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EQUILIBRIUM BETA LIMIT AND ANOMALOUS TRANSPORT STUDIES OF HELICAL SYSTEMS

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EQUILIBRIUM BETA LIMIT AND ANOMALOUS TRANSPORT STUDIES OF HELICAL SYSTEMS

ABSTRACT

Equilibrium magnetic surfaces and anomalous transport of helical systems are studied. The equilibrium beta limit, which is defined by the breaking of magnetic surfaces due to the finite beta effect, is investigated for two types of helical systems. The results indicate that the breaking often imposes severer limitation on beta than the Shafranov shift. However, if we properly choose controllable parameters, such as the vertical field B_v, we can obtain a high beta equilibrium, such as $\bar{\beta} \geq 5\%$. Furthermore, a simple method is proposed, by which the breaking can be actively suppressed and fairly high beta equilibria with clearly nested magnetic surfaces can be realized. In order to study anomalous transport based on the fluid type turbulence in heliotron/torsatron six-field equations are derived from the two fluid model. A new type of the η_i mode coupled to the g mode is found in the presence of unfavarable magnetic curvature. For the drift resistive interchange turbulence the Botzmann relation between the density and the potential fluctuation spectrum is observed in the weak collisional regime.

We discuss two physical aspects of toroidal helical systems; one is on the breaking of magnetic surfaces in finite beta equilibria and on methods to suppress it (Part A), and the other is on anomalous transport (Part B).

PART A EQUILIBRIUM BETA LIMIT OF HELICAL SYSTEMS AND SUPPRESSION OF MAGNETIC SURFACE BREAKING

(T.HAYASHI, A.TAKEI, N.OHYABU, T.SATO)

1.Introduction

The equilibrium beta limit of a helical system is conventionally defined by the amount of the Shafranov shift, such as $\Delta_s(\beta) < \frac{a_p}{2}$. However, we must care about the breaking of magnetic surfaces due to the finite pressure effect, since it is well known that a non-axisymmetric toroidal finite beta equilibrium does not necessarily regularly nest magnetic surfaces. The boundary region of a helical system is ergodic even in a vacuum field. Therefore, the physical issue in this paper is to investigate how large the boundary ergodic region expands in a finite beta equilibrium and to look for methods to suppress it.

The origin of the appearance of magnetic islands in a finite beta equilibrium of toroidal helical systems is attributed to the plasma current which is induced to satisfy the equilibrium force balance condition $\mathbf{j} \times \mathbf{B} = \nabla p$. The resonant field, which can be produced by the plasma current for the case of nonaxisymmetric torus, causes the appearance of magnetic islands inside the plasma. When an island is induced, the pressure profile is significantly modified near the island. Thus, a consistent analysis between the $\mathbf{j} \times \mathbf{B} = \nabla p$ condition and the island formation is required.

In order to analyze quantitatively the total 3D effect of the plasma current on rational surfaces, we have developed a 3D equilibrium code (HINT).[1][2] In the following, we show results of HINT on Heliotron/Torsatron and Helias configurations and propose a simple method to suppress the breaking.

2. l=2 Heliotron/Torsatron

In order to understand the general tendency of the "fragility" of magnetic surfaces in a finite beta equilibrium, we have executed parameter survey for several kinds of physical parameters. The M (pitch period number) dependency of the breaking of magnetic surfaces is shown in Fig.1(a) for the l=2 heliotron configuration. This survey was made under conditions that the pitch parameter $\gamma_c \equiv \frac{M}{l}\frac{a_c}{R_c}$ is fixed to be 1.3, the vacuum magnetic axis is at the helical coil center, external quadrupole component $B_q = 0$, and the helical coil has no modulation. The broken line in Fig.1 indicates a tentative beta limit at which the outer region of about 30 % of the minor radius becomes ergodic. In general, the breaking has a tendency to be suppressed as M increases. For low M (low aspect ratio) configurations, however, we find that the breaking can be improved by properly choosing several free parameters.[2]

Figure 1(b) shows the effect of the external vertical field B_v, which controls the radial position of the vacuum magnetic axis. As is shown in Fig.1(b), the inward shift of the magnetic axis is favorable to suppress the breaking, and in fact, we can obtain the high beta equilibrium (such as $\bar{\beta} \geq 5\%$) keeping clearly nested surfaces by a small inward shift for the $M = 10$ configuration. As for the effect of B_q, vertically elliptic shaping of the surfaces is favorable to suppress the breaking. Another survey indicates that the breaking is significantly improved by decreasing $\gamma_c(= 1.2)$, or is also improved by making the helical coils positively modulated.

