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Experiments and Applications of Soliton Physics

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Experiments and Applications of Soliton Physics

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Solitons are everywhere in the nature. The present lecture surveys various soliton phenomena, after giving the mathematical foundation to define solitons. Laboratory devices for the studies of plasma soliton phenomena are described together with experimental results. The most interesting application of soliton physics is illustrated in the discussion of soliton propagation in optical fibers. Topics on chaotic behavior in nonlinear dynamical systems will be discussed briefly in concluding remarks.

Keywords, soliton, double plasma device, optical fiber

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1 Solitons are Everywhere

In the month of August 1834, while riding on horseback beside a narrow channel, a Victorian naval architect John Scott Russell encountered with the singular and beautiful phenomena which he called the Wave of Translation.[1] When the boat he was observing suddenly stopped, the mass of water in the channel "accumulated round the pow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded smooth and well defined heap of water". Its original form of some thirty feet long and a foot to a foot and a half in height was preserved while it was rolling on at a rate of some eight or nine miles an hour and was running over the distance of one or two miles.

Over several years after his first encounter with the solitary wave, Russell had carried out series of extensive experiments. The speed of propagation of a solitary wave in a channel of uniform depth h is given by

$$v = \sqrt{g(h+\eta)} \tag{1}$$

η " being the height of the crest of the wave above the plane of repose of the fluid and g the measure of gravity."[2] Yet, it took another 50 years until Korteweg and de Vries[3] succeeded to give theoretical foundation for the solitary wave observed by Russell. They derived an equation, which bears their names, governing small but finite amplitude shallow-water waves.

The Korteweg-Vries equation (the K-dV equation in short)

$$\frac{\partial}{\partial t} q + q \frac{\partial}{\partial x} q + \delta^2 \frac{\partial^3}{\partial x^3} q = 0 \tag{2}$$

was the sleeping beauty in deep forest of nonlinear science until Gardner and Morikawa(4) discovered in 1960 that the K-dV equation describes propagation of collision free hydromagnetic wave. In the mean time, Boussinesq(5) also derived a nonlinear evolution equation for long waves with small but finite amplitude. The Boussinesq equation

$$\frac{\partial^2}{\partial t^2} y = s^2 \frac{\partial^2}{\partial x^2} \left(y + \frac{3}{2h} y^2 + \frac{1}{3} h^2 \frac{\partial^2}{\partial x^2} y \right) \tag{3}$$

admits solitary wave solutions travelling either along the positive or the negative x direction.

In quest of light in the dark forest of nonlinear science, the advent of high speed electronic computer was the critical event. Challenging to resolve the Fermi-Pasta-Ulam recurrence phenomena,(6) in 1965 Zabusky and Kruskal(7) discovered that solitary wave solutions of the K-dV equation, eq.(2),

$$q(x,t) = A \operatorname{sech}^2\left(\sqrt{\frac{A}{12\delta^2}} \left(x - \frac{1}{3} A t \right) \right) \tag{4}$$

retain their original forms through collision processes governed by the K-dV equation, and were led to call the solitary wave "soliton".

Looking around the nature, we are much surprised to find such localized disturbances that exhibit remarkable stabilities and tendency to preserve their original structure. One of such examples can be illustrated by a series of pendulums connected by linear springs.[8] Denoting the twisted angle of the n-the pendulum by θ_n , we have

$$\frac{\partial^2}{\partial t^2}\theta_n = \omega_0^2(\theta_{n+1} - 2\theta_n + \theta_{n-1}) - \kappa^2 \sin\theta_n \qquad (5)$$

which is reduced to the sine-Gordon equation in the continuum limit as follows,

$$\frac{\partial^2}{\partial t^2}\theta - s^2\frac{\partial^2}{\partial x^2}\theta = -\kappa^2 \sin\theta \qquad (6)$$

Eq.(6) has a travelling kink (+) and anti-Kink (-) solution,

$$\theta_{K,A} = 4\tan^{-1}\left[\exp\left\{\pm\left(\frac{\kappa}{s}\right)\frac{x-ut}{\sqrt{1-(u/s)^2}}\right\}\right] \qquad (7)$$

As it will be discussed in the next section, the K-dV equation, eq.(2), and the sine-Gordon equation, eq.(6), belong to the Ablowitz-Kaup-Newell-Segur scheme[9] of the inverse scattering transformation. Hence, eq.(4) and eq.(7) represent the soliton solutions of these nonlinear evolution equations. As for the Boussinesq equation, eq.(3), Zakharov[10] proposed a set of Lax-pair equations, though it is difficult to solve by the inverse scattering transformation method. N-soliton solution of the Boussinesq equation, however, is constructed by Hirota's bilinear transformation method.[11]

In order to assure that solitons are everywhere, and to indicate the wide scope of soliton physics, we conclude the present section by listing the physical problems for which the sine-Gordon equation plays the key role in the following. Frenkel and Kontorova[12] discussed the propagation of a slip dislocation in a one-dimensional crystal by eq.(6). Fergason and Brown[13] examined a splay wave along the lipid membrane of a biological cell. Eq.(6) is noted as being relevant to the description of the propagation of ultra-short optical pulses through a two-level atomic system,[14] dynamics of the Bloch wall[15] and magnetic flux propagation in a Josephson junction.[16] It will be worth to mention that Perring Skyrme[17] have discussed nonlinear theories of elementary particles on the basis of the travelling kink and anti-kink solution of eq.(7).

2 Mathematical Foundation of Soliton Physics

Inspired by the discovery of unique behavior of a solitary wave solution of the K-dV equation, Gardner, Greene, Kruskal and Miura[18] challenged to explore secret of the soliton, and succeeded to discover an ingenious mathematical method, called the inverse scattering transformation, for solving the Korteweg-de Vries equation. For nearly five years after their invention, the inverse scattering method had been believed to be effective only for the Korteweg-de Vries equation. Then, suddenly around 1972, people realized the inverse scattering transformation method is not a fluke at all. Now, various schemes of the inverse scattering transformation provide us the firm mathematical foundation of soliton physics.

Generalizing the scheme of Ablowitz-Kaup-Newell-Segur,[19] we consider the eigenvalue problem,

$$\begin{aligned} \frac{\partial}{\partial x} u_1 + F(\lambda) u_1 &= G(\lambda)\, q(x,t)\, u_2 \\ \frac{\partial}{\partial x} u_2 - F(\lambda) u_2 &= G(\lambda)\, r(x,t)\, u_1 \end{aligned} \qquad (8)$$

where F(λ) and G(λ) are functions of the eigenvalue λ. Together with eq.(8), we assume that the eigenfunctions u_1 and u_2 evolve in time according to the temporal evolution equation,

$$\begin{aligned} \frac{\partial}{\partial t} u_1 &= A(\lambda, q, r) u_1 + B(\lambda, q, r) u_2 \\ \frac{\partial}{\partial t} u_2 &= C(\lambda, q, r) u_1 - A(\lambda, q, r) u_2 \end{aligned} \qquad (9)$$

where A, B and C depend on λ and functionals of the potentials q and r and of their spatial derivatives in the arbitrary order. It is quite natural to request that

$$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial x} u_i\right)=\frac{\partial}{\partial x}\left(\frac{\partial}{\partial t} u_i\right), \quad i=1.2 \tag{10}$$

The key point to squeeze out the soliton is to postulate that the eigenvalue λ does not change in time while the potentials q(x.t) am r(x.t) change their shape in time. We set the condition

$$\frac{\partial}{\partial t}\lambda = 0 \tag{11}$$

Then, we find that A, B and C should satisfy the following set of equations,

$$\frac{\partial}{\partial x}A - G(\lambda)(rB - qC) = 0 \tag{12.a}$$

$$G(\lambda)\frac{\partial}{\partial t}q - \frac{\partial}{\partial x}B - 2F(\lambda)B - 2G(\lambda)qA = 0 \tag{12.b}$$

$$G(\lambda)\frac{\partial}{\partial t}r - \frac{\partial}{\partial x}C - 2F(\lambda)C - 2G(\lambda)rA = 0 \tag{12.c}$$

For given expressions of F(λ) and G(λ), we construct the functions A,B and C from eqs. 12.a)-c), and obtain the nonlinear evolution equation for q and r , which are the soliton equation

We notice that for the choice of the set of equations,

$$F(\lambda) = \lambda, \qquad G(\lambda) = 1 \tag{13}$$

eps. (8), (9) and (12) are reduced to the A-K-N-S scheme. They have shown that setting r =const, they get the K-dV equation, eq.(2), and for the choice of r =-q (real), determining expressions of A, B and C, they reduced the modified K-dV equation

$$\frac{\partial}{\partial t} q + q^2 \frac{\partial}{\partial x} q + \frac{\partial^3}{\partial x^3} q = 0 \tag{14}$$

For the choice of r =-q* (complex), we get the cubic nonlinear Schrodinger equation,

$$i \frac{\partial}{\partial t} q + \frac{\partial^2}{\partial x^2} q + |q|^2 q = 0 \tag{15}$$

In particular, setting

$$A = \frac{1}{4\lambda} \cos\theta \qquad B = C = \frac{1}{4\lambda} \sin\theta \tag{16}$$

and

$$q = -r = -\frac{1}{2}\theta_x \quad . \tag{17}$$

we obtain

$$\frac{\partial^2}{\partial t \partial x} \theta = \sin\theta \tag{18}$$

which is the sine-Gordon equation in the light cone coordinates x $=(\xi - \tau)/2$, t $=(\xi + \tau)/2$. Thus, we confirm that the kink and anti-kink solution of eq.(7) is nothing but the soliton.

For the choice of

$$F(\lambda) = -i\alpha\lambda^2 - \sqrt{2\beta}\,\lambda \tag{19.a}$$

$$G(\lambda) = \alpha\lambda - i\sqrt{\beta/2} \tag{19.b}$$

with $r = \pm q^*$, we[20] could determine the functions A, B and C and integrable nonlinear evolution equation as

$$i\frac{\partial}{\partial t}q + \frac{\partial^2}{\partial x^2}q \mp i\alpha\frac{\partial}{\partial x}(|q|^2 q) \pm \beta|q|^2 q = 0 \tag{20}$$

which could be called the modified nonlinear Schrodinger equation. Eq.(20) describes nonlinear propagation of the Aefven wave.[21] It is also relevant in the discussion of deformed continuous Heisenberg ferromagnet[22] and in the study of two-photon self-induced transparency and ultrashort light pulse propagation in an optical fiber.[23]

Furthermore, we[24] have still other integrable nonlinear evolution equations for the choice of

$$F(\lambda) = i\lambda \tag{20.a}$$

$$G(\lambda) = \lambda \tag{20.b}$$

For the real potential $r = \pm q$, with the expressions of A, B, and C determined from eqs. 12.a)-c), we get

$$\frac{\partial}{\partial t}q + \frac{\partial^2}{\partial x^2}\left(\frac{q_x}{(1 \mp q^2)^{3/2}}\right) = 0 \tag{21}$$

While for the complex potential $r = \pm q^*$, eqs. 12.a)-c) determines the functions A, B and C with the integrable nonlinear evolution equation

$$i\frac{\partial}{\partial t}q \mp \frac{\partial^2}{\partial x^2}\left(\frac{q}{\sqrt{1 \mp |q|^2}}\right) = 0 \qquad (22)$$

The above choices do not exhaust possible existence of other integrable nonlinear evolution equations. For the choice of

$$F(\lambda, q) = \frac{i}{2}\lambda^2 + |q|^2 \qquad (23.a)$$

$$G(\lambda) = \lambda \qquad (23.b)$$

with $r = \pm q^*$, we can construct another kind of the derivative nonlinear Schrodinger equation

$$i\frac{\partial}{\partial t}q + \frac{\partial^2}{\partial x^2}q \pm 2i|q|^2\frac{\partial}{\partial x}q = 0 \qquad (24)$$

which has been shown to be integrable by Chen, Lee and Liu[25] some years ago.

We conclude the present section by referring to the generalization of Morris and Dodd[26] to expand the two components eigenvalue problem to the three components eigenvalue problem. Their covariant formalism allows to adapt the method to solve the n-component derivative nonlinear Schrodinger equation.

3 Experimental Device for Plasma Soliton Studies

Various observations of space plasma phenomena suggest many evidences of manifestation of plasma solitons. Kennel[27] noticed that the Alfuen soliton[28] may account for the sharp shocklets detected by ISEE-3 when it encountered the interplanetary shock on November 12th, 1978, and also the magnetic field shocklets detected in the Comet Giacobini-Zinner shock interaction region on September 11th, 1985. Temerin and his collaborators[29] have observed solitary waves in the auroral plasma with an artificial satellite. It is important, however, to carry out controlled experiments with good accuracy on laboratory plasmas, so that we can develop quantitative analysis of experimental data referring to the relevant soliton theory.

One of the standard experimental setup is the double plasma device[30], using the plasma produced by the filament discharge in the argon gas at a pressure of $(1\text{-}5) \times 10^{-5}$ Torr. The discharge voltage and current could be taken in the range of (40-50) Volt and (50-100) mA. Typical plasma parameters are electron temperature $T_e \approx (1.5\text{-}3)$ eV, and electron number density $N_e \approx (10^8\text{-}10^9)$ c.c.. Ion temperature T_i is much lower than electron temperature, $T_i / T_e \ll 1$. A chamber of (50-80) cm in diameter and (100-120) cm in length is divided by a fine-meshed metallic grid biased at -20 Volt, or is separated into a driver and a target section with a floating grid. Multipole walls with permanent magnets in a full line cusp configuration improve plasma confinement, so that the discharge could be maintained at low neutral pressure. This ensures that large volume of plasma is collision free. Fig. 1 shows a typical arrangement of the device.

After the pioneering work of Ikezi and his collaborators on the formation of ion-acoustic solitons,(31) various extensions of experimental studies were undertaken. As for the planar soliton experiments, measurements of the soliton velocity and the soliton width are compared with the theoretical values predicted for the cold ion K-dV equation. Since a finite T makes the dispersion of the ion acoustic wave less dispersive, the width of solitons becomes narrower and the soliton velocity becomes faster than the cold ion K-dV soliton. This is accord with experimental results of Ikezi.(32) For the larger value of density amplitude, however, it has been noted also that the cubic nonlinear correction appears to be important to account the observed soliton structure.(33)

Considering cylindrical and spherical symmetric geometries, Maxon and Viecelli(34) have reduced the following equation,

$$\frac{\partial}{\partial t} q + \frac{n}{2t} q + q \frac{\partial}{\partial x} q + \frac{1}{2} \frac{\partial^3}{\partial x^3} q = 0 \qquad (25)$$

where n =1 and 2 for cylindrical and spherical geometries, respectively. Modifying the double plasma device, Hershkowitz and Romesser[35] performed experiments on propagation of cylindrical symmetric ion-acoustic waves. Fig.2 illustrates the cylindrical double plasma device used by Nagasawa and Nishida.[36] Tsukabayashi and his collaborators[37] carried out similar experiment and compared their result with computer simulation result. Observing the $\pi/2$ phase change at the center of collapse, they found that the convergence at the center of the inward moving soliton is not a penetrating collision but a reflection of the pulse. In Fig.3, we show the experimental result.

With regards geometrical effects on the one dimensional K-dV soliton, Kadomtsev and Petviashvili[38] have examined the effect of transverse perturbation, obtaining

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial t} q + q \frac{\partial}{\partial x} q + \frac{\partial^3}{\partial x^3} q\right) + \alpha \frac{\partial^2}{\partial y^2} q = 0, \quad \alpha = \pm 1 \qquad (26)$$

which is called the K-P equation in short. The K-P equation is one of the rare cases of multi-dimensional soliton equation.[39] Oblique interactions of solitons were investigated by Miles in the study of oblique reflection of shallow water waves on a rigid wall.[40] A beautiful photograph of Toedtemier[41] illustrates oblique interaction between two shallow water solitons. Experimental studies on oblique interactions of two ion acoustic solitons were carried out by Nishida and Nagasawa,[42] Tsukabayashi and Nakamura,[37] and Khazei, Bulson and Lonngren.[43]

4 Solitons in Optical Fibers

To conclude the present lecture, we will discuss soliton propagation on optical fibers as one of the most interesting application of soliton physics. The study of nonlinear optical effects in silica fibers has been of challenging subject in connection with the development of fiber-optic communication lines, generation of extremely short pulses, and soliton lasers. Inspired by the pioneering work of Hasegawa and Tappert,[44] experimental research on the propagation of optical solitons continues intensively. Mollenauer, Stolen and Gordon[45], applying the cubic nonlinear Schrodinger equation, have analyzed the experimental observation of pico second behavior of the optical pulse envelope. Further experimental research[46] has revealed that asymmetric modulation of the output pulse spectrum occurs in the femtosecond range.

The one-dimensional wave equation for a linearly polarized optical wave pulse is

$$\frac{\partial^2}{\partial x^2}E - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}D_L = \frac{n_0 n_2}{c^2}\frac{\partial^2}{\partial t^2}\left(|E|^2 E\right) \qquad (27)$$

where E and D are the electric field and the displacement field, respectively. The subscript L refers to its linear part, while n_2 characterizes the nonlinear part of the refractive index n , which is written as

$$n(\omega, |E|^2) = n(\omega) + n_2|E|^2 \qquad (28)$$

The electric field is assumed in the form

$$E(x,t) = q(x, t) \exp\left[i\,(k_0 x - \omega_0 t)\right] \tag{29}$$

where q (x, t) is a complex amplitude, $k_0 = \omega_0 n_0 / c$ is the wave number, ω_0 is the frequency, and $n_0 = n(\omega_0)$. Transforming to coordinate system moving with the group velocity of the carrier wave pulse, Tzoar and Jain(47) have reduced eq. (27) to the modified nonlinear Schrodinger equation.

$$i\frac{\partial}{\partial \xi} q - \frac{1}{2}\frac{\partial^2}{\partial \tau^2} q - R|q|^2 q \mp i\gamma \frac{\partial}{\partial \tau}\left(|q|^2 q\right) = 0 \tag{30}$$

where $\xi = x$, and $\tau = (t - v_g^{-1} x)/(\partial^2 k/\partial \omega_0^2)^{1/2}$ with the group velocity $v_g = \partial\omega/\partial k)_0$ at $\omega = \omega_0$. The coefficients R and γ are defined by

$$R = -(n_2/c)\,\omega_0 \tag{31.a}$$

$$\gamma = \frac{n_2}{c} \Big/ \left(\frac{\partial^2 k}{\partial \omega_0^2}\right)^{1/2} \tag{31.b}$$

The last term proportional to γ is important to describe the short-pulse propagation over long distance. In the studies of soliton propagation along optical fibers, the role of space variable ξ and temporal variable τ is interchanged in the usual formulation of the soliton physics in terms of the inverse scattering transformation.

The fact that the modified nonlinear Schrodinger equation, eq.(30), is the integrable soliton equation has not been recognized by researchers in the field of opto-electronics. Golovchenko and his collaborators[48] reported a numerical analysis of decay of optical solitons, regarding the last term of eq.(30) as a perturbation term to the cubic nonlinear Schrodinger equation. They have shown that the initial pulse for a three-soliton bound state of the cubic nonlinear Schrodinger equation breaks into two peaks rather than into three one - soliton components. This is contradiction to the complete integrability of the modified nonlinear Schrodinger equation, eq.(30). We[49] have carried out the careful numerical analysis of eq.(30) with the same parameter as used by Golovchenko.

With regard to a stationary pulse propagation, we can write the solution of eq.(30) as

$$q(\tau, \xi) = \rho(z) \exp\left\{ i\left[K\xi - \Omega\tau - \theta(z) \right] \right\} \quad (32)$$

where $z = \tau - M\xi$ with the boundary conditions $\rho = 0$, $d\rho/dz = 0$, $d^2\rho/dz^2 = 0$ and $d\theta/dz = 0$ in a limit $z \to -\infty$. The boundary conditions specify constants Ω and K as

$$\Omega = M \quad (33.a)$$

$$K = \frac{1}{2}M^2 - \frac{1}{2}(R - \gamma M)\rho_0^2 - \frac{1}{8}\gamma^2\rho_0^4 \quad (33.b)$$

The amplitude $\rho(z)$ and the phase modulation $\theta(z)$ are given as

$$\rho(z) = \rho_0 \left[(2-\nu)\cosh^2(\mu z) + \nu - 1 \right]^{-1/2} \tag{34.a}$$

$$\theta(z) = \frac{3}{2}\frac{\gamma}{\mu}\rho_0^2 (1-\nu)^{-1/2} \tan^{-1}\left[\sqrt{1-\nu}\, \tanh(\mu z) \right] \tag{34.b}$$

With the following definitions of μ and ν

$$\mu^2 = (R - \gamma M)\rho_0^2 - \gamma^2 \rho_0^4 \tag{35.a}$$

$$\nu = (R - \gamma M)\rho_0^2 \mu^{-2} \tag{35.b}$$

The peak height ρ_0 and the rate of time delay M are the characteristic parameters to define the stationary pulse.

In order to compare the results of numerical experiment with the analytical prediction, taking R = 1, we observe the -evolution of the "initial" pulse $q(\tau, \xi=0) = \mathrm{sech}(\tau)$ by numerical integration of eq.(30). The asymptotic shape is expected to approach the stationary pulse given by eq.(34-a) We observed that the peak position $\tau_0(\xi) = M\xi$ and determined the rate of delay M as 0.0996, 0.244 and 0.477, for the values of γ = 0.1, 0.25 and 0.5, respectively. At the same time, the asymptotic stationary peak amplitude ρ_0 are measured to be 0.995, 0.976 and 0.896 for the above values of γ, respectively.

Next, increasing the values of R to 4 and 9, we examined effects of the derivative term on the two- and three-soliton bound states. On the contrary to the results of Golovchenko et al.,we observed that the initial pulses for R = 4 and 9 split into two and three components. We show in Fig.4 the results of numerical observation of the splitting process of the sech-type initial pulse for the value of R = 9. In Tables I and II, we list observed values of the asymptotic peak height and the rate of time delay for the values of γ = 0.1, 1.0, 3.0, and 5.0.

γ	0.1	1.0	3.0	5.0
$\rho_0^{(1)}$	1.666	1.647	1.183	0.382
$\rho_0^{(2)}$	1.000	0.953	0.586	0.291
$\rho_0^{(3)}$	0.333	0.306	0.174	0.132

Table I. Asymptotic Peak Heights for R = 9

γ	0.1	1.0	3.0	5.0
$M^{(1)}$	0.1667	1.647	3.546	1.892
$M^{(2)}$	0.0999	0.957	1.755	1.275
$M^{(3)}$	0.0333	0.307	0.496	0.414

Table II. Rate of Time Delay for R = 9

In order to account for these observed results, we [50] have carried out the rigorous analytic calculation by solving the Wadati-Konno-Ichikawa inverse scattering problem for the sech-type potential. We obtained the following results;

i) the number of solitons associated with the initial pulse $q(\tau, \xi=0) = \text{sech}\,(\tau)$ is determined as

$$N = \textit{integer part of} \left(\sqrt{R} + \frac{1}{2}\right) \qquad (36.a)$$

ii) the peak height of the n-th component soliton is

$$\rho_0^{(n)} = \left| 2 - \left\{ 2\left[\sqrt{(R\gamma^{-2} - 1)^2 - (2n-1)^2\gamma^{-2}} + 1 - R\gamma^{-2} \right] \right\}^{1/2} \right| \qquad (36.b)$$

iii) the rate of time delay of the n-th component soliton is

$$M^{(n)} = \gamma \rho_0^{(n)} \qquad (36.c)$$

We conclude the present section by showing in Figs. 5 and 6 the dependence of the peak height $\rho_0^{(n)}$ and the rate of time delay $M^{(n)}$ on the values of γ, together with the observed results.

5 Concluding Remarks

The discovery of the inverse scattering transformation method in 1967 is undoubtedly one of the most elegant contributions to mathematical physics in the 20th century. In the present lecture, we have tried to convince the audience that *solitons are everywhere* and *physics of solitons are expanding its territory.*

We should notice,however, that it is only the one side of the problem inspired by Fermi, Pasta and Ulam. The other side of the story is the quest of *chaos*. Since Fermi,Pasta and Ulam failed to observe the energy equipartition among the normal modes, intensive studies on the chaotic behavior have been undertaken by many mathematicians and physicists. In this connection, returning to eq. (5), we examine its static solution,

$$\theta_{n+1} - 2\theta_n + \theta_{n-1} = (\kappa^2/\omega_0^2)\sin\theta_n \tag{37}$$

of which continuum limit is

$$\frac{d^2}{dx^2}\theta = (\kappa^2/\omega_0^2)\sin\theta \tag{38}$$

Introducing an action variable I by

$$I_{n+1} = \theta_{n+1} - \theta_n \tag{39}$$

we can transform eq.(37) into

$$I_{n+1} = I_n + (\kappa^2/\omega_0^2)\sin\theta_n \tag{40.a}$$

$$\theta_{n+1} = \theta_n + I_{n+1} \tag{40.b}$$

Eqs. (40.a) and b) are nothing but the celebrated standard map[51]. More than one hundred papers have been published with referring to this simplest set of equations. The standard map poses the canonical problem on the studies of intrinsic stochasticity of low dimensional Hamiltonian systems. At the same time, it would be worth to note that Aubry[52], Bak[53], and many others have been working with the set of eqs.(40.a) and b) in the context of condensed matter physics. Discussions on the discretized integrable nonlinear evolution equations are the critical issues under the light of computational physics. Here, we give few references on such subjects.[54]-[56]

Since the time allocated to the present lecture is limited, we are not able to explore these fascinating topics in detail. The studies of chaos in dissipative systems, intrinsic stochasticity in conservative systems and many other topics are the current issue of nonlinear physics. We hope to have a chance to discuss on these subjects in the next Tropical College on Applied Physics.

References

1) Russell, J. S. (1838); Report of the committee on waves, the 7th Meeting of British Association for the Advancement of Science, Liverpool, p.417-496.

2) Russell, J. S. (1844); Report on waves, the 14th Meeting of British Association for the Advancement of Science, London, p.311-390.

3) Korteweg, D. J. and G. de Vries; Phil. Mag. 39 (1895) p.422-443.

4) Gardner, C. S. and G. K. Morikawa; Courant Inst. Math. Sci. Res. Rep. NYO-9082, (1960) New York University.

5) Boussinesq, J.; Compte Rendus Acad. Sci. Paris 72 (1871) p.755-759.

6) Fermi, E., J. Pasta and S. Ulam; Los Alamos Rep. LA-1940, (1955); reproduced in Nonlinear Wave Motion, A. C. Newell, ed., American Mathematical Society, Providence, RI.

7) Zabusky, N. J. and M.D. Kruskal; Phys. Rev. Lett. 15 (1965) p.240-243.

8) Barone, A., F. Esposito, C. J. Magee, and A. C. Scott; Riv. Nuvo Cimento, 1 (1971) p.227-267, see also Rnbinstein, J.; J. Math. Phys. 11 (1970) p.258-266.

9) Ablowitz, M. J., D. J. Kaup, A. C. Newell and H. Segur; Stud. Appl. Math. 53 (1974) P.249-315.

10) Zakharov, V. E.; Sov. Phys. JETP, 38 (1974) p.108-110.

11) Hirota, R.; J. Math. Phys. 14 (1973) p.810.

12) Frenkel, J. and T. Kontorova; Journ. Phys. USSR 1 (1939) p.137-.

13) Fergason, J. L. and G. H. Brown; Journ. Am. oil Chemists Soc. 45 (1968) p.120-.

14) Arrechi, J. T., G. L. Masserini and P. Schwendiman; Riv. Nuovo Cimento. 1 (1969) p.181.

15) Enz, U.; Helv. Phys. Acta 37 (1964) p.245-, see also Feldtkeller, E.; Phys Stat. Sol. 27 (1968) p.161-.

16) Josephson, B. D.; Àdv. Phys. 14 (1965) p.419-. see also A. Barone; Journ. Appl. Phys. 42 (1971) p.2747-.

17) Perring, J. K. and T. H. R. Skyrme; Nucl. Phys. 31 (1962) p.550.

18) Gardner, C. S., J. M. Greene, M. D. Kruskal and R. Miura; Phys. Rev. Lett. 19 (1967) p.1095-1097. and Comm. Pure Appl. Math. 27 (1974) p.97-133.

19) Ablwitz, M. J. D. J. Kaup, A. C. Newell and H. Segur; Stud. Appl. Math. 53 (1974) p.249-315.

20) Wadi, M., K. Konno and Y. H. Ichikawa; J. Phys. Soc. Japan 46 (1979) p.1965-.

21) Ichikawa, Y. H., K. Konno, M Wadati and H. Sanuki; J. Phys. Soc. Japan 48 (1980) p.279-.

22) Porsezian, K., K. M. Tamizhmani, and M. Lakshmanan; Phys. Lett. A. 124 (1987) p,159.

23) Zabolotskii, A. A.; Phys. Lett. A 124 (1987) p. 500-.

24) Wadati, M., K. Konno and Y. H. Ichikawa; J. Phys. Soc. Japan 47 (1979) p.1698-.

25) Chen, H. H., Y. C. Lee and C. S. Liu; Phys. Scripta 20 (1979) p.490-492.

26) Morris, H. C. and R. K. Dodd; Phys. Scripta 20 (1979) p.505-508.

27) Kennel, C. F.; Adv. Space Res. 6 (1986) p.5.

28) Ichikawa, Y. H. and Y. Abe; Suppl. No. 94 (1988) p128-137. Prog. Theoret. Phys.

29) Temerin, M., K. Cerny, W. Lotko and F. S. Mozer; Phys. Rev Lett. 48 (1982) p.1175-.

30) Christensen, T. and N. Hershkowitz; Phys. Fluids. 20 (1977) p.840-.

31) Ikezi, H., R. J. Taylor and D. R. Baker; Phys. Rev. Letters. 25 (1970) p.11-14.

32) Ikezi, H.; Phys. fluids 16 (1973) p.1667-1675.

33) Konno, K. and Y. H. Ichikawa; J. Phys. Soc. Japan 37 (1974) p.1631.

34) Maxomn S. and J. Viecelli; Phys. Rev Lett., 32 (1974) p.4; also Phys. Fluids, 17 (1974) p. 1614.

35) Hershkowitz, N and T. Romesser; Phys. Rev. Lett. 32 (1974) p.581-.

36) Nagasawa. T. and Y. Nishida; Plasma Phys: 23 (1981) p.575-595.

37) Tsukabayashi, I. and Y. Nakamura; Phys. Lett. 85A (1981) p.152-.

38) Kadomtsev, B. B. and V. I. Perwisahvili; Dokl. Akad. Nauk SSSR, p.753. [Sov. Phys.-Dokl. 15 (1970) p.539]

39) Zakharov, V. E. and A. B. Shabat; Funct. Anal. Priloz 8 (1974) p.43.

40) Miles, J. W.; J. Fluid. Mech. 79 (1979) p. 171-.

41) Abolwitz, M. J. and H. Segur; Solitons and the Inverse Scattering Transform (SIAM, Philadelphia, 1981) p.291.

42) Nishida, Y. and T. Nagasawa; Phys. Rev. Lett 45 (1980) P.1626.

43) Khazei, M., J. Bulson and K. E. Lonngren; Phys. Fluids 25 (1982) p.759-.

44) Hasegawa, A. and f. Tappert; Appl. Phys. Lett 23 (1973) p.142.

45) Mollenauer, L. F., R. H. Stolen, and J. P. Gordon; Phys. Rev Lett. 45 (1980) p.1095.

46) Nakatsuka, H., K. Grischkowsky, and A. C. Balant; Phys. Rev. Lett. 47 (1981) p.910.

47) Tzoar, N. and M. Jain; Phys. Rev. A 23 (1981) p.1266.

48) Golovchenko, E. A., E. M. Dianov, A. M. Prokhorov, and V. N. Serkin; Pis'ma Zh. Eksp. Teor. Fiz. 42 (1985) p.74. [Soviet Phys. JETP Lett. 42 (1985) p.88]

49) Ohkuma, K., Y. H. Ichikawa, and Y. Abe; Opt. Lett. 12 (1987) p.516-518.

50) Ohkuma, K., Y. H. Ichikawa, and Y. Abe; to be published.

51) Chirikov, B. V; Phys. Rep. 52 (1979) p.263-379, Green, J.; J. Math. Phys. 20 (1979) p. 1183, Ichikawa, Y. H., T. Kaminura, and T. Hatori; Physica D 29 (1987) p.247-253.

52) Aubry, S and P. Y. Le Dearon; Physica D 8 (1983) P.381-422.

53) Bak, P. ; Rep. Prog. Phys. 45 (1982) p.587-629.

54) Ross, K. A. and C. J. Thompson; Physica A 135 (1986) p.551-558.

55) Friedman, A. and S. P. Auerbach; UCRL-96178 (1989)

56) Goedde, C. G., A. J. Lichtenberg and M. A. Lieberman; Physica D 41 (1990) p.341-355.

Captions of Figures

Fig. 1 Schematic of the double plasma device. Whole chamber wall is covered with many permanent magnets.

Fig. 2 Cylindrical double plasma device.

Fig. 3 Spatial evolution of solitons converging to ($t< 0$) and diverging from ($t>0$) the center ($t=0$) where the collapse of solitons occurs.

Fig. 4 Spatial evolution of the initial pulse $q(\tau, \xi=0)= \mathrm{sech}(\tau)$ for $R= 9$ and $\gamma=0.25$.

Fig. 5 γ-dependence of the peak height $\rho_0^{(n)}$ for $R = 9$. Black circles are the numerically observed asymptotic values.

Fig.6 γ-dependence of the rate of time delay $M^{(m)}$ for $R = 9$. Black circles are the numerically observed values.

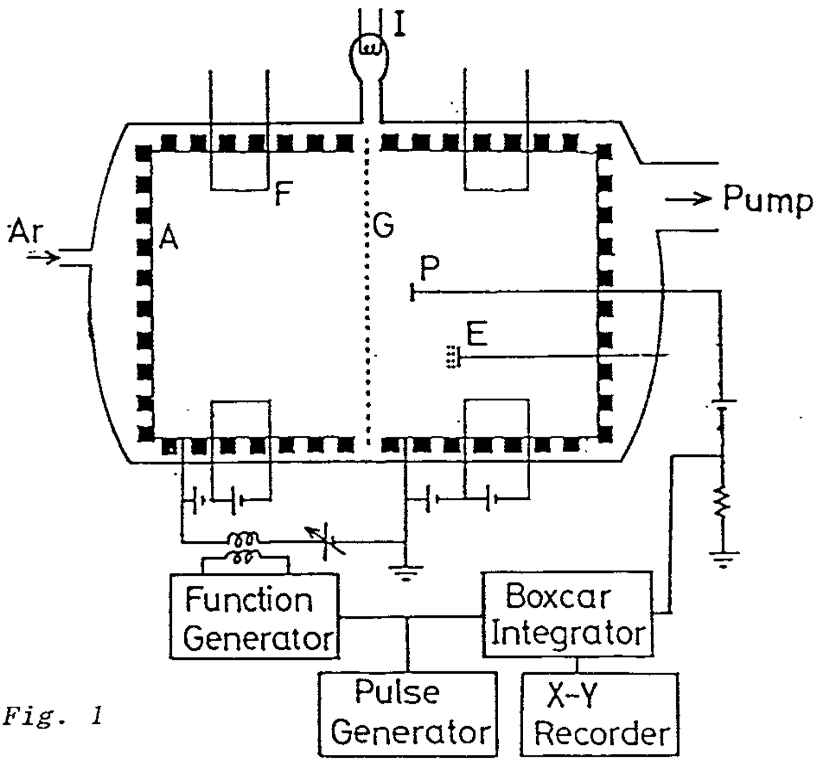

I
F
G
Ar
A
Pump
P
E
Function Generator
Boxcar Integrator
Pulse Generator
X-Y Recorder

Fig. 1

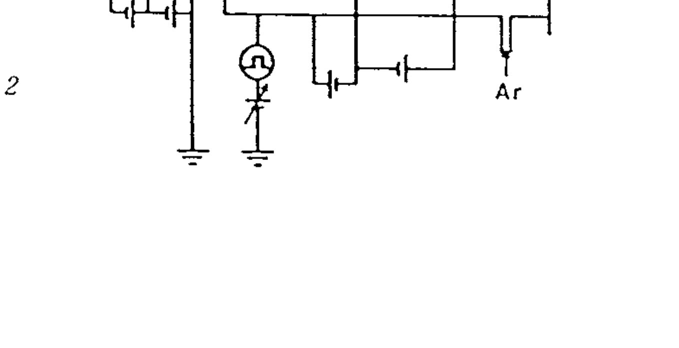

Filament
Probe
Farady Cup
to Pump
to Oscilloscope
Ar

Fig. 2

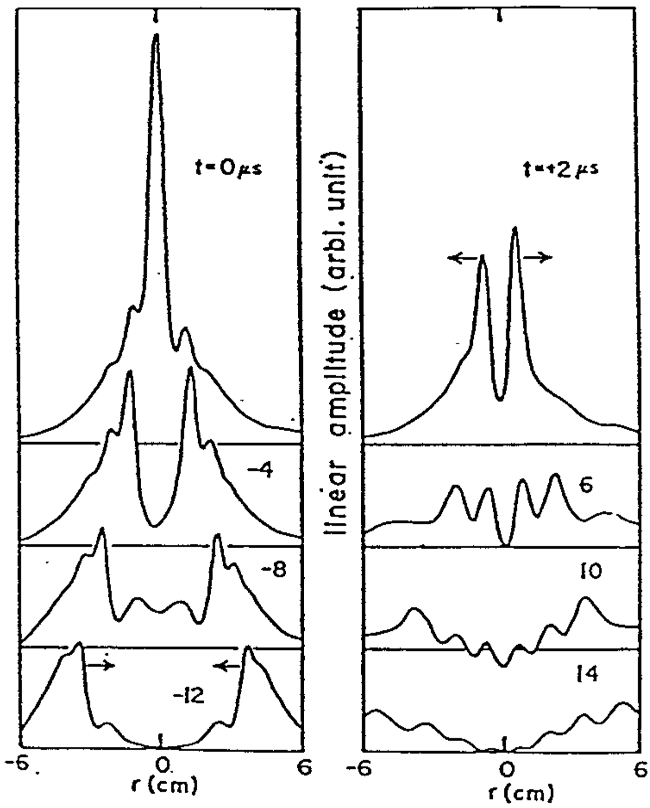

t=0μs
-4
-8
-12
-6
0
6
r (cm)
linear amplitude (arbi. unit)
t=+2μs
6
10
14
-6
0
6
r (cm)

