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S.-I. Itoh and K. Itoh

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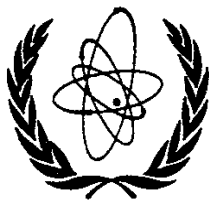
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## Modelling of Improved Confinements

- Peaked Profile modes and H-mode -

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Abstract

Theoretical models on improved confinement with peaked density profile and on H-mode are discussed. A model theory of inward pinch and peaked density profile of ohmic discharges in tokamak is presented. Ion anomalous viscosity in the presence of sheared toroidal rotation causes the drift across the magnetic field. The ratio of viscosity to diffusion coefficient, Prandtl number, determines structures of  $E_r$  and density profile in a stationary state. In viscous plasmas, peaked profiles of both density and rotation velocity are possible without central particle/momentum sources. Resultant rotation of the core relative to the edge is in the counter direction to plasma current. Reduction of edge neutrals can induce further density peaking.

H-mode transition model is further developed. The radial derivative of  $E_r$ ,  $E_r'$ , changes ion losscone loss through squeezing poloidal gyroradius and electron loss by affecting micro-instabilities. Charge neutrality condition predicts cusp type catastrophe in the relation between the gradient and flux at edge. At the transition,  $E_r'$  jumps to negative and fluctuations and anomalous fluxes reduce simultaneously. Combined model predicts that the core density profile is flattened during H-phase.

## 1. Introduction

Recently various improved modes have been found other than H-mode in tokamaks. In H-mode the steep gradient is established only near the edge<sup>1)</sup>. Other improved modes are characterized by peaked density/ion temperature profiles<sup>2,3)</sup>. We first present a model of peaked density profile of the core associated with radial electric field,  $E_r(r)$ , showing that anomalous viscosity in the presence of sheared toroidal rotation induces an inward flow of particles. The extended model of H-mode based on bifurcation of  $dE_r(r)/dr$  associated with poloidal rotation is presented. Combining both models, the effect of edge pedestal to the core profile is discussed.

## 2. Peaked Profile Modes

Density continuity and charge conservation in the presence of the diffusion determines the structure of  $E_r(r)$  and density profile,  $n(r)$ . To obtain  $E_r(r)$ , electron and ion fluxes are assumed to be anomalous due to drift-type microturbulence of frequency,  $\omega$ , and poloidal wave number,  $m$ . Applying quasilinear theory, we have<sup>4)</sup>,

$$\Gamma_{s,a} = -D_s n_s \left[ \frac{n_s'}{n_s} (1 + \alpha_s \eta_s) - \left( \frac{eE_r}{T_s} - \left\langle \frac{r\omega}{m} \right\rangle \frac{eB_t}{cT_s} \right) \right] \quad (1)$$

where  $\eta = d \ln T / d \ln n$ ,  $\alpha_s$ ; numeric and  $\langle \omega r / m \rangle$ ; spectrum averaged phase velocity, suffix shows particle species,  $s$ . Radial ion flux includes an anomalous viscosity flow,  $\Gamma_{i,v} = (qR/Z_e r B) n_i m_i \nabla \mu_{\perp} \nabla U_{\phi}$ , ( $\mu_{\perp}$ : viscosity)<sup>5)</sup> and the momentum loss by charge exchange,

$\Gamma_{i,cx} = (qR/ZerB)n_i m_i U_\phi n_0 \langle \sigma_{cx} v \rangle$ , where the toroidal rotation,  $U_\phi = (qT_i R/ZerB)[n_i'/n_i(1+c_i n_i) - ZeE_r/T_i] + qRU_p/r$ , is derived from the radial momentum balance. The charge conservation equation,  $\Gamma_{e,tot} = \Gamma_{i,a} + \Gamma_{i,v} + \Gamma_{i,cx}$  and the continuity  $\nabla \cdot \Gamma_{e,i,tot} = S_{e,i}$  are basic equations.  $\Gamma_{e,tot} = \Gamma_{e,a}$  is assumed.

In the absence of beam or pellet, particle sources,  $S_{e,i}$ , and  $\Gamma_{i,cx}$  are localized near the edge and we assume that  $S_e = S_i$  ( $S = n_0 n_e \langle \sigma_i v \rangle$ ; where  $\langle \sigma_i v \rangle$  is the ionization rate). We introduce the normalized electric field,  $E (= a^2 e E_r / r T_i)$ , and its shear part  $\{E\} (= E - \text{const})$ . We take  $n=0$  with  $T_e = T_i$  and  $U_p \rightarrow 0$ .  $D, \mu_i$  and  $q$  are set to be numeric. In the core, where  $n_0 \neq 0$ ,  $\Gamma_{e,tot} = 0$  gives the relation  $-2(\ln n)' = \{E\} + C_1$  in  $y (= r^2/a^2)$  coordinate.  $C_1 (\propto \langle \omega/m \rangle)$  is assumed to be constant, which is determined by boundary conditions. The condition,  $\Gamma_{e,tot} = \Gamma_{i,tot}$  gives,

$$-v_a \{E\} + v_\mu \frac{1}{y} \frac{d}{dy} \{E\} = 0 \quad (2)$$

where  $v_\mu = (1/Z+1)q^2 R^2 \mu_i / a^4$ ,  $v_a = (1+Z)D_i / \rho_i^2$  and  $\rho_i$  is toroidal gyro-radius.\*\*

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\*\*footnote: If we evaluate  $\langle \omega/m \rangle$  by the local dispersion relation, then we have  $\{E\} = -2C_2(\ln n)'$  ( $C_2 = 1/(1-\Lambda_0)$ ,  $\Lambda_0 = I_0 e^{-b}$ ,  $b = k_i^2 \rho_i^2$ ,  $I_0$  being the 0-th order modified Bessel's function) and  $v_\mu$  is redefined as  $(C_2/Z+1)q^2 R^2 \mu_i / a^4$ . The qualitative nature of the solution in the following does not change.

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The solution is given as,  $\{E\} = A_1 I_0(\sqrt{v_a/v_\mu} y)$ , and  $\{E\} + C_1 = -$

$2(\ln n)'$ . The profile of  $E_r$  is dictated by the parameter  $v_\mu/v_\alpha$ , which indicates the diffusion Prandtl number<sup>6</sup>). When the higher order derivative of the pressure is neglected,  $v_\mu/v_a$  reduces to  $(\mu_i/D_i)(q^2R^2\rho_i^2/a^4)$ .

Neutrals in the outer region ( $a-\Delta < r < a$ ,  $\Delta$ : penetration length of neutrals) affect the momentum balance. For the boundary condition at  $r=a$ ,  $dn/dr/n|_{r=a} = 1/\lambda_n$ , we obtain

$$\{E\} = \left\{ \left( \frac{a}{\lambda_n} - \frac{a^2 \langle S \rangle}{2D_e} \right) \Big|_{r=a} - C_1 \right\} \frac{I_0(\sqrt{v_a/v_\mu} r^2/a^2)}{I_0(\sqrt{v_a/v_\mu})} \quad (3)$$

in the limit of  $\Delta \ll a$  where  $\langle S \rangle = \int n_e n_0 \langle \sigma_i v \rangle dy / n_e y$ .

We plot  $n(r)$  for various values of  $v_\mu/v_a$  in Fig.1a. For fixed line averaged density,  $v_\mu/v_a$  is changed from 0.01 to 100.  $A_0 = (a/\lambda_n - C_1 - a^2 \langle S \rangle / D_e)$  is chosen to be 10. The more viscous plasma has the more peaked profile. Viscous frictional force to the toroidal rotation and the poloidal magnetic field causes the FxB drift of ions. Only if the rotation flow is counter to the current direction the inward drift occurs. The force differently acts on electrons, and  $E_r$  piles up. Electrons also drift inward. The peaked profile can be thus sustained without particle source. The peaking is expected if  $v_\mu/v_a$  is large, i.e., if  $\mu_i/D$ ,  $R/a$ ,  $\rho_i/a$  is large or  $I_p$  is small. Radial profile of toroidal rotation is given by  $U_\phi/v_{Ti} = (\rho_p/a)A_1 I_0(\sqrt{v_\mu/v_a} y) + \text{const.}$  and is shown in Fig.1b. We plot the value of  $(U_\phi - U_\phi(a-\Delta))/v_{Ti} A_0 (\rho_p/a)$ . The rigid rotation part is determined by the boundary condition. Associated with the density peaking, the rotation profile also

peaks. We show the effect of boundary condition on  $n(r)$  in Fig.1c. As  $A_0$  becomes large, the density peaks. When gas-puff is reduced, edge source is expected to become small. The reduction of  $\langle S \rangle$  reduces the momentum loss of the rotation, which can increase the density gradient at  $r=a-\Delta$ . And  $A_0$  rises if  $\lambda_n$  remains similar. This causes the peaking of core density. The SOC/IOC transition<sup>2)</sup> is attributed to this mechanism.<sup>7)</sup> When  $v_u/v_a$  is small (less viscous), the reduction of  $\langle S \rangle$  is found to have little effect to cause density peaking. Actual value of  $v_u/v_a$  is not known, however, if we apply a quasilinear theory to drift wave fluctuations<sup>5)</sup> this ratio becomes order of unity to 10. Recent measurement of toroidal rotation<sup>8)</sup> supports this estimate.

### 3. H-mode

We consider the edge of circular plasma. Particle flux can be bipolar near edge<sup>9)</sup>. The relation  $\Gamma_{e,NA} = \Gamma_{i,NA}$  determines the ambipolar electric field,  $E_r$ . The bipolar flux of ions  $\Gamma_{i,NA}$  comes from losscone loss. In the presence of  $E_r'$ , the banana width changes by the factor  $G \equiv 1/\sqrt{1-u_g + C\epsilon}$ <sup>10,11)</sup>, where  $u_g = \rho_p E_r' / v_{Ti} B_p$ . Taking  $u_g$  corrections, we have  $\Gamma_{i,NA} = (F n_i v_i \rho_p G / \sqrt{\epsilon}) \exp[-X^2]$ , where  $v_i$  is the ion-ion collision frequency,  $X = \rho_p e E_r / T_i$  and  $F$  is a numerical coefficient ( $\sim 0(1)$ ).

The microscopic mode stability is affected by  $E_r'$ . The collisionless trapped particle drift instability, for example, is stabilized if the condition  $u_g < -u_c$  is satisfied, ( $u_c^2 = 8\sqrt{2\epsilon}(4-u_c)$ ). We employ an analytic estimation of the growth rate and transport

coefficient as  $\gamma \sim \gamma_0 \sqrt{1+u_g/u_c}$  and  $D_e = D_{e0} \sqrt{1+u_g/u_c}$ , where suffix 0 denotes cases of  $E_r' = 0$ . The bipolar electron flux near edge,  $\Gamma_{e,NA}$ , being originated from the convection of excited waves<sup>9)</sup>, is  $\Gamma_{e,NA} = n_e D_{e0} \sqrt{1+u_g/u_c} (\lambda - X)$ ,  $\lambda = -(T_e T_i)^{\rho_p} (n_e'/n_e + \alpha_{T_e}'/T_e + eBr/T_e \langle \omega/m \rangle)$ . Equating  $\Gamma_{e,NA}$  and  $\Gamma_{i,NA}$ , we have the refined equation, (  $d \equiv \sqrt{\epsilon} D_e / v_i F \rho_p^2$  )

$$d \sqrt{1+u_g/u_c} (\lambda - X) = G \exp[-X^2]. \quad (4)$$

The poloidal bulk viscosity gives the relation as  $U_p/v_{Ti} \propto \rho_i \nabla T_i / T_i$ <sup>12)</sup>, i.e.,  $X = -\lambda_i - U_\phi/v_{Ti}$ , where  $\lambda_i = -\rho_p (n_i'/n_i + c_i T_i'/T_i)$  and  $c_i$  is a numerical coefficient (O(1)). Since the toroidal flow in the scrape-off layer is damped<sup>13)</sup>, we neglect  $U_\phi/v_{Ti}$  and have  $X = -\lambda_i$ . The solution is obtained only in the negative  $E_r$  region.

In Fig.2, the solution of  $E_r'$ , the relative transport coefficient and the resultant flux are shown for  $\lambda = \lambda_i$ . At the threshold value of  $\lambda$ ,  $\lambda_{c1}$ ,  $E_r'$  jumps from positive to negative, and  $D_e/D_{e0}$  and  $\Gamma/\Gamma_0$  show transitions to smaller values.

Combining the models in Sec.2 and 3, we finally note the effect of the H-mode to the core density profile. The transport barrier effectively increases  $\langle S \rangle / D_a$  ( i.e., local diffusion is reduced ) and reduces  $A_0$ . Profile of the density and  $U_\phi$  of the core plasma are flattened owing to the decrease of  $A_0$ . The reduced momentum loss increases the rigid toroidal rotation.

#### 4. Summary and Discussions

We have presented a model of the improved confinement with peaked density profile and that of the H-mode transition.



Inward pinch induced by the ion viscosity is studied; it is predicted that the peaking occurs by the reduction of the edge source if the values of  $v_{\mu}/v_a$  is large enough. This mechanism explains SOC/IOC transition. The bifurcation of  $E_r'$  at the edge is predicted. The anomalous diffusivity and the associated flux reduce simultaneously.  $E_r'$  turns to negative and the negative  $E_r$  becomes more negative. Combining these models, the core density profile is predicted to be flattened in H-mode.

The inward pinch is studied in the absence of the external momentum source and  $\nabla T_i$ . The extension of the present model to the various peaked profile modes are left for future study.