3. Helias

The Helias configuration [3], in contrast with the heliotron configuration, has a very low shear, but the Pfirsch-Schlüter current is optimized to be small. The island size induced by the finite pressure effect is determined by the competition between these effects. The way islands appear for the Helias is different from that for the heliotron case; the number of relevant dangerous rational surfaces is much smaller, but the size of islands is much bigger when they appear. The results indicate that the dangerous rational surface, that is the $\iota = 5/6$ surface, comes into the plasma region as beta increases, and m=6 islands are formed on the surface. However, the size of the island is not so serious as to destroy the whole plasma. When we further increase beta, the 5/6 surface is again removed away from the plasma region, and a high beta equilibrium with clear surfaces can be achieved. In this process, it turns out that the pressure profile plays an important role: A broader profile is favorable. This is because the magnetic axis shift becomes smaller and the change of iota profile is smaller, whereby the number of relevant dangerous rational surfaces can be suppressed.

4. Suppression of breaking by adding a simple extra coil

As is stated above, the axisymmetric external poloidal fields, such as B_v, can be used to suppress the breaking of surfaces. One problem of this method, however, is that such an external field significantly changes the physical properties of the configuration, such as the well depth, and usually the stability is deteriorated when the breaking is improved. Here we propose another way to suppress the breaking. Islands which appear on a rational surface whether in a vacuum field or in a finite beta field of a heliotron configuration have the following empirical properties; 1) the island size is noticeably larger on the outer side of the torus, and 2) islands appear in phase at the outside of the torus when islands are induced on several rational surfaces simultaneously. By taking advantage of these properties, we can devise a set of simple extra coils enough to suppress induced islands. This method is studied by using the Cary-Hanson technique [4] to measure the island size.

Figure 2(a) shows an example of induced islands which appear in a finite beta equilibrium ($\bar{\beta} \sim 4\%$) of a $M = 10/l = 2$ heliotron configuration. As is shown in Fig.2(b), these islands are clearly suppressed by adding an extra coil, so that clear magnetic surfaces are recovered in the outer ergodic region as is seen in Fig.2(a). It is interesting to note that the required extra coil current is only about 3% of the helical coil current. One important advantage of this method is that the physical properties, such as the well depth and the ι profile, are very slightly changed when the extra coil field is imposed. Thus, this method can provide an efficient and powerful way to remedy magnetic islands for any reasonable beta value and simultaneously can improve the equilibrium beta limit.

PART B FLR-MHD EQUATIONS AND ANOMALOUS TRANSPORT STUDIES OF HELIOTRON/TORSATRON (M.WAKATANI, H.SUGAMA, M.YAGI, K.WATANABE, B.G.HONG, W.HORTON)

We derived FLR(Finite Larmor Radius)-MHD equations by retaining the FLR effects through the collisionless viscosity of ions and using the stellarator expansion from the two fluid mode. The resultant six-field equations describe almost all fluid type instabilities in heliotron/torsatron.

The FLR-MHD equations are written in the toroidal coordinates (r, θ, ξ) by using the Poisson bracket,

$$n_0 m_i \left(\frac{\partial}{\partial t} \nabla_\perp^2 F + [F, \nabla_\perp^2 F] \right) - \frac{1}{\omega_{ci}} \nabla_\perp \cdot [P_i, \nabla_\perp F]$$
$$= \frac{1}{c} B_0 \nabla_\parallel J_\parallel + \nabla(P_e + P_i) \times \nabla\Omega \cdot \hat{\xi}, \tag{1}$$

