ISSN 0915-633X

NATIONAL INSTITUTE FOR FUSION SCIENCE

Anomalous Viscosity due to Drift Wave Turbulence

S.-I. Itoh

(Received – June 11, 1990)

NIFS-33 June 1990

RESEARCH REPORT
NIFS Series

NAGOYA, JAPAN

Anomalous Viscosity due to Drift Wave Turbulence

Sanae-I. Itoh

National Institute for Fusion Science, Nagoya 464-01 Japan

Abstract

Anomalous ion, electron viscosities and off-diagonal elements of transport matrix in the presence of low frequency microturbulence are calculated. Quasilinear theory is used in the electrostatic limit. Symmetric matrix is obtained including the off-diagonal terms. The electron and ion components in transport matrix apparently decouple in the drift wave approximation.

Particle, momentum and energy fluxes are presented in terms of three thermodynamical forces, where the gradient of parallel flow is included. Particle and energy fluxes are driven by density gradient, temperature gradient, radial electric field and convection due to the fluctuations. Anomalous electron viscosity is found to be very small. Ion anomalous viscosity, heat conductivity and off-diagonal elements are comparable to the coefficients for electrons.

The transport equations of ions show that off-diagonal element reduces the heat conductivity and even causes the anomalous heat pinch effect. Anomalous viscosity gives rise to additional heating on ions due to viscous damping.

Keywords;anomalous viscosity, transport matrix, off-diagonal drift wave, rotation

1. Introduction

Throughout the 1970s and early 1980s, the neoclassical theory of ion heat transport had been considered to be in reasonable agreement with some experiments[1]. The same was not true for momentum transport; the neoclassical theory gives viscosities[2,3] which are smaller than experimentally observed values by one or two orders of magnitude[4]. In order to overcome the discrepancy, the neoclassical theory of ion transport in rotating plasma has been formulated, in which the flow is allowed to be of the order of the ion thermal speed[3]. However, the obtained viscosities, either in banana regime or in Pfirsch-Shluter regime, are not able to explain the presently observed momentum transport in tokamaks. An attempt to explain the enhanced transport based on classical gyroviscosity[5] has been done but has been refuted[6]. Quite recent theoretical progress in neoclassical theory of toroidal momentum transport has been made to explain the experimentally observed values of viscosities[7]. In the presence of moderate concentration of low energy ions, the predicted value of viscosity is found to be increased.

The measurements of ion temperature profile as well as the rotation profile have been recently done in several toroidal devices[8-12]. Not only ion momentum transport but also ion energy transport have been found to be anomalous. Experiments have revealed the high T_i mode, in which the peaked ion temperature profile is observed associated with the reduction of the effective ion thermal conductivity. Such discharges are often reported[8-12] that the strong shear of the toroidal rotation velocity is established in the core. The study on the anomalous viscosity and on the relation with the ion energy confinement is required.

Various anomalous transport theories have been developed[13-22]. Most of them are concerned with estimates of particle diffusivity and the heat conductivity due to the nonlinear consequences of low frequency instabilities. The rest of transport elements have not been usually discussed. Some attempts to calculate the off-diagonal elements of the transport matrix, such as terms of anomalous Ware pinch[23] and anomalous bootstrap current[24], have been done for drift wave fluctuations. However, the formulation has not been clear about the consistency concerning on the Onsager's symmetry. The complete transport matrix has been formulated by quasilinear theory and elements are calculated in toroidal geometry[25]. The calculation of the anomalous viscosity has been done for a special case[9,26], however, the complete formulation of the anomalous viscosity and transport matrix are left undone.

In this paper, we calculate anomalous viscosities in the presence of low frequency drift wave type fluctuations. Quasilinear theory is used and elements of the transport matrix are obtained in the electrostatic limit. We include the radial gradients of parallel flows as a thermodynamical diagonal terms. The matrix is symmetry by a proper choice of thermodynamical forces. It is found that the electron component and ion component in transport matrix decouple in a usual low frequency limit, that the electron part corresponds to the results by Shaing[25], and that the ion anomalous viscosity as well as ion heat conductivity are large, which are also comparable to electron anomalous transport coefficients. Thus the momentum transport is considered to be anomalous whose confinement time is comparable to ion energy confinement time[26]. It should be noted that the off-diagonal elements of the ion transport matrix are large, and the shear of the rotation velocity and the ion temperature gradient influence each other.