Fig. 3

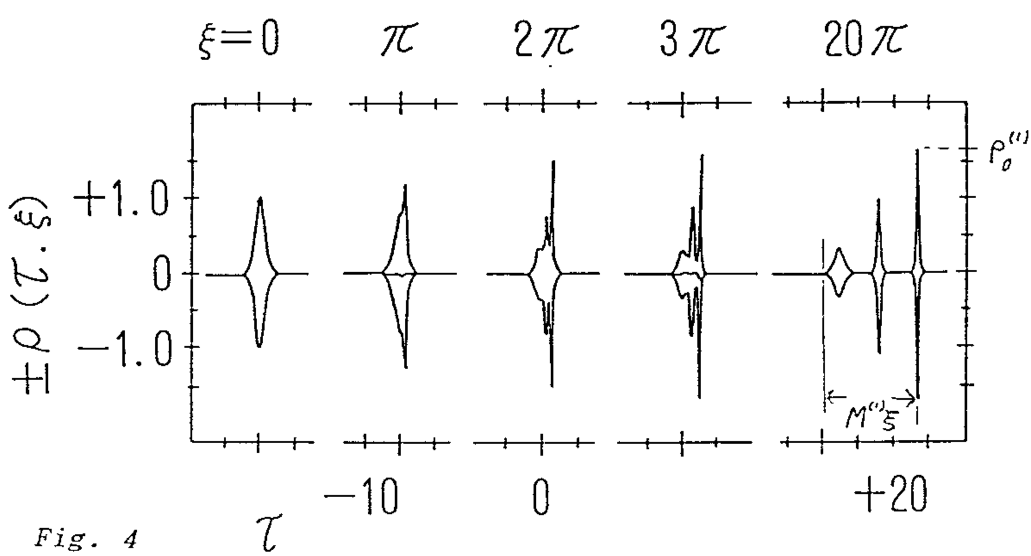
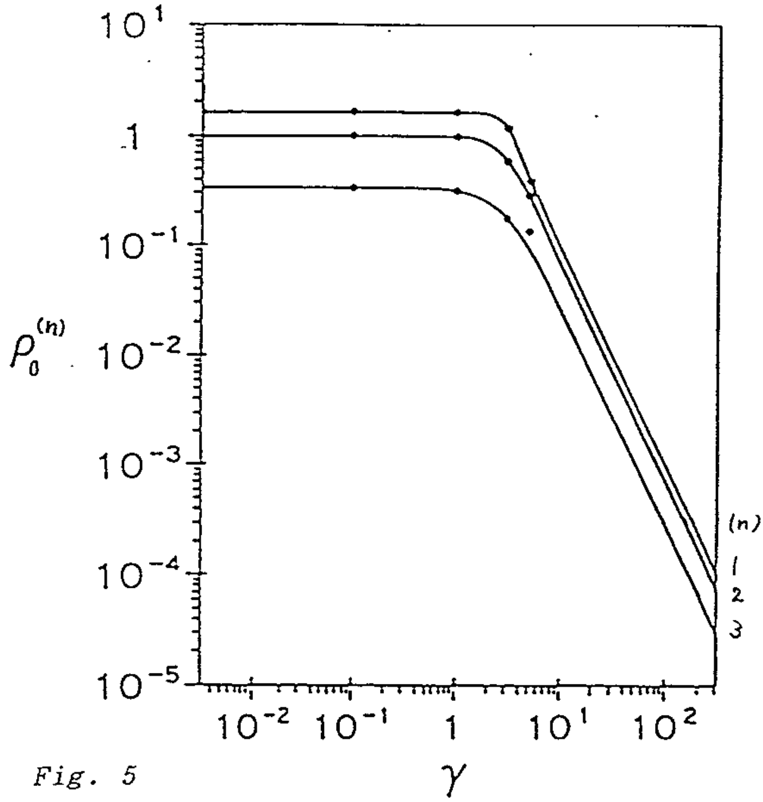
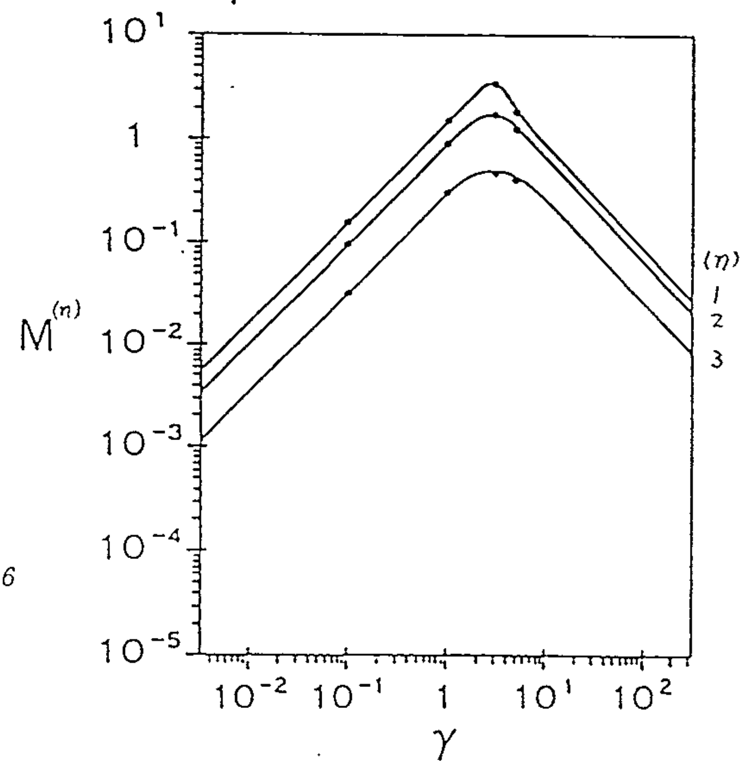

ξ=0
π
2π
3π
20π
±ρ(τ, ξ)
+1.0
0
−1.0
−10
0
+20
τ

Fig. 4



10^1
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$\rho_0^{(n)}$
10^{-2} 10^{-1} 1 10^1 10^2
γ
(n)
1
2
3

Fig. 5



10^1
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$M^{(n)}$
10^{-2} 10^{-1} 1 10^1 10^2
γ
(η)
1
2
3

Fig. 6