$$\frac{1}{c}\frac{\partial A}{\partial t} = -\nabla_\parallel \left(\phi - \frac{P_e}{n_0 e} \right) - \eta_\parallel J_\parallel, \tag{2}$$

$$n_0 m_i \left(\frac{\partial v_\parallel}{\partial t} + \frac{c}{B_0}[\phi, v_\parallel] \right) = -\nabla_\parallel (P_e + P_i), \tag{3}$$

$$\frac{\partial}{\partial t}\left(\frac{n}{n_0} - \frac{B_\beta}{B_0} \right) + \frac{c}{B_0}\left[\phi - \frac{P_e}{n_0 e}, \frac{n}{n_0} - \frac{B_\beta}{B_0} \right]$$
$$= -\frac{c}{B_0}\left[\phi - \frac{P_e}{n_0 e}, \Omega \right] - \nabla_\parallel \left(v_\parallel - \frac{J_\parallel}{n_0 e} \right) - \frac{\eta_\perp c^2}{4\pi} \nabla_\perp^2 \frac{B_\beta}{B_0}, \tag{4}$$

$$\frac{\partial}{\partial t}\left(\frac{P_e}{\gamma_e P_{e0}} - \frac{n}{n_0}\right) + \frac{c}{B_0}\left[\phi - \frac{P_e}{en_0}, \frac{P_e}{\gamma_e P_{e0}} - \frac{n}{n_0}\right] = 0, \tag{5}$$

$$\frac{\partial}{\partial t}\left(\frac{P_i}{\gamma_i P_{i0}} - \frac{n}{n_0}\right) + \frac{c}{B_0}\left[\phi + \frac{P_i}{en_0}, \frac{P_i}{\gamma_i P_{i0}} - \frac{n}{n_0}\right] = 0, \tag{6}$$

where $F = \frac{c}{B_0}(\phi + \frac{P_i}{n_0 e})$, $P_e + P_i = -B_0 B_\beta/4\pi$, $J_\parallel = -c\nabla_\perp^2 A/4\pi$, $\nabla_\parallel = \partial/\partial\xi + (\nabla\psi \times \hat{z}/B_0)\cdot\nabla$, $\psi = A + \overline{\nabla\Phi \times \nabla \int_0^{\xi} d\tilde{\xi}\, \Phi \cdot \hat{\xi}}$ and $\Omega = 2r\cos\theta/R_0 + \overline{|\nabla\Phi|^2}/B_0^2$. Here $B_h = -\nabla\Phi$ denotes external stellarator fields and the bar means the average over one field period length in the ξ direction. Also P_{e0}, P_{i0}, n_0 and B_0 are constant and γ_e and γ_i are specific heat ratio for electrons and ions. Equations (1) $\sim$ (6) for six variables $\{F, A, v_\parallel, n, P_e, P_i\}$ have the energy conservation relation,

$$\frac{d}{df}\int dV\left\{\frac{n_0 m_i}{2}(v_\parallel^2 + |\nabla_\perp F|^2) + \frac{1}{8\pi}\left((B^\beta)^2 + |\nabla_\perp A|^2\right) + \frac{1}{2}\frac{(P_e + P_i)^2}{\gamma_e P_{e0} + \gamma_i P_{i0}}\right\}$$
$$= -\int dV\left\{\eta_\parallel J_\parallel^2 + \eta_\perp(\frac{c}{4\pi}|\nabla_\perp B^\beta|^2)\right\}. \tag{7}$$

We studies the following instabilities which are candidates to explain the edge turbulence and the anomalous transport in Heliotron E:

(i) drift resistive interchange mode by using the two-field equations for $\{\phi, n\}$ in the electrostatic limit with assumptions of $v_\parallel \to 0$, $P_i \to 0$, $\beta_e = 8\pi P_e/B_0^2 \to 0$, $P_e = nT_{e0}$ and T_{e0} =const.[5]

(ii) η_i modes by using the five-field equations for $\{\phi, A, n, v_\parallel, P_i\}$ with an assumption of $P_e = nT_{e0}$ or the four-field equations for $\{\phi, n, v_\parallel, P_i\}$ in the electrostatic limit.[6], [7]

