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SELF-ORGANIZING MAGNETOHYDRODYNAMIC PLASMA

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Abstract

In a resistive magnetohydrodynamic (MHD) plasma, both the magnetic energy and the magnetic helicity dissipate with the resitive time scale. When sufficiently large free magnetic energy does exist, however, an idial current driven instability is excited whereby magnetic reconnetction is driven at a converging point of induced plasma flows which does exist in a bounded compressible plasma. At a reconnection point excess free energy (entropy) is rapidly dissipated by ohmic heating and lost by radiation, while magnetic helicity is completely conserved. The magnetic topology is largely changed by reconnection and a new orderd structure with the same helicity is created. It is discussed that magnetic reconnection plays a key role in the MHD self-organization process.

Keywords

relaxation, self-organization, computer simulation, magnetohydrodynamics, magnetic reconnection, magnetic helicity

1. Introduction

Should the second thermodynamic law reign over any system, how dull would be the universe. Or, we should say, there would exist no universe of our understanding. Since entropy can decrease locally and transiently, ordered structures like galaxies, stars and planets are formed here and there in the universe and ordered organic creatures like human beings are also created on a planet like earth. These facts indicate that the law of entropy increase can at least locally be violated under certain conditions.

Suppose a local system where an excess free energy is stored and entropy can be thrown away. Then, an ordered structure can be created, at least, transiently. Though very vague, self-organization can be defined as a dynamical process in which an ordered structure can be created in an open, local system where disorder is nonlinearly thrown away.

The magnetohydrodynamic plasma is a good system to study the self-organizing process. We start with showing one typical example of self-organizing plasmas which is obtained by a three-dimensional magnetohydrodynamic(MHD) simulation. Then, by carefully examining what is occuring in the dynamical process, we try to extract key physical processes governing highly tangled nonlinear self-organizing process and come up with a unified theoretical model of magnetohydrodynamic self-organization.

2. Typical Example of MHD Self-Organization

Fig. 1 shows three snapshots of a self-organizing magnetohydrodynamic plasma [1]. We take a cylinder with rectangular cross section in which a magnetized plasma is filled. We assume periodicity along the axis (z) and that plasma is confined by a perfect conductor with a toroidal system in mind.

The initial condition is a force-free equilibrium. Neither plasma flow nor pressure exists in the initial state. Furthermore, we make an important assumption that ohmically heated plasma is instantaneously cooled by radiation loss. This is equivalent to assuming that entropy produced during the dynamical process is thrown away from the system. We note that in an actual magnetically confined fusion device the pressure is usually small compared with the magnetic energy (low β approximation) and that heated plasma can be cooled by radiation loss. Therefore, our assumption is not so artificial but acceptable.

The initial force-free equilibrium includes free magnetic energy because plasma itself carries currents. In the present case, the free magnetic energy is sufficiently large that an ideal current-driven (helical kink) instability arises. Accordingly, plasma (magnetic field configuration) is deformed. Because of resistivity, whatever small it is, magnetic reconnection is driven at places where antiparallel field condition is realized. At reconnection points where antiparallel field condition is satisfied currents are largely enhanced because of $\mu_0 \boldsymbol{J} = \nabla \times \boldsymbol{B}$. Therefore, free magnetic energy is strongly dissipated there due to enhanced ohmic dissipation $\eta \boldsymbol{J}^2$. Simultaneously reconnection changes magnetic topology.

If there exists a minimum energy magnetic field configuration other than the vacuum field configuration, it is expected that the system realizes a new ordered magnetic field configuration with the minimum energy.

Fig. 1 depicts a trace of a constant axial (toroidal) magnetic field intensity. The upper panel shows the initial equilibrium configuration, the second an intermediate configuration where the initial magnetic structure is torn in peaces at converging points of plasma flows by reconnection driven by an ideal helical kink instability. The bottom panel shows a clear helical structure which is established after disordered magnetic energy is dissipated by ohmic heating which in turn is thrown away from the system by radiation loss.

Summing up, we can say that a simple ordered structure (a helical structure in this case) is created from a once disordered structure (middle panel) when the entropy produced during the process is swept out from the system.

3. Characteristics of MHD Self-Organization

The above example suggests that the magnetohydrodynamic self-organization requires :

1) Existence of a sufficient free (magnetic) energy that can drive a current-driven (helical kink) instability.

2) Plasma flows induced by the current-driven instability drive reconnection at their converging points whereby magnetic topology is changed.

3) Disordered magnetic energy (entropy) is swiftly removed. Actually, disordered magnetic energy is transferred into thermal energy through ohmic heating enhanced by driven reconnection which, in turn, is removed from the system by radiation. This process takes place in the magnetohydrodynamic time scale rather than the resistive time scale.

Let us investigate more elaborately the self-organization process by carefully examing the evolutions of physical quantities. Fig. 2 shows the time evolutions of the magnetic energy W and the magnetic helicity K which are defined by

$$W = \int_V \frac{\boldsymbol{B} \cdot \boldsymbol{B}}{2\,\mu_0} dV \tag{1}$$

$$K = \int_V \frac{\boldsymbol{A} \cdot \boldsymbol{B}}{2\,\mu_0} dV \tag{2}$$

where $\nabla \times \boldsymbol{A} = \boldsymbol{B}$ ($\boldsymbol{A}$: vector potential). Note that the initial state at this example is different from that of Fig. 1. As one sees in this figure, the helicity decreases slowly and smoothly, while the energy exhibits a stepwise decrease.

Incidentally, the evolutions of W and K are given by

$$\frac{\partial W}{\partial t} = -2 \int \eta \boldsymbol{J} \cdot \boldsymbol{J} \, dV \tag{3}$$

$$\frac{\partial K}{\partial t} = -2 \int \eta \boldsymbol{J} \cdot \boldsymbol{B} \, dV \qquad (4)$$

where η is the resistivity. These equations indicate that, generally speaking, both W and K decrease with the time scale of ohmic dissipation.

The fact that W and K behave very differently during the self-organization process must tell us some important physical law underlying the process. It is natural to infer that the driven reconnection process must hold the key. So let us examine the difference between $\boldsymbol{J} \cdot \boldsymbol{J}$ and $\boldsymbol{J} \cdot \boldsymbol{B}$ arround the reconnection point.

The left side panel of Fig. 3 shows that there exists an antiparallel field point in a cross section (poloidal plane) and the right hand panel shows that at the poloidally antiