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Fokker-Planck Equation in the presence of Anomalous Diffusion

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Abstract

Spatial and temporal evolution of the distribution function is theoretically discussed in the presence of the rf waves and microscopic fluctuations. Using the quasilinear formulation, Fokker-Planck equation is derived in terms of the parallel and perpendicular energies and the distance across the magnetic surface. The energy dependence of the anomalous spatial diffusion term is studied for the case of the drift wave fluctuations. Cases for the intense ICRF heating is also discussed.

Key words: Fokker-Planck equation, Anomalous Diffusion, RF waves, Drift waves,

§1 Introduction

The high power heating experiments in large tokamaks have revealed the important role of the loss of energetic particles. Nonthermal energetic particles often appear in confinement experiments. The generation and confinement of the fusion products such as the tritium burn-up experiments[1-2], the intense ICRF (ion cycrotron range of frequencies) heating experiments[3], and the rf current-drive in low density plasma[4] are the typical examples. In these situations, the relative magnitude of the loss rate of the energetic particles to the thermal particles is the key parameter. If there are mechanisms to cause the selective loss of high energy particles, then the heating/current-drive efficiency may degrade as the increments of the heating density and plasma parameters. On the contrary, if the mechanisms which cause the presently-observed anomalous loss do not affect the transport of the high energy particles, then the future improvement of the plasma parameter would enhance the heating/current-drive efficiency.

The transport analysis on the rf-heated plasma has been performed intensively.[5-6] In studying the transport in the presence of nonthermal particles, it is usually assumed that the nonthermal component is represented by the fluid with higher temperature and smaller density, and the spatial transport of this hot component is independently treated from the thermal component. This multi-fluid approach seems to succeed in simulating the present high power experiments[5-6], where a simple model of diffusion, such that the spatial loss is neglected for the hot component, is taken. Nevertheless, more firm basis for the choice of the transport coefficient is necessary to have a better understanding of the heated plasma. In some cases, it is reported that the existence of a small but finite value of the diffusion of hot component simulates the experiments better.

The temporal evolution of the distribution function has been studied in many literatures[5-12] both theoretically and numerically. The energy dependence of the velocity diffusion is known to be the key parameter which dictates the energy partition of the absorbed energy and the content of the nonthermal particles.[8] The collisional drag term becomes small for high energy particles, so that a small but finite value of the spatial loss would affect the asymptotic form of the distribution function, thus changing the heating and current-drive efficiency.[13-17] In studying the competition between the heating, collisional drag and spatial diffusion, the energy dependence of the spatial diffusion term must be determined based on the fundamental basis. The flux which is driven by the rf wave has been discussed in literatures.[18-21] The formulation which includes both heating and anomalous loss is necessary.

The purpose of this article is to develop the quasilinear theory for an evolution of the distribution function in the presence of the rf heating/current-drive and the microscopic fluctuations. The energy dependence of the spatial diffusion term is discussed for the case of the drift wave fluctuations.

In principle, the fluctuation level is influenced by the existence of the rf wave or nonthermal components. In this article, we assume that the total energy content of the nonthermal part is small and that the fluctuation spectrum is given to be fixed. Averaging over the gyrophase, we obtain the three-dimensional Fokker-Planck equation; variables are the perpendicular and parallel energies and the distance across the magnetic surface. In usual applications, the nonthermal component often appears either in the parallel direction (such as the current-drive) or in the perpendicular direction (such as cyclotron heating). In these cases, the equation can be reduced to two-dimensional, i.e., one spatial variable and one velocity variable, allowing direct numerical computations.

§2 Quasilinear Equation for Distribution Function

2.1 Model

The z-axis is taken in the direction of the strong magnetic field, and the (y,z) plane coinsides with the magnetic surface. The x-axis is taken in the direction of the spatial gradient. In this article, we neglect the particle drift and trapping effects owing to the megnetic field inhomogeneity.