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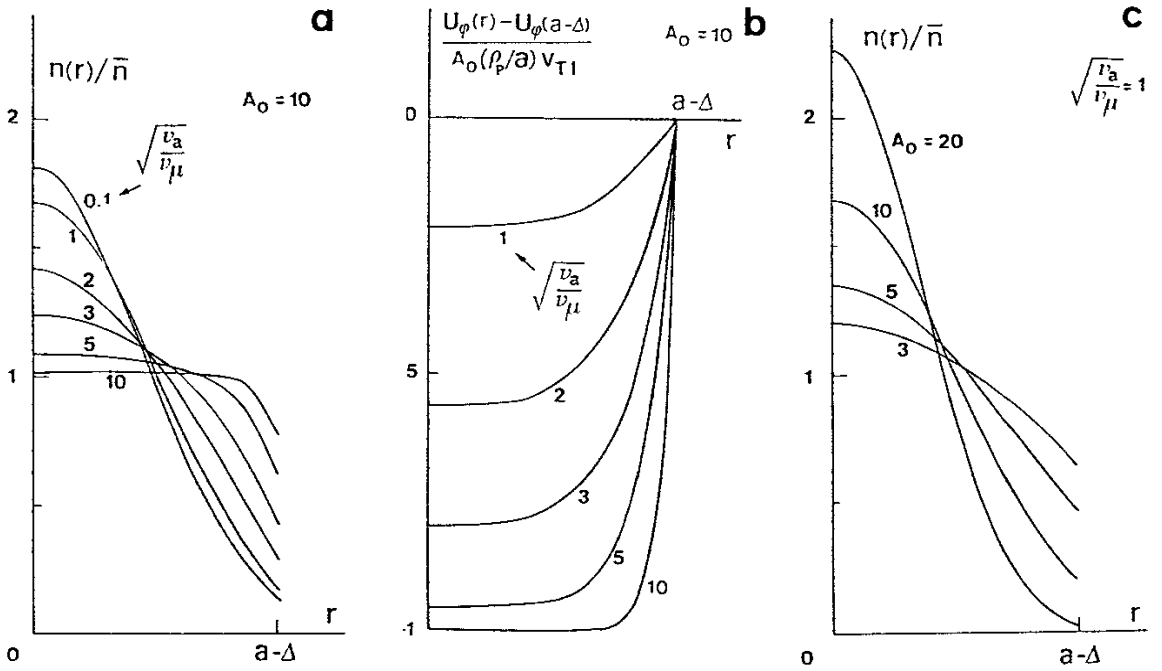


Fig. 1 Density profiles for various values of  $\sqrt{v_a/v_\mu}$  (from 0.1 to 10) (1a), and flow velocity ( normalized to  $v_{T1}\rho_p/a$  ) (1b) for fixed value of  $A_0 (=10)$  (1a). Density profiles for various values of  $A_0$  ( 3, 5, 10, 20 ) for the fixed values of  $v_a/v_\mu (= 1)$  (1c).

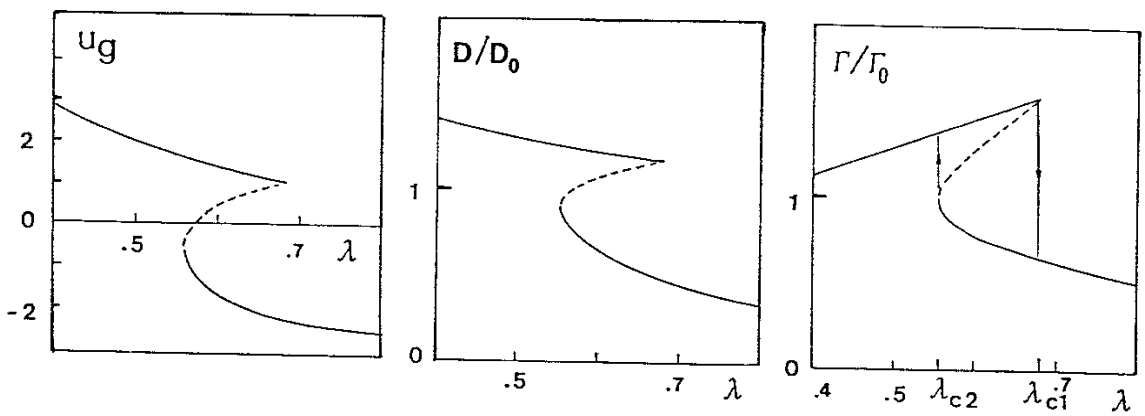


Fig. 2 Radial gradient of  $E_r(u_g)$ , transport coefficient ( $D_e/D_{e0}$ ) and flux ( $\Gamma/\Gamma_0$ ) are shown as a function of  $\lambda$ . In the region of  $\lambda_{c2} < \lambda < \lambda_{c1}$ , solutions become multi-valued. Transition takes place at  $\lambda = \lambda_{c1}$  (L $\rightarrow$ H) and  $\lambda = \lambda_{c2}$  (H $\rightarrow$ L).

Key words : Peaked profile, Inward pinch, H-mode, Improved mode  
Radial electric field