For the case (i) the unstable modes localized at the mode resonant surface with growth rate $\gamma \propto \eta_\parallel s^{-2}$ and $\omega \simeq \omega_g$ were obtained in the semi-collisional regime, which is different from the g mode in the RMHD, $\gamma \propto \eta_\parallel^{1/3} s^{-2/3}$, where ω_g is a curvature drift frequency and s is a shear parameter. Figure 1 shows poloidal mode number spectra of density and potential fluctuations for large $\eta_\parallel$ (right side) and small $\eta_\parallel$ (left side) cases obtained by the nonlinear calculation in the cylindrical geometry.[5] For the small $\eta_\parallel$ case, $\tilde{n}/n_0 \simeq e\tilde{\phi}/T_e$ or the Boltzmann relation is seen and the particle transport decreases significantly compared to the large $\eta_\parallel$ case. This implies that the particle transport increases toward the edge according to the increase of $\eta_\parallel$ or deviation from the Boltzmann relation.

Figure 2 shows the linear growth rate of η_i mode in the cylindrical plasma model of Heliotron E [5] as a function of $\eta_i \equiv d\ln T_i/d\ln n_0$. When ν_e or $\eta_\parallel$ is finite, the $\ell = 0$ mode is dominant and its growth rate is larger than the collisionless limit, where ℓ is a radial mode number. For the finite ν_e the η_i mode couples to the g mode and an anomalous ion thermal transport is predicted by the mixing length theory.

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Figure Captions

Fig.1 (a) M dependency and (b) B_v control of the breaking of magnetic surfaces for $l = 2$ heliotron configuration. Lines indicate a tentative beta limit at which the outer region of about 30 % of the minor radius becomes ergodic.

Fig.2 Magnetic surfaces of $l = 2$ heliotron configuration (a) before and (b) after the suppression of magnetic islands by use of the proposed simple extra coil.

Fig.3 Poloidal mode number spectra of $E^n_m = \sum_n \int |\tilde{n}_{mn}|^2 dV$ and $E^\phi_m = \sum_n \int |\tilde{\phi}_{mn}|^2 dV$ for small $\eta_\parallel(\nu_{ei}/\Omega_e = 2.1 \times 10^{-4})$ and large $\eta_\parallel(\nu_{ei}/\Omega_e = 1.05 \times 10^{-2})$, where Ω_e is an electron cyclotron frequency.

Fig.4 Growth rate of η_i mode with $(\mathrm{m}, \mathrm{n}) = (1, 1)$ for $\nu_{ei}/\Omega_e = 0$ and $\nu_{ei}/\Omega_e = 5 \times 10^{-4}$. ℓ is a radial mode number.