The constitution of this paper is as follows. In Sec.2, the general formula of anomalous fluxes, which are derived from Klimontovich formula, are presented and the fluxes are calculated in the presence of inhomogeneities of density, parallel flow, temperature and the electrostatic potential. Applying the flux-friction relationship, we obtain the matrix elements. In Sec.3, the set of ion transport equations which includes the ion momentum balance is presented. Summary and discussions are given in the final section.

2. General formula and Representation of fluxes

In this section , we present our theoretical model and formulations and calculate the anomalous particle, momentum and energy fluxes due to drift wave type turbulence. We consider a cylindrical plasma immersed in a strong magnetic field in the z-direction. The poloidal magnetic field is assumed to be small. The plasma has inhomogeneities of the density, the parallel flow and the temperature in the radial direction.

We start our analysis with the coarse-grained momentum and energy balance equations which are derived from Klimontovich equation. When the gyroradius ρ is small as compared with inhomogeneity scale lengths, we have[25],

$$m_\sigma n_\sigma\left(\frac{\partial \mathbf{V}_\sigma}{\partial t}+\mathbf{V}_\sigma\cdot\nabla\mathbf{V}_\sigma\right)-\frac{q_\sigma}{c}\Gamma_\sigma\times\mathbf{B}+\nabla p_\sigma = q_\sigma\langle\hat{n}_\sigma\tilde{\mathbf{E}}+\frac{1}{c}\tilde{\Gamma}_\sigma\times\tilde{\mathbf{B}}\rangle$$

$$+\,n_\sigma q_\sigma \mathbf{E}-\nabla(\overleftrightarrow{\mathbf{P}}_\sigma-p_\sigma\overleftrightarrow{\mathbf{I}}) \qquad (1)$$

$$\partial/\partial t(3p_\sigma)+\nabla\cdot Q_\sigma+2\overleftrightarrow{\mathbf{P}}_\sigma\cdot\nabla\mathbf{V}_\sigma = q_\sigma\langle(\hat{\Gamma}_\sigma-\hat{n}_\sigma\mathbf{V}_\sigma)\cdot\tilde{\mathbf{E}}\rangle \qquad (2)$$

for each particle species, where $\Gamma_\sigma = n_\sigma V_\sigma$ represents the particle flux defined by $\int v f_\sigma\, dv$, $p_\sigma = n_\sigma T_\sigma$ ($\equiv \int m_\sigma (v_i - V_i)^2 f_\sigma\, dv$) is the pressure, $\mathbf{V}_\sigma$ is the mean velocity, $Q_{\sigma,k} = 3T_\sigma$

$\Gamma_{\sigma,k} + \Sigma Q_{\sigma,kii}$ is the heat flux, $Q_{\sigma,ijk}$ ($\equiv \int m_\sigma (v_i - V_i)(v_j - V_j)(v_k - V_k) f_\sigma dv$) is the heat conduction, q_σ is the charge, **E** is the static electric field, $\overleftrightarrow{P}$ is the stress tensor and $\overleftrightarrow{I}$ is the unit tensor. Here i,j,k stand for the three coordinates r, θ, z, tilde shows the fluctuation quantity, the bracket < > indicates the coarse-grained space-time average and $q_\sigma \langle \Gamma \cdot \mathbf{E} \rangle$ is the energy exchange rate between particles and fluctuations. The kinetic energy of the mean flow is also included. The suffix of the particle species, σ, is suppressed in the following unless we especially note it. The representations of the radial transport fluxes of the particle, momentum and heat in terms of cross correlation of fluctuation quantities are[25),

$$\Gamma_r = \frac{c}{B} \langle (\tilde{n}\tilde{\mathbf{E}} + \frac{1}{c}\tilde{\Gamma} \times \tilde{\mathbf{B}})_\theta \rangle \qquad (3)$$

$$P_{zr} = \frac{mc}{B} \langle (\tilde{\Gamma}_z - \tilde{n}V_z)(\tilde{\mathbf{E}} + \frac{V \times \tilde{\mathbf{B}}}{c})_\theta + (\tilde{\Gamma}_\theta - \tilde{n}V_\theta)(\tilde{\mathbf{E}} + \frac{V \times \tilde{\mathbf{B}}}{c})_z \rangle$$

$$+ \frac{1}{B} \langle \tilde{B}_r (\tilde{P}_{zz} - \tilde{P}_{\theta\theta}) + \tilde{B}_\theta \tilde{P}_{\theta r} - \tilde{B}_z \tilde{P}_{zr} \rangle \qquad (4)$$

