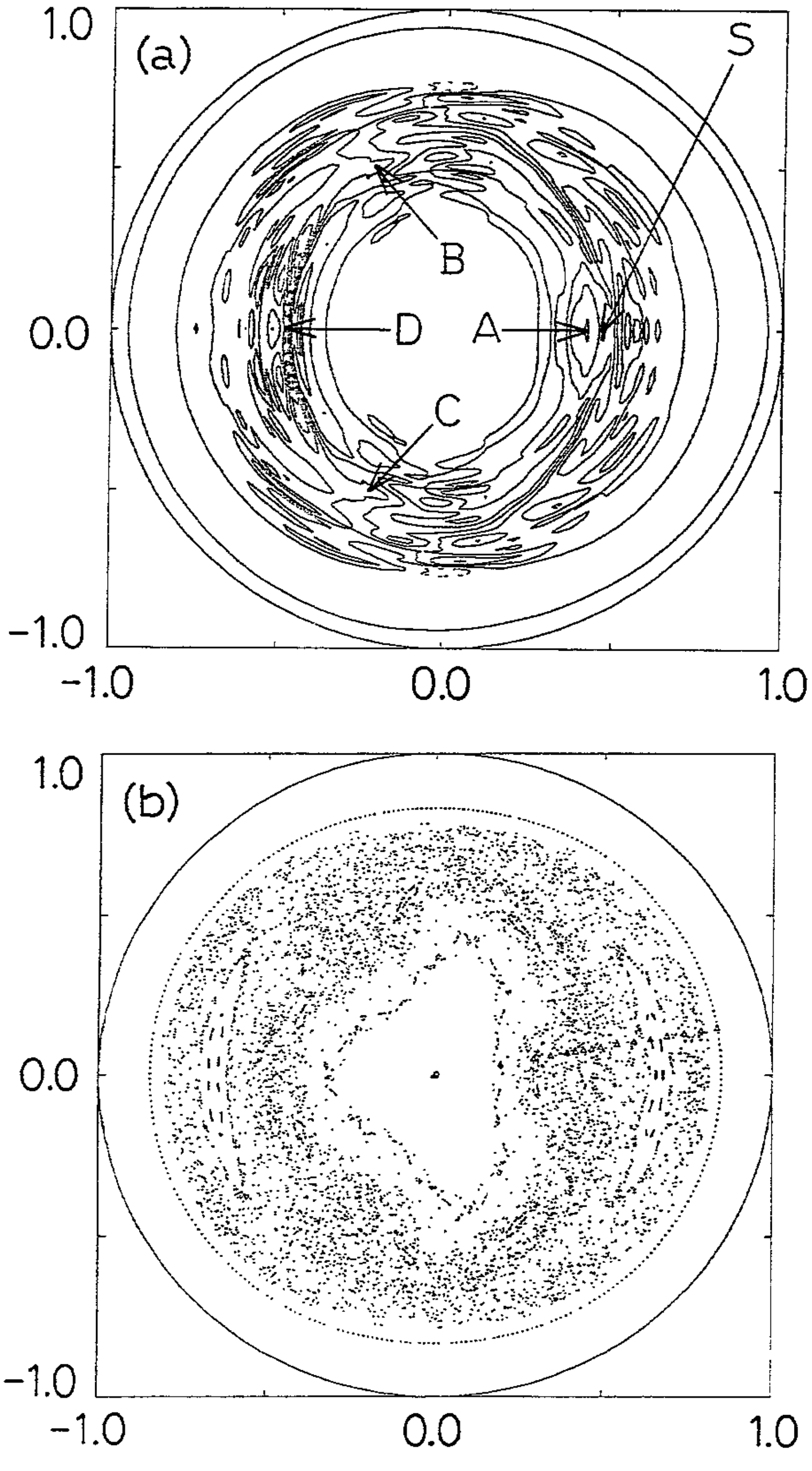
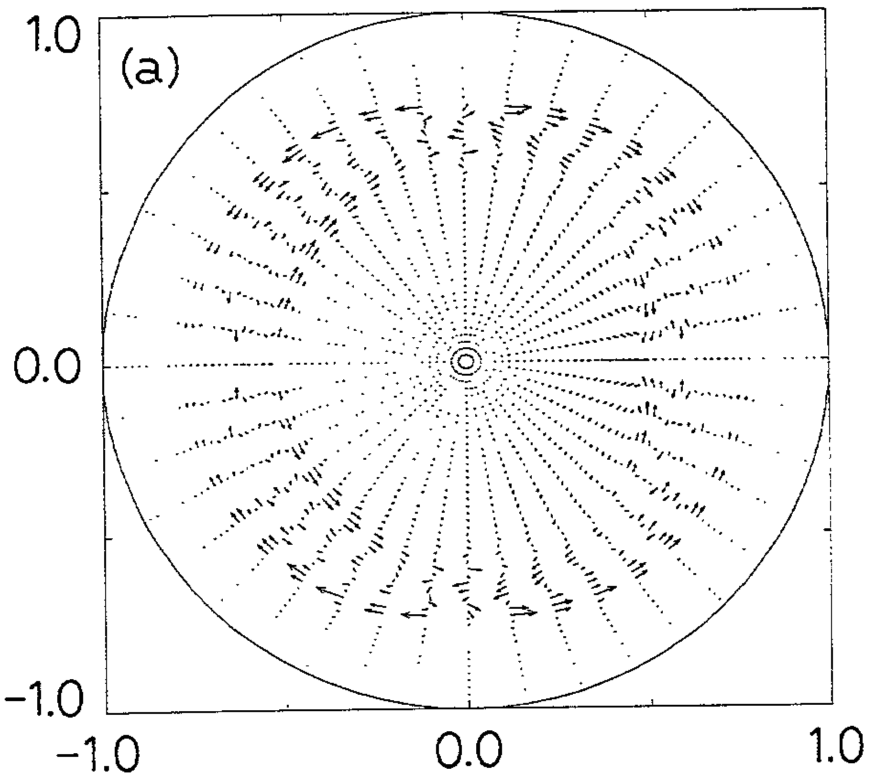