The distribution function f is assumed to be expressed in terms of the sum of slowly varying part, f_0, oscillating part with rf frequency, $\tilde{f}_{rf}$, and the fluctuation part, $\tilde{f}_d$, as

$$f = f_0 + \tilde{f}_{rf} + \tilde{f}_d, \qquad (1)$$

where we employ the time scale separation between them. The slowly varying part f_0 (the equribrium distribution function) is expressed as $f_0(W_\perp, W_\parallel, p)$ in terms of the particle energies $W_\perp$ and $W_\parallel$ (suffix $\perp$ and $\parallel$ represent parpendicular and parallel, respectively) and the normalized canonical momentum $p = x + v_y/\Omega$. x is the coordinate of the guiding center, and Ω is the cyclotron frequency, eB/m (e and m are the particle charge and mass, respectively). The phase of the gyromotion is given as $\theta = \alpha - \Omega t$, and the distribution function f_0 does not depend on α.

The temporal evolution of the slowly varying part of f is derived by using the quasilinear theory[22]. Introducing the collision operator C[f], we have

$$\frac{\partial f_0}{\partial t} = - \frac{e}{m} \langle (\tilde{\vec{E}}^* + \vec{v}\times\tilde{\vec{B}}^*)\nabla_v(\tilde{f}_{rf}+\tilde{f}_d)\rangle + C(f_0) \qquad (2)$$

where $\vec{E}$ and $\vec{B}$ are the rf and fluctuation components of the field, the asterix indicates the complex conjugate, and ‹› stands for the average over the oscillation period. The oscillating field is expressed as

$$(\tilde{\vec{E}},\tilde{\vec{B}}) = (\vec{E},\vec{B})_{rf} + (\vec{E},\vec{B})_d. \qquad (3)$$

We assume that the rf wave and microscopic fluctuations do not directly interact to each other, and that they are subject to change associated with the evolution of the equilibrium distribution function f_0. Standing on this assumption, we take the averages of the rf field part and fluctuation part independently and obtain

$$\partial f_0/\partial t = Q + L + C, \qquad (4)$$

$$Q = -\frac{e}{m} \langle (\vec{E}^* + \vec{v}\times\vec{B}^*)_{rf}\nabla_v\tilde{f}_{rf}\rangle \qquad (5)$$

$$L = -\frac{e}{m} \langle (\vec{E}^* + \vec{v}\times\vec{B}^*)_d\nabla_v\tilde{f}_d\rangle \qquad (6)$$

where Q and L are the phase space diffusions due to rf wave and microscopic fluctuations, respectively.

2.2 Quasi-linear Operator

To obtain the distribution functions f_{rf} and f_d, we assume that the collision is neglected compared to the oscillation frequency ω. The wave vector is denoted by $\vec{k}$. They are claculated from the Vlasov equation as,

$$\frac{\partial \tilde{f}}{\partial t} + \vec{v}\cdot\nabla\tilde{f} + \frac{e}{m}(\vec{v}\times\vec{B})\nabla_v\tilde{f} = -\frac{e}{m}(\tilde{\vec{E}}+\vec{v}\times\tilde{\vec{B}})\nabla_v f_0 \tag{7}$$

Noting the relation

$$\nabla_v f_0 = [m\vec{v}\ \partial/\partial W_\perp + mv_{\parallel}(\partial/\partial W_{\parallel} - \partial/\partial W_\perp) + \hat{y}\partial/\partial p] f_0 \tag{8}$$

we have

$$(\tilde{\vec{E}}+\vec{v}'\times\tilde{\vec{B}})\nabla_v f_0 = (\vec{E}\cdot\vec{v}')L_1 f_0 + (1-\vec{k}\cdot\vec{v}'/\omega)L_2 f_0 \tag{9}$$

where L_1 and L_2 are defined as

$$L_1 \equiv m\partial/\partial W_\perp' + mk_z v_z'(\partial/\partial W_\parallel' - \partial/\partial W_\perp')/\omega + (k_y/\omega)\partial/\partial p' \tag{10-1}$$

$$L_2 \equiv mE_z v_z'(\partial/\partial W_\parallel' - \partial/\partial W_\perp')/\omega + E_y\partial/\partial p' \tag{10-2}$$

The argument $\vec{v}'$ is the particle velocity at $t'=t-\tau$. Substituting Eq.(9) into Eq.(7), we have

$$\tilde{f} = \frac{e}{m}\Sigma_{\ell,j}J_j(\zeta)a_\ell \int d\tau e^{(k_{/\!/}v'_{/\!/}-\omega+\ell\Omega)\tau+i(\ell-j)(\psi-\theta)}L_1 f_0$$

$$+ \frac{e}{m}\frac{L_2}{\omega}f_0 \tag{11}$$

In Eq.(11), a_ℓ is defined as

$$a_\ell = a_+J_{\ell+1}(\zeta) + a_-J_{\ell-1}(\zeta) + a_zJ_\ell(\zeta) \tag{12-1}$$

$$a_+ = \frac{v_\perp}{2}(E_x-iE_y)\exp(+i\psi), \tag{12-2}$$

$$a_- = \frac{v_\perp}{2}(E_x+iE_y)\exp(-i\psi), \tag{12-3}$$

$$a_z = v_zE_z \tag{12-4}$$

where ψ is defined as $k_x=k\ \cos\psi$, $\zeta=k\ v\ /\Omega$, and J_ℓ is the ℓ-th order Bessel function of the first kind. Using Eqs.(9) and (11), the quasilinear term, $-(e/m)\langle(\vec{E}^*+\vec{v}\times\vec{B}^*)\nabla_v f\rangle$, is calculated as,

$$\frac{\partial f_0}{\partial t} =(\frac{e}{m})^2\ \Sigma_\ell L_1 a_\ell^* a_\ell \mathrm{Im}(\frac{1}{k_{/\!/}v_{/\!/}-\omega+\ell\Omega})L_1 f_0 \tag{13}$$

Note that the terms containing L_1L_2, L_2L_1 and L_2L_2 do not appear in Eq.(13). Since the term which contains L_2 in Eq.(11) is a nonresonant part, this does not contribute to the modification of f_0.

The qusilinear term consists of the derivatives with respect to the velocity and space coordinates. In the absence of the inhomogeneity, Eq.(13) reduces to the usual representation for the quasilinear velocity space diffusion.

§3 Reduced Set of Equations

3.1 Rf Heating Term

The quasilinear operator Eq.(13) is the extension of the usual velocity diffusion operator[8,22] which has been derived in the absence of the spatial diffusion. Here the contributions from the spacial diffusion and from the velocity diffusion are compared for the case of $\omega \simeq \ell\Omega$ (ℓ=1,2,3...). The case for the low frequency fluctuations, the frequency range of which is the drift frequency ω_*, is discussed in the next subsection. In this high frequency range, the typical heating term in Eq.(13) is given as,

$$\left.\frac{\partial f_0}{\partial t}\right)_h \simeq e^2\frac{\partial}{\partial W}a_-^* a_- J_{\ell-1}^2 \,\mathrm{Im}\left(\frac{1}{k_\parallel v_\parallel-\omega+\ell\Omega}\right)\frac{\partial}{\partial W}f_0 , \tag{14}$$

and the spatial diffusion term is given as,

$$\left.\frac{\partial f_0}{\partial t}\right)_{dif} =(-)\frac{e^2}{m}\frac{k_y^2}{\ell^2\Omega^4}\frac{\partial}{\partial x}a_-^* a_- J_{\ell-1}^2 \,\mathrm{Im}\left(\frac{1}{k_\parallel v_\parallel-\omega+\ell\Omega}\right)\frac{\partial}{\partial x}f_0 . \tag{15}$$

We compare the contributions, taking the ratio of Eq.(15) to Eq.(14), i.e.,

$$\eta \equiv (\partial f_0/\partial t)_{dif}/(\partial f_0/\partial t)_h , \tag{16}$$

which is evaluated as

$$\eta \simeq \frac{k_y^2 f_0}{m^2 a^2 \ell^2 \Omega^4} \left(\frac{\partial^2 f_0}{\partial W^2}\right)^{-1} \tag{17}$$

where we introduce a typical length of a spatial inhomogeneity a and write $\partial^2 f_0/\partial x^2 \simeq f_0/a^2$.

When the heating power is small (and/or in the initial phase of the heating), the tail distribution is not generated. In this case the velocity derivative can be estimated as

$$\partial f_0/\partial W \simeq -f_0/T, \tag{18}$$

where T is the plasma temperature. The ratio η is obtained as

$$\eta \simeq k_y^2 \rho^4/a^2 \ell^2 \tag{19}$$

where ρ is the gyro radius. We see that this value is much smaller than unity.

When the heating density is high, and the heating time is longer than the slowing down time by collisions, τ_s, the tail distribution developes. In the stationary state in which the heating and the slowing down balance with each other, the energy derivative satisfies the relation[8)]

$$\partial f_0/\partial W \simeq -f_0/(1+\xi)T. \tag{20}$$

for the fundamental heating, $\ell=1$. In Ref.(8), the parameter ξ (the normalized heating density) is defined as

$$\xi \equiv P_{heat}\tau_s/nT, \tag{21}$$

P_{heat} is the heating density per unit volume, and n is the number density of the species which interacts with the launched rf wave. In this case, we have

$$\eta \simeq \{(1+\xi)k_y\rho^2/a\}^2. \tag{22}$$

This term can be of the order of unity if the value of ξ reaches the value of $a/k_y\rho^2$. Considering the heating level of present experiments, $\xi \sim 10$, we see that $\eta \ll 1$. We therefore conclude that the spatial loss due to the rf heating wave itself gives a small correction, and that it can be neglected at least in the usual heating level[21]. In the 0th order argument the operator Q reduces to

$$Q[f_0] = e^2\Sigma_\ell[\partial/\partial W_\perp + k_z v_z(\partial/\partial W_\parallel - \partial/\partial W_\perp)/\omega]\delta(k_\parallel v_\parallel - \ell\omega)a_\ell^* a_\ell$$

$$[\partial/\partial W_\perp + k_z v_z(\partial/\partial W_\parallel - \partial/\partial W_\perp)/\omega]f_0 \tag{23}$$

and contains only the derivative with respect to the velocity.

3.2 Diffusion due to Microscopic Fluctuations

The contribution of the real space diffusion to the modification of f_0 becomes large if the frequency is much lower than the cyclotron frequency. We treat this case separately. We derive the simplified form of the operator L in the case of the electrostatic fluctuations.

We keep the $\ell=0$ component in Eq.(13). The fluctuating electric field is expressed in terms of the electrostatic potential as $\vec{E} = -\nabla\tilde{\phi}$. The temporal evolution due to the fluctuations is given as

$$\partial f_0/\partial t = \Sigma \pi e^2 J_0^2 |\phi|^2 L_0 \delta(k_\parallel v_\parallel - \omega) \hat{L}_0 f_0, \tag{24}$$

where the summation is taken over the wave spectrum and the operators L_0 and $\hat{L}_0$ are defined as

$$L_0 = k_\parallel v_\parallel \partial/\partial W_\parallel + (k_y/m\Omega)\partial/\partial x. \tag{25-1}$$

$$\hat{L}_0 = \omega\, \partial/\partial W_\parallel + (k_y/m\Omega)\partial/\partial x. \tag{25-2}$$

In deriving Eq.(24), we assume that the real frequency of the fluctuating mode, ω_r, is large compared with the imaginary part, ω_i (the typical value of the growth rate). We only keep the resonant contributions in the framework of the quasilinear theory. If the equilibrium distribution is given by the maxwellian distribution, equation (24) reduces to the usual representation for the fluctuation driven diffusion term[23].

We derive a reduced form of Eq.(24) by imposing assumptions for the mode spectrum. We assume that the spectrum ϕ can be separated as

$$\tilde{\phi}(k_\perp, k_{/\!/}) = \phi\bar{\phi}(k_\perp)\bar{\phi}(k_{/\!/}), \tag{26}$$

where $\bar{\phi}(k_\perp)$ and $\bar{\phi}(k_{/\!/})$ are normalized, i.e., the average of them over the spectrum is unity and $\langle\tilde{\phi}(k_\perp,k_{/\!/})^2\rangle = \phi^2$. Using Eq.(26), Eq.(24) turns out to be

$$L[f_0] = \pi\frac{e^2}{m^2}|\phi|^2\frac{\omega_i}{\omega_r^2}[\langle k_{/\!/}^2\rangle\frac{\partial}{\partial v_{/\!/}}H_0(v_\perp)\frac{\partial}{\partial v_{/\!/}} + \frac{k_0}{\Omega}\langle k_{/\!/}^1\rangle\frac{\partial}{\partial v_{/\!/}}H_1(v_\perp)\frac{\partial}{\partial x}$$

$$+\frac{k_0}{\Omega}\langle k_{/\!/}^1\rangle\frac{\partial}{\partial x}H_1(v_\perp)\frac{\partial}{\partial v_{/\!/}} + \frac{k_0^2}{\Omega^2}\langle k_{/\!/}^0\rangle\frac{\partial}{\partial x}H_2(v_\perp)\frac{\partial}{\partial x}]f_0, \tag{27}$$

where functions $\langle k_{/\!/}^n\rangle$ and H_n are defined as

$$\langle k_{/\!/}^n\rangle = \int d\omega\int dk_{/\!/}\, k^n\bar{\phi}^2(\omega,k_{/\!/})\delta(k_{/\!/}v_{/\!/}-\omega), \tag{28-1}$$

and

$$H_n(v_\perp) = \int dk_\perp k_\perp^{n+1}\bar{\phi}^2(k_\perp)J_0^2(k_0v_\perp/\Omega)k_0^{-n}, \tag{28-2}$$

where k_0 is the typical perpendicular wave number of the mode.

The origin of micrscopic fluctuations are considered to be drift waves, and the wave vector is mainly oriented to the perpendicular direction with respect to the magnetic field.

Note that the first term in Eq.(27) is the diffusion term in velocity space due to low frequency modes, the second and the third terms contribute to the anomalous viscosity and the forth term is the spatial diffusion term. When we apply the result to the case of ion cyclotron heating, the first term is small compared with that of the heating term, Eq.(14). If we neglect the viscosity effect on the change of f_0, we have

$$L[f_0] = \pi\frac{e^2}{m^2}|\phi|^2\frac{\omega_i}{\omega_r^2}\frac{k_0^2}{\Omega^2}\frac{\partial}{\partial x}H_2(v_\perp)\frac{\partial}{\partial x}f_0 . \tag{29}$$

The energy dependence of the operator L is determined by that of the function H_n. The small $k_0 v_\perp/\Omega$ limit and the asymptotic formula for the large $k_0 v_\perp/\Omega$ limit can be obtained. Taking the Taylor series and asymptotic expansion of J_0, we have

$$H_n \to \begin{cases} h_n & (v_\perp \to 0), \\ g_n\Omega/\pi k_0 v_\perp & (v_\perp \to \infty). \end{cases} \tag{30}$$

The numerical coefficients h_n and g_n are evaluated as

$$h_n = \int dk_\perp k_\perp^{n+1} \bar{\phi}^2(k_\perp) k_0^{-n},$$

$$g_n = \int dk_\perp k_\perp^{n} \bar{\phi}^2(k_\perp) k_0^{-n+1}. \tag{31}$$

The simplified form of the diffusion term is derived from Eq.(30). Taking the order estimate of $\omega_r \simeq \omega_*$, we have

$$L[f_0] \simeq \frac{\omega_i}{\kappa^2}\left(\frac{e\phi}{T}\right)^2 \frac{\partial}{\partial x} H_2(v_\perp) \frac{\partial}{\partial x} f_0 \ . \tag{32}$$

or we have

$$L[f_0] \simeq D \frac{\partial}{\partial x} H_2(v_\perp) \frac{\partial}{\partial x} f_0 \ . \tag{33}$$

where the coefficient D is defined as

$$D = \frac{\sqrt{\pi}}{2} \frac{\omega_i}{\kappa^2}\left(\frac{e\phi}{T}\right)^2 \ . \tag{34}$$