$$\Sigma Q_{rii} = \Sigma_i [\frac{c}{B} \langle \hat{p}_{ii} (\tilde{E} + \frac{V \times \tilde{B}}{c})_\theta \rangle + \langle (\tilde{p}_{i\theta} (\hat{E} + \frac{V \times \tilde{B}}{c})_i \rangle - \frac{B}{nc} p_{ii} \Gamma_r$$

$$+ \frac{1}{n} p_{i\theta} \langle (\hat{n}\tilde{E} + \hat{\Gamma} \times \tilde{B}/c)_i \rangle + \frac{1}{c} \langle \tilde{B}_r \hat{Q}_{zii} - \tilde{B}_z \tilde{Q}_{rii} \rangle \quad (5)$$

In order to relate the fluctuations of the plasma quantities to the electromagnetic fluctuations, we assume that the nonlinearlity is weak and that we can expand the fluxes with respect to the fluctuation spectrum and take only the lowest order terms. Hereafter we shall consider this situation and calculate the fluxes in the approximation linear with respect to the fluctuation spectrum. For simplicity, we restrict ourselves to the fluctuations with frequencies much lower than the ion cyclotron frequency, Ω_i, that is, so called drift wave type fluctuations.

The plasma is assumed to be in a steady state and the unperturbed velocity distribution f_σ is chosen to be a function of the energy, $m_\sigma v^2/2 + q_\sigma \Phi$, where Φ is the static potential and canonical moments in the θ and parallel-directions, $P_\theta = m_\sigma r(v_\theta + r\Omega_\sigma/2)$ and $P_\parallel = m_\sigma v_\parallel$, as

$$f_0 = (\frac{m}{2\pi})^{3/2} \frac{n(X)}{T(X)\sqrt{T_\parallel(X)}} \exp[-\frac{mv_\perp^2}{2T_\perp(X)} - \frac{m(v_\parallel - U_\parallel(X))^2}{2T_\parallel(X)} + \frac{q\Phi}{T}] \quad (6)$$

where r is expanded around r_0 with respect to gyroradius and a new coordinate X as $X = r-r_0+v_\theta/\Omega_\sigma$ is used.

The expressions of the fluxes in terms of spectral function is obtained by taking the linear response to the fluctuation and by performing the path integral along the unperturbed orbit. Substituting Eq.(6) into Eqs.(3),(4) and (5), we obtain,

$$\Gamma_r = \frac{c}{B}\Sigma_{\mathbf{k}}\int\frac{d\omega}{2\pi}\left(-i\frac{q}{k_\theta T_\perp}\right)\mathbf{G} \tag{7}$$

$$P_{zr} = \frac{mc}{B}\Sigma_{\mathbf{k}}\int\frac{d\omega}{2\pi}\left(-i\frac{q}{k_\theta T_\perp}\right)(\omega/k_\parallel-U_\parallel)\mathbf{G} \tag{8}$$

$$Q_{rzz} = -T_\parallel\Gamma_r + \frac{mc}{B}\Sigma_{\mathbf{k}}\int\frac{d\omega}{2\pi}\left(-i\frac{q}{k_\theta T_\perp}\right)(\omega/k_\parallel-U_\parallel)^2\mathbf{G} \tag{9}$$

where

$$\mathbf{G} \equiv I_{11}(D-D^*)\Lambda n/2,$$

$$\langle E_\theta(\mathbf{k},\omega)E_\theta(\mathbf{k}',\omega')\rangle = 2\pi\delta_{\mathbf{k},\mathbf{k}'}\delta(\omega+\omega')I_{11},$$

$$D\cdot F = \left(\frac{T_\perp}{T_\parallel}\frac{\hat\omega-k_\parallel U}{\hat\omega} - \frac{T_\perp k_\theta \partial}{\Omega m\hat\omega \partial x}\right)\cdot\left[\frac{\hat\omega}{\sqrt{2T_\parallel/m}\,|k_\parallel|} Z\left(\frac{\hat\omega-k_\parallel U}{\sqrt{2T_\parallel/m}\,|k_\parallel|}\right)F\right],$$

where, $\hat\omega = \omega-\omega_E$, $\omega_E = k_\theta E_r c/B$.[28] The radial derivative in D operates on density, parallel velocity, temperature as well as the static potential, which is included in terms of ω_E. D^* is given by replacing (ω,**k**) in the expression for D by $(-\omega,-\mathbf{k})$; $\Lambda(b) = I_0(b)e^{-b}$, $\Lambda'(b) = \partial\Lambda(b)/\partial b$, $b=(T_\perp/m)(k_\perp{}^2/\Omega^2)$ and I_0 is the zeroth order modified Bessel function of the first kind; $Z(\xi)$ is the plasma dispersion function. The explicit forms of fluxes in the presence of parallel flows are (for $T_\parallel = T_\perp$)