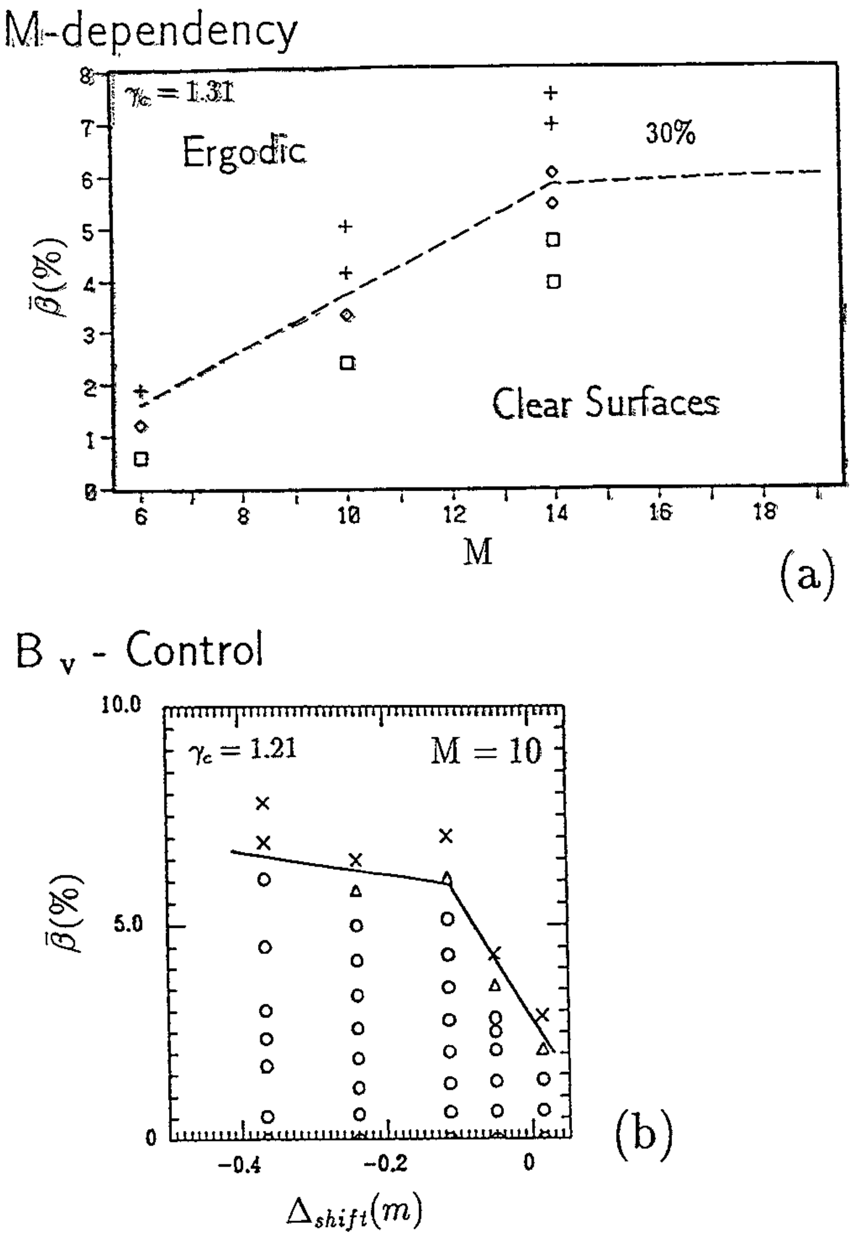

M-dependency
$\gamma_c = 1.31$
Ergodic
30%
Clear Surfaces
$\bar{\beta}(\%)$
M
(a)
B v - Control
$\gamma_c = 1.21$
M = 10
$\bar{\beta}(\%)$
$\Delta_{shift}(m)$
(b)

Fig.1

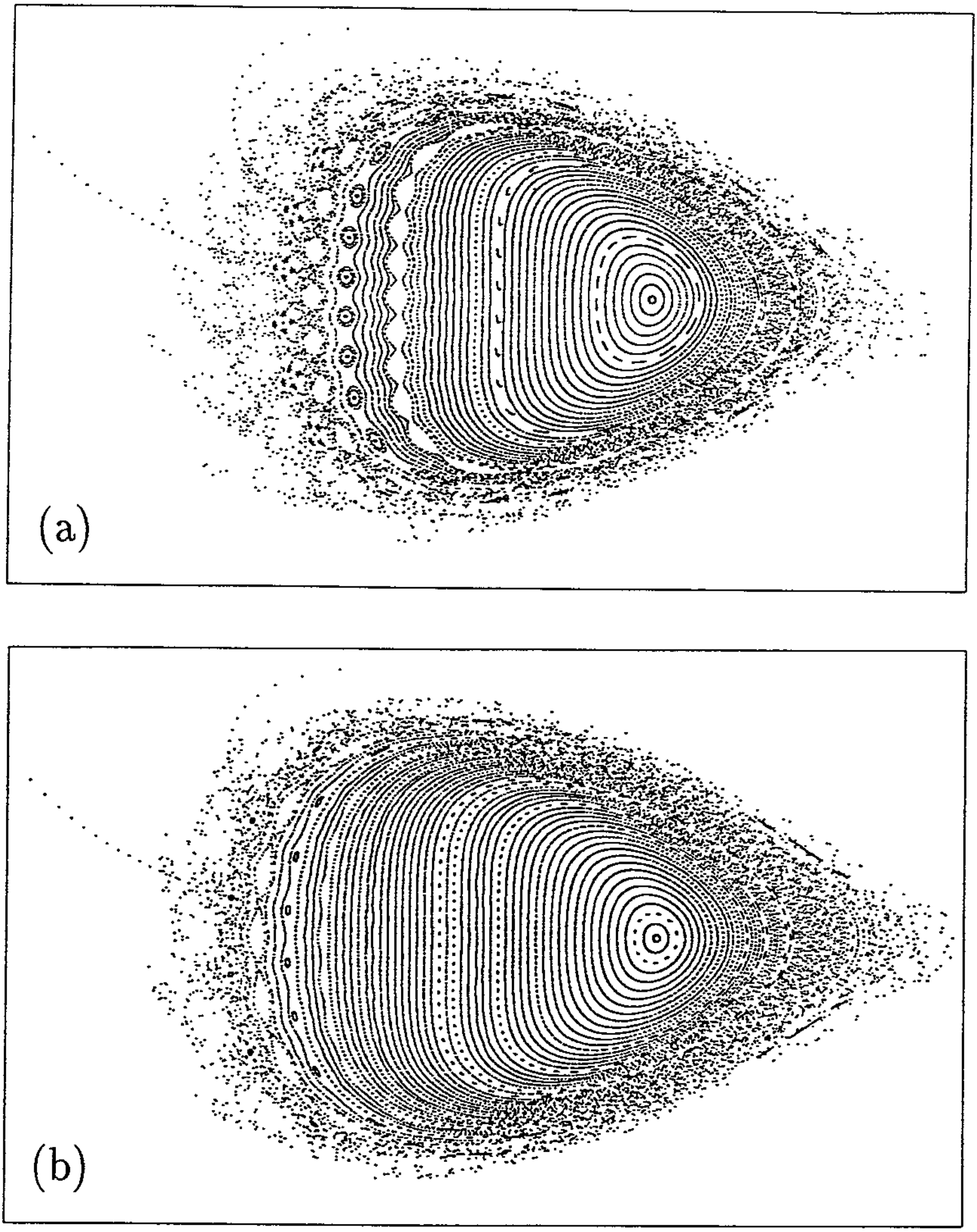


Fig.2

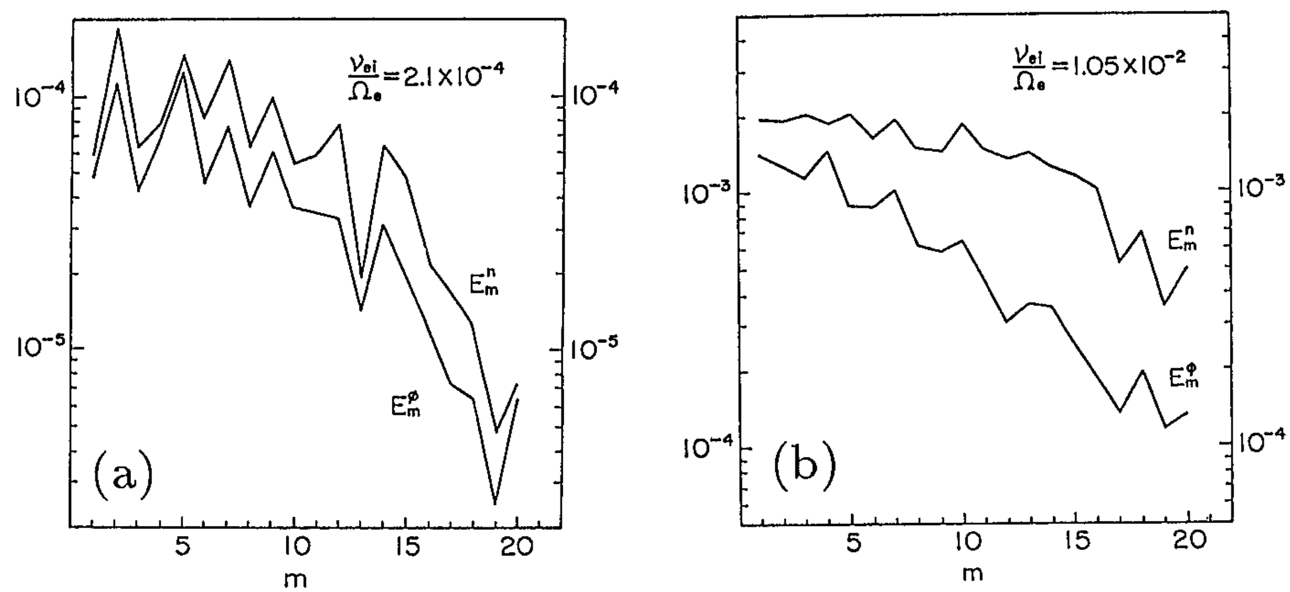
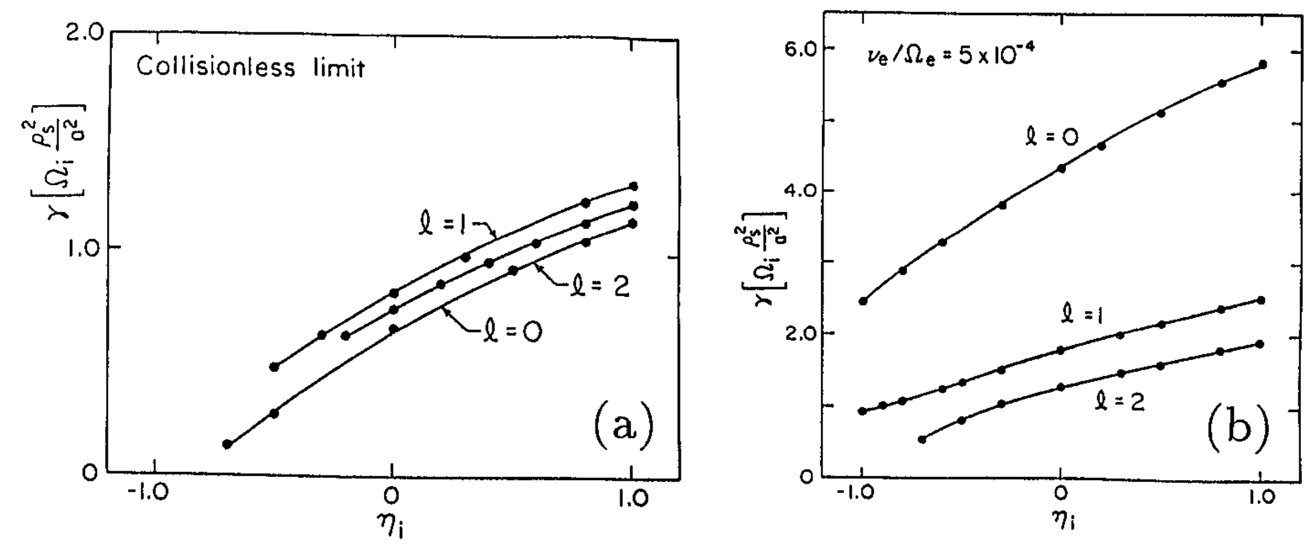

$\frac{\nu_{ei}}{\Omega_e}=2.1\times10^{-4}$
10^{-4}
10^{-5}
E_m^n
E_m^ϕ
(a)
5
10
15
20
m
$\frac{\nu_{ei}}{\Omega_e}=1.05\times10^{-2}$
10^{-3}
10^{-4}
E_m^n
E_m^ϕ
(b)
m

Fig.3



Collisionless limit
ℓ = 1
ℓ = 2
ℓ = 0
(a)
$\gamma\left[\Omega_i \frac{\rho_s^2}{a^2}\right]$
η_i
2.0
1.0
0
-1.0
0
1.0
$\nu_e/\Omega_e = 5\times10^{-4}$
ℓ = 0
ℓ = 1
ℓ = 2
(b)
$\gamma\left[\Omega_i \frac{\rho_s^2}{a^2}\right]$
η_i
6.0
4.0
2.0
0
-1.0
0
1.0

Fig.4