In the usual estimation of the fluctuation level, $e\phi/T$ is estimated as κ/k_0, where $1/\kappa$ is the typical scale length of the density gradient, reducuing to $D \approx \omega_i/{k_0}^2$.

If the wave spectrum has a peak at $k_y = k_0$ with the half width of the order of k_0, and the spectrum function $\phi^2(k_\perp)$ is approximated by a Gaussian distribution,the integral H_n is written as

$$H_n(v_\perp)=\int d\psi(\sin\psi)^{n+1}\int(2/\sqrt{\pi})s^{2+n}\exp(-s^2)J_0^2(k_0 v_\perp s/\Omega)ds. \quad (35)$$

Function $H_2(v_\perp)$ is given as

$$H_2(v_\perp) = {}_2F_2(1/2,5/2;\ 1,1;\ -k_0^2 v_\perp^2/\Omega^2) \quad (36)$$

where ${}_2F_2$ is the generalized hypergeometric function. The approximated formula, which satisfies Eq.(30) is given as

$$H_2(v_\perp) \simeq \bar{H}_2(k_0 v_\perp/\Omega) = (1-g_2)\exp(-\beta b) + g_2/\sqrt{1+b} \quad (37\text{-}1)$$

$$b = k_0^2 v_\perp^2/\Omega^2 \quad (37\text{-}2)$$

$$g_2 = 4/3\pi\sqrt{\pi} \quad (37\text{-}3)$$

$$\beta = (5-2g_2)/(4-4g_2). \quad (37\text{-}4)$$

Figure 1 compares H_2 (solid line) with $\bar{H}_2$ (dashed line). If the spectrum is peaked near $k_\perp \simeq 0$ and has a typical half width of the order of k_0, H_2 is approximately given as

$$H_2(v_\perp) \simeq \hat{H}_2(k_0 v_\perp/\Omega) = \partial/\partial b[be^{-b}I_0(b)], \quad (38)$$

where I_0 is the 0-th order modified Bessel function of the first kind.

§4 Summary and Discussion

In this article, we obtain the reduced set of equations which describe the evolution of the distribution function in the presence of rf heating and spatial loss. The level of the microscopic fluctuations is treated as a given parameter, the energy dependences of the heating term Q, loss term L and collision term C are determined.

The reduced set of equations are

$$\partial f_0/\partial t = Q[f_0] + L[f_0] + C[f_0], \tag{39-1}$$

$$Q[f_0] = e^2\Sigma_\ell[\partial/\partial W_\perp + k_z v_z(\partial/\partial W_\parallel - \partial/\partial W_\perp)/\omega]\delta(k_\parallel v_\parallel - \omega)a_\ell^* a_\ell$$

$$[\partial/\partial W_\perp + k_z v_z(\partial/\partial W_\parallel - \partial/\partial W_\perp)/\omega]f_0 \tag{39-2}$$

and

$$L[f_0] \simeq \frac{\partial}{\partial x} D\, H_2(v_\perp)\frac{\partial}{\partial x} f_0 \ , \tag{39-3}$$

with the equation of the wave propagation as,

$$\nabla\times\nabla\times\tilde{E}_{rf} + (\omega/c)^2\tilde{E}_{rf} = i\mu_0(\overleftrightarrow{\sigma}\tilde{E}_{rf} + \tilde{J}_{ant}) \tag{40}$$

where $\tilde{J}_{ant}$ is the current on the rf antenna. The conductivity tensor for the rf wave $\overleftrightarrow{\sigma}$ is calculated if f_0 is given. The explicit form of $\overleftrightarrow{\sigma}$ in terms of f_0 is given in Ref.[12] and is not reproduced here. To make a closure of this set equations (39) and (40), the charge neutrality condition is to be incorporated.

This set of equations yields the basis to analyze the evolution of the plasma under the strong and localized heating. The competitions between the heating, slowing down and spatial loss are to be studied. The loss influences the heating and current-drive efficiency, the partition of the absorbed energy to plasma species and the deposition profile. The more quantitative comparison with the experiment would be possible.

Note that the second and the third terms of Eq.(27) contribute to an anomalous viscosity due to micro-turbulence. The relative magnitude to the diffusion coefficient is roughly estimated to be $\langle k_{\parallel} \rangle \Omega / \kappa v_{th} \langle k_0 \rangle$ for the maxwellian distribution. For usual tokamak operations, $\langle k_{\parallel} \rangle / \langle \kappa_0 \rangle$ is order of $(\kappa \rho_i)^2 \kappa L_s$ where L_s is the magnetic shear length.[24,45] For the mode of $\langle k_0 \rangle \sim \rho_i^{-1}$, the ratio becomes $(\rho_i/\rho_e)\kappa^2 \rho_i L_s$ for electrons and $\kappa^2 \rho_i L_s$ for ions. The value of viscosity for electrons can easily be order of the diffusion and the anomalous flux is affected by the gradient of the parallel flow velocity. Futhermore, if we consider the case of current-drive, the tail formation of the electron distribution is influenced by the anomalous viscosity. The anomolous viscosity under the strong heating and/or current-drive are to be examined. Analysis is left for our future work.

Assumption of the independence of the fluctuation from the rf wave (the validity of time scale separation) must be examined in applying the result to experiments. The effect of the fluctuations on the rf wave propagation has been discussed[26,27]. If this process would be important, the coupling must be first included in the wave propagation equation, (40). The other simplification is that the rf wave itself does not affect the nature of the micro-turbulence. Finally, it is noted that the analysis is done in a slab geometry. In the toroidal geometry, the toroidally trapped particle exists and which can influence on the evolution of the pitch angle dependence of f_0[7]. This kind of effect is not incorporated here. The relevance of these assumptions must be examined through the comparison study of the experiment with the theoretical calculations.

By choosing the particular form of the loss term, the effect of the spatial loss on the change of f_0 has been studied by analytic and numerical calculations. The asymptotic form of the Fokker-Planck equation is written as[16)]

$$\frac{1}{u^2}\frac{\partial}{\partial u}\left[\frac{\xi+1}{2}u^2\frac{\partial f}{\partial u}+u^3 f\right]-\frac{\tau_s}{\tau_0}u^s f \simeq 0 \tag{41}$$

for the case of the fundamental cyclotron resonance, where $u=v/v_{th}$, τ_0 is the typical time scale associated with the spatial loss, index s indicates the asymptotic form of H_2 and the limit of $\xi \gg 1$ and $u \gg (m_i/m_e)^{1/6}$ has been discussed. We see that $s=-1$ is appropriate for the case of drift wave fluctuations. The WKB solution $f \propto \exp(\int q du)$ is given as

$$f \sim f_a \equiv \exp[-u^2/(\xi+1)], \tag{42}$$

where the relation (30) is used. The distribution function is characterized by $\partial/\partial W \simeq 1/(\xi+1)T$, which is not explicitly affected by the spatial loss (operator L). The number density of the tail component is reduced in comparison with the case without a spatial loss. In the case of higher harmonic heating, either the effect of spatial loss or the finite gyroradius effect on the power absorption is to be included to obtain the stationary state distribution[9]. The quantitative comparison with the heating experiment by using the formula derived in this article would be discussed in a separate paper.

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Figure Caption

Fig. 1 The approximated formula Eq.(37) (dashed line) is compared to the value of Eq.(36) (solid line).

Fig.1