$$\Gamma_r = \frac{nc^2}{B^2}\Sigma_{\mathbf{k}} \int \frac{d\omega}{2\pi}\left(\frac{I_{11}\Lambda \,\mathrm{Im}Z_i}{\sqrt{2T_\parallel/m}\,|k_\parallel|}\right)\left[\frac{-dn_i}{n_i dr} - \frac{qd\Phi}{T_i dr} - \frac{(\omega-k_\parallel U_{i\parallel})qB}{k_\theta\ cT_i}\right.$$

$$\left. -(\omega/k_\parallel - U_\parallel)\frac{m_i\partial U_{i\parallel}}{T_i\partial r} - \left(\frac{b\Lambda'}{\Lambda} + \frac{m_i\omega^2}{2T_i k_\parallel{}^2} - \frac{1}{2}\right)\frac{\partial T_i}{T_i\partial r}\right] \qquad (7')$$

$$\frac{P_{zr}}{m_i} = \frac{nc^2}{B^2}\Sigma_{\mathbf{k}} \int \frac{d\omega}{2\pi}\frac{I_{11}\Lambda\,\mathrm{Im}Z_i}{\sqrt{2T_\parallel/m}\,|k_\parallel|}\left(\frac{\omega}{k_\parallel} - U_{i\parallel}\right)\left[\frac{-dn_i}{n_i dr} - \frac{qd\Phi}{T_i dr} - \frac{(\omega-k_\parallel U_{i\parallel})qB}{k_\theta\ cT_i}\right.$$

$$-(\omega/k_{\parallel}-U_{\parallel})\frac{m_i}{T_i}\frac{\partial U_{i\parallel}}{\partial r}-\left(\frac{b\Lambda'}{\Lambda}+\frac{m_i\omega^2}{2T_ik_{\parallel}^2}-\frac{1}{2}\right)\frac{\partial T_i}{T_i\partial r}] \quad (8')$$

$$\frac{Q_{r,i}}{T_i}=\frac{nc^2}{B^2}\Sigma_k\int\frac{d\omega}{2\pi}\frac{I_{11}\mathrm{Im}Z_i}{\sqrt{2T_{\parallel}/m'}\,k_{\parallel}}[(b\Lambda'-\Lambda-\Lambda'+\frac{m_i^2}{2T_i}\Lambda(\frac{\omega}{k_{\parallel}}-U_{i\parallel})^2-\frac{\Lambda}{2})[\frac{-dn_i}{n_idr}$$

$$-\frac{qd\Phi}{T_idr}-\frac{(\omega-k_{\parallel}U_{i\parallel})qB}{k_\theta cT_i}-(\omega/k_{\parallel}-U_{\parallel})\frac{m_i}{T_i}\frac{\partial U_{i\parallel}}{\partial r}-\left(\frac{m_i\omega^2}{2T_ik_{\parallel}^2}-\frac{1}{2}\right)\frac{\partial T_i}{T_i\partial r}]$$

$$+[b\{(2b^2-2b+1)\Lambda+(4b-2b^2+2)\Lambda'\}-b\Lambda'\{\frac{m_i}{2T_ik_{\parallel}}(\frac{\omega}{} -U_{i\parallel})^2\}-\frac{1}{2}\}]\frac{1}{}\frac{1}{T_i}\frac{dT_i}{dr}]$$

$$(9')$$

For electron components, $U_{e\parallel}$ can be replaced by $J_{\parallel} = ne(U_{i\parallel}-U_{e\parallel})$. There appear terms which are proportional to $\nabla J_{\parallel}$, corresponding to so-called hyper-viscosity. In the approximation for usual drift wave fluctuations, this viscosity is found to be small.

Equations (7'),(8') and (9') are approximated by the following ordering. The averaged phase velocity in the parallel direction is greater than the ion thermal speed and smaller than the electron thermal speed, that is $v_i^2 \ll (\omega/k_\parallel)^2 \ll v_e^2$. Perpendicular wave length is larger than ion gyroradius, $k\rho_i < 1$ and the magnitudes of parallel mean flows, $U_{i\parallel}$, $U_{e\parallel}$, are at most ion thermal speed. In this ordering, the electron component decouples from the ion component for the proper choice of the thermodynamical force. By using the flux-friction relationships of the electron parallel momentum and heat flux, $-\langle neE_\parallel B\rangle - \langle neE_\parallel^A B\rangle + \langle BF_{1\parallel}\rangle = 0$ and $-\langle ne(3T_e - \langle T_e\rangle)E_\parallel B\rangle + \langle T_e\rangle\langle BF_{2\parallel}\rangle = 0$, where $E_\parallel^A$ is is the parallel electric field as a thermodynamic force, anomalous bootstrap current and the modification of the plasma conductivity (as well as the anomalous Ware pinch) have been consistently obtained by Shaing in toroidal geometry[25]. The explicit expressions for $F_{1\parallel}$ and $F_{2\parallel}$ are $F_{1\parallel} = \ell_{11}(U_{i\parallel}-U_{e\parallel}) + 2\ell_{12}(Q_{e\parallel}/p_e)/5$, $F_{2\parallel} = -\ell_{21}(U_{i\parallel}-U_{e\parallel}) - 2\ell_{22}(Q_{e\parallel}/p_e)/5$, where $\ell_{11}=nm_e\nu_{ei}$, $\ell_{12}=\ell_{21}=1.5\ell_{11}$, $\ell_{22}=4.66\ell_{11}$ and ν_{ei} is the electron-ion collision frequency. The electron components of our result correspond to the cylindrical version of ref(25), therefore we do not repeat them here.

The ion transport matrix is strongly modified by the presence of $\nabla U_{i\parallel}$, even in the drift-wave approximation. The transport matrix is given in the symmetric form as,

$$\begin{pmatrix} \Gamma_r \\ P_{zr}/m_i v_i \\ Q_r/T_i \end{pmatrix} = \begin{pmatrix} D_{0i} & AV_0 & \mu_{0i} \\ AV_0 & \mu_{0i} & AV_1 \\ \mu_{0i} & AV_1 & \chi_{0i} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \tag{10}$$

$$D_{0i} = \frac{nc^2}{B^2}\Sigma_{\mathbf{k}} \int \frac{d\omega}{2\pi} \frac{I_{11}\Lambda\, \mathrm{Im}Z_i}{\sqrt{2T_{\parallel}/m}\,|k_{\parallel}|} \equiv \frac{nc^2}{B^2}\Sigma_{\mathbf{k}} \int \frac{d\omega}{2\pi}\nu_{k,\omega}$$

$$AV_0 = \frac{nc^2}{B^2}\Sigma_{\mathbf{k}} \int \frac{d\omega}{2\pi}\left(\frac{\omega}{k_{\parallel}V_i}\right)\nu_{k,\omega}, \qquad \mu_{0i} = \frac{nc^2}{B^2}\Sigma_{\mathbf{k}} \int \frac{d\omega}{2\pi}\left(\frac{\omega}{k_{\parallel}V_i}\right)^2$$

$$\chi_{0i} = \frac{nc^2}{B^2}\Sigma_{\mathbf{k}}\frac{-\Lambda'}{\Lambda} \int \frac{d\omega}{2\pi}\left(\frac{\omega}{k_{\parallel}V_i}\right)^4\nu_{k,\omega}, \quad AV_1 = \frac{nc^2}{B^2}\Sigma_{\mathbf{k}} \int \frac{d\omega}{2\pi}\left(\frac{\omega}{k_{\parallel}V_i}\right)^3\nu_{k,\omega},$$

$$X_1 = \left[-\frac{dn_i}{n_i dr} - \frac{qd\Phi}{T_i dr} - \left\langle\frac{\omega}{k_\theta}\right\rangle\frac{qB}{cT_i} + \frac{dT_i}{2T_i dr}\right],$$

$$X_2 = -2(dU_t/dr)/v_i, \qquad X_3 = -(dT_i/dr)T_i,$$

where $d\Phi/dr$ in the force X_1 can be replaced by $(U_tB_p-U_pB_t)/c-\nabla p_i/qn_i$ in the presence of U_p and/or U_t [25,28]. If the mean flows U_p and U_t as well as the mean fluctuation frequency, $\langle\omega\rangle$, are simultaneously zero, then the thermodynamical forces X_1 and X_3 reduce to X_3. It should be noted that the determinant of the transport matrix is positive definite.

We further consider the values of matrix elements. The relative values of μ_{0i} to D_{0i} is $\langle\langle(\omega/k_\parallel v_i)^2\rangle\rangle$, where $\langle\langle\ \rangle\rangle$ is the average over the fluctuation spectrum. The relative value of χ_{0i} to μ_{0i} is also $\langle\langle(\omega/k_\parallel v_i)^2\rangle\rangle$. This value can become appreciably large for a certain spectrum profile. If the spectrum width of the parallel wave number $k_\parallel$ is order of ω/v_i, then the matrix elements, D_{0i}, μ_{0i} and χ_{0i} are the same order. The off-diagonal elements, AV_0 and AV_1 are odd functions of $k_\parallel$, whose contributions are generally expected to be small in the absence of a strong radial inhomogeneity of the spectrum profile. However, if it exists, momentum flux and the heat flux much more strongly couple to each other due to these off-diagonal elements, AV_1, in addition to the viscosity term μ_{0i}. This case is discussed in ref.-[24] in the study of anomalous bootstrap current. In the following arguments, we approximate, for the simplicity, that these off-diagonal contributions of odd functions are negligiblly small.

3. Ion Transport Equations

The results obtained in Sec.2. are used to construct the transport equations. Electron transport equations can be derived from Ref.[25]. We here show the ion transport equations as,

$$\frac{\partial}{\partial t} n_i + \frac{1}{r}\frac{\partial}{\partial r} r\Gamma_{r,i} = S_i \tag{11}$$

$$\frac{\partial}{\partial t} n_i m_i U_t + \frac{1}{r}\frac{\partial}{\partial r} r P_{zr,i} = M_i + M_e \tag{12}$$

$$\frac{3}{2}\frac{\partial}{\partial t} n_i T_i + \frac{1}{r}\frac{\partial}{\partial r} r Q_{r,i} = -P_{zr,i}\frac{\partial U_t}{\partial r} + Q_{ie} \tag{13}$$

where S_i is the particle source, M_i and M_e are the momentum input rates from the source to the ions and electrons, respectively, and Q_{ei} is the collisional energy exchange rate between electrons and ions. Collisions between ions and electrons have been included, however, they canceled in the total momentum equation. Fluxes are obtained from the matrix (10). For the case where we neglect AV_0 and AV_1, we obtain,

$$\Gamma_r = D_{0i}\left[-\frac{dn_i}{n_i dr} - \frac{qd\Phi}{T_i dr} - \left\langle\frac{\omega}{k_\theta}\right\rangle\frac{qB}{cT_i}\right] - (\mu_{0i} - D_{0i}/2)\frac{dT_i}{T_i dr} \quad (14)$$

$$P_{zr,i} = -2m_i n_i \mu_{0i} dU_t/dr \quad (15)$$

$$\frac{Q_{r,i}}{T_i} = \mu_{0i}\left[-\frac{dn_i}{n_i dr} - \frac{qd\Phi}{T_i dr} - \left\langle\frac{\omega}{k_\theta}\right\rangle\frac{qB}{cT_i}\right] - (\chi_{0i} - \mu_{0i}/2)\frac{dT_i}{T_i dr} \quad (16)$$

For actual applications, we need some approximations with respect to the fluctuation spectrum. Further consideration and the comparison study with experiments are left for future. The decay time of the toroidal rotation has been measured in experiment and the observed value is comparable to the energy confinement time[2]. The fact indicates us that the anomalous viscosity, μ_{0i}, is order of the D_{0i} and χ_{0i}.

The transport equations, (11)-(16), show that the existence of the anomalous viscosity affects not only the momentum transport but also the particle and the heat transports. Particle and energy fluxes are driven by the density gradient, the ion temperature gradient, the radial electric field and the convection of the fluctuations. This fact is also true for the electron particle flux[25], provided that the force X_1 of electrons has the different sigh of charge.

As is shown by Eq.(14), the particle inward pinch due to ion temperature gradient becomes small if the value of μ_{0i} is the order of D_{0i}. This is different from the electron flux dependence on the electron temperature gradient, where the anomalous electron viscosity is very small. With respect to the energy flux, the finite value of μ_{0i} reduces the heat conductivity and even causes an anomalous heat pinch effect.

Let us consider the case, as an example, where particle source is located near the plasma periphery. In this case, we have the condition of $\Gamma_i \sim 0$ in the core region. This gives the relation

$$\left[\frac{dn_i}{n_i dr} + \frac{qd\Phi}{T_i dr} + \langle\frac{\omega}{k_\theta}\rangle\frac{qB}{cT_i}\right] = (\mu_{0i} - D_{0i}/2)\frac{dT_i}{T_i dr}$$

from Eq.(14). Substituting this relation into Eq.(16), we have

$$Q_{r,i} = (\chi_{0i} - \mu_{0i}{}^2/D_{0i})\nabla T_i. \qquad (17)$$

The off-diagonal term has large contribution, if the value of μ_{0i}/D_{0i} becomes of the order unity. The effective thermal conductivity, the ratio of $Q_{r,i}$ to ∇T_i becomes smaller than χ_{0i}. The ratio in Eq.(17) becomes minimum when $\chi D \sim \mu^2$ holds. It is noted that the value of $(\chi_{0i} - \mu_{0i}{}^2/D_{0i})$ is positive definite for any spectral profile due to the Schwaltz's inequality, which is consistent with the thermodynamic law.

Anomalous viscosity explicitly affects the ion energy balance equation (13), that is, the viscous damping of the toroidal flow causes the energy input to ions in addition to the exchange rate of electrons, Q_{ie}, and/or the other heating source. If the value of μ_{0i} is the order of χ_{0i}, then this extra input would become the same order of the ion conductive loss.

4. Summary and Discussions

In summary, we present the formula of anomalous ion and electron viscosities due to low frequency microturbulence. In the framework of quasilinear theory, the expressions in electrostatic limit are obtained for the given distribution function, f_0, which is assumed to be close to the maxwellian distribution. Applying the drift wave approximation and estimating the parallel flows observed in present day experiment, the ion anomalous viscosity is found to be large, even if the rotation speed is less than the ion thermal speed. The ion particle flux, the momentum flux, as well as the energy flux are calculated in terms of the spectrum, and are shown as the function of thermodynamical forces, X_1, X_2, and X_3. Particle and energy fluxes are driven by the ion density gradient, the ion temperature gradient, the radial electric field and the convection due to the fluctuations. The electric field in X_1 can be replaced by using the relation, $d\Phi/dr= (U_t B_p - U_p B_t)/c - \nabla p_i/qn_i$ in the presence of U_p and/or U_t.

We also obtain the result that the anomalous electron viscosity is very small, therefore, our result for electron transport reduces to the one obtained by Shaing[25] even in the presence of the toroidal rotation.

The transport equation of ions are presented, including the momentum balance equation in toroidal direction. With respect to the energy flux, the finite value of μ_{0i} reduces the heat conductivity and even causes the anomalous heat pinch effect. Anomalous viscosity explicitly affects the ion energy balance equation (13), that is, the viscous damping of the toroidal flow causes an additional energy input to ions. If the value of μ_{0i} is the order of χ_{0i}, this extra input would become the same order of the ion conductive loss. It is shown that the ion energy flux can be reduced to $(\chi_{0i} - \mu_{0i}{}^2/D_{0i})\nabla T_i$ in the plasma core. The effective conductivity reduces to $\chi_{0i} - \mu_{0i}{}^2/D_{0i}$ due to the finite value of μ_{0i}, and the reduction becomes noticiable when $\chi_{0i} \sim \mu_{0i}{}^2/D_{0i}$ (i.e., $\mu_{0i}/D_{0i} \sim O(1)$). It has been suggested that the density peaking is explained by the large value of Prandtle number, μ_{0i}/D_{0i}[29]. The model theory may also explain the peaked ion temperature profile associated with the peaked profile modes like IOC, if we use the result obtained in this paper.

The transport study by using the assumed values of coefficients, in which the spectrum profile is assumed, and the actual application to the experimental results are left for future. The analysis may reveal the roles of viscosity and the electrostatic potential on the inward particle pinch[29] and the heat pinch.

Present paper deals with the case where the velocity distribution function is not far from the Maxwellian. In the cases of high frequency wave heating as well as of the current drive, the distribution function distorts from the maxwellian. In such a case, the ordering in this paper is no more valid, and the other terms appear. The electron anomalous viscosity may also appear. This kind of analysis should be started from the Fokker-Planck equation. The Fokker-Planck equation in the presence of anomalous transport due to micro fluctuations has been formulated for radio frequency heating cases[30]. The combined study is left for future.

Acknowledgements

The author would like to express sincere gratitude to Drs. K. Itoh, K.C. Shaing and A. Fukuyama for stimulating discussions. Acknowledged are also due to Dr. N. Nakajima for fruitful discussions. Without his suggestion, this work would not be completed. The work is partly supported by Grant-in-Aid for Scientific Research for MoE, Japan.

References

1) M. Greenwald, D. Gwinn, S. Milora et al, Phys. Rev. Lett., **53**, 352 (1984).

2) M.N. Rosenbluth, P.H. Rutherford, J.B. Taylor et al, in *Plasma Physics and Controlled Nuclear Fusion Research*, 1970 (IAEA, Vienna,1971), Vol.1, p495.

3) F.L. Hinton and S.K. Wong, Phys. Fluids **28**, 3082 (1985).

4) S. Suckewer, H.P. Eubank, R.J. Goldston, et al, Phys. Rev. Lett., **43**, 207 (1979).

5) W.M. Stacy,Jr., and D.J. Sigmar, Phys. Fluids **27**,2076 (1984) ; **28**, 2800 (1985).

6) J.W. Connor, S.C. Cowley, R.J. Hastie and L.R. Pan, Plasma Phys. Controlled Fusion **29**, 919 (1987); **31**, 1451 (1989) ; **31**, 1469 (1989).

7) A.A. Ware, Phys. Rev. Lett., **64**, 2655 (1990).

8) O. Gehre and ASDEX Team, Phys. Rev. Lett., **60**, 1502 (1988).

9) R.J. Fonck, R. Howell, K. Jaehnig, et al, Phys. Rev. Lett., **63**, 520 (1989); S.D. Scott, P.H. Diamond, R.J. Fonck et al, Phys. Rev. Lett. **64**, 531 (1990).

10) H. Weisen, M.von Hellermann, A. Boileau, et al, Nucl. Fusion **29**, 2187 (1989) ; K. Ida and JFT-2M gr. private communication.

11) K.H. Burrell, R.J. Groebner, H. St.John and R.P. Seraydarian, Nucl. Fusion **28**, 3 (1988).

12) R. Taylor, M.L. Brown, B.D. Fried, et al, Phys. Rev.

Lett. **63**, 2365 (1989).

13) W. Horton, in *Handbook of Plasma Physics*: Basic Plasma Physics II (Elsevier, New York, 1984), Vol.2, p.383.

14) B.B. Kadomtsev and O.P. Pogutse, in *Plasma Physics and Controlled Nuclear Fusion Research*, 1984 (IAEA, Vienna, 1985), Vol.2, p.69.

15) T. Ohkawa, Plasma Physics and Controlled Fusion **9**, 127 (1985).

16) W.M. Tang, Nucl. Fusion **26**, 1605 (1986).

17) R. Dominguez and R.E. Waltz, Nucl. Fusion **27**, 65 (1987).

18) B.A. Carreras, P.H. Diamond, M. Murakami et al, Phys. Rev. Lett., **50**, 503 (1983).

19) S.P. Hirshman and K. Molvig, Phys. Rev. Lett., **42**, 648 (1979).

20) A. Hasegawa and M. Wakatani, Phys. Fluids **26**, 2779 (1983).

21) P.W. Terry, P.H. Diamond and T.S. Hahm, Phys. Rev. Lett., **57**, 1899 (1986).

22) P.N. Guzdar, C.S. Liu, J.Q. Dong and Y.C. Lee, Phys. Rev. Lett., **57**, 2818 (1986).

23) S.-I. Itoh, J. Phys. Soc. Jpn **51**, 3345 (1982).

24) S.-I. Itoh and K. Itoh, Phys. Lett. **A127**, 267 (1988).

25) K.C. Shaing, Phys. Fluids **31**, 2249 (1988).

26) N. Mattor and P. Diamond, Phys. Fluids **31**, 1180 (1988); W. M. Tang, G. Rewoldt and S.T. Hahm; a talk presented at US-Japan workshop on Comparison of Theoretical and Experimental Transport in Toroidal Systems (Oct. 1989)

Nagoya Japan.

27) T. Tange, S. Inoue, K. Itoh et al, J. Phys. Soc. Jpn. **46**, 266 (1979).

28) R.D. Hazeltine, S.M. Mahajan and D.A. Hichcock, Phys. Fluids **24**, 1164 (1981).

29) S.-I. Itoh, " A Model of Peaked Density Profile and Inward Pinch" NIFS report-8 (National Institute for Fusion Science (1990).

30) S.-I. Itoh, A. Fukuyama and K. Itoh, "Fokker-Planck Equation in the presence of Anomalous Diffusion" NIFS report-29 (1990), submitted to Nucl Fusion.