Fig.4



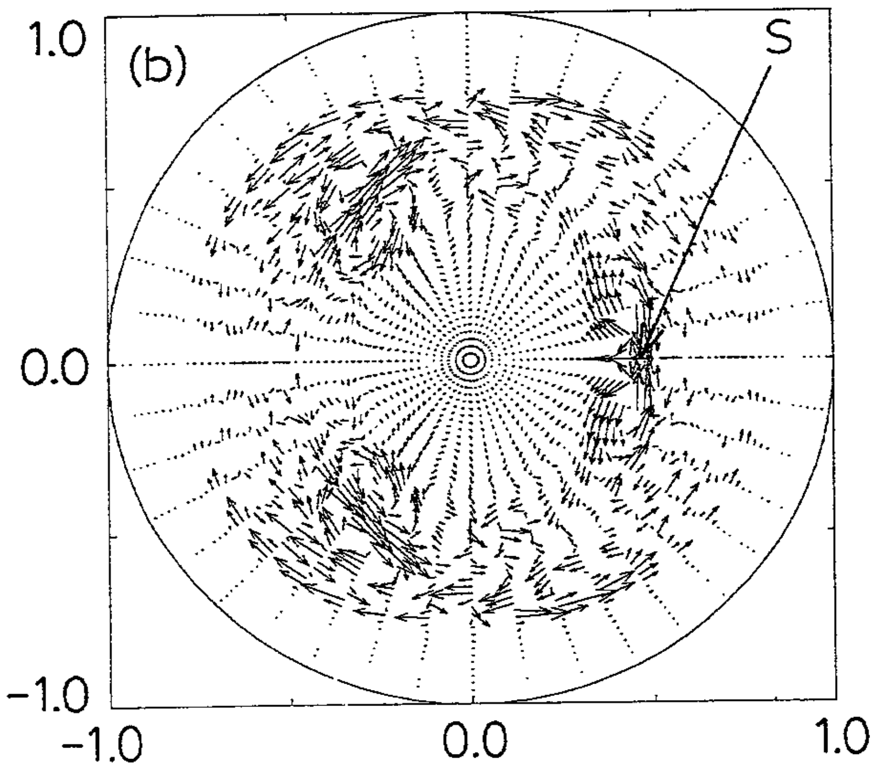


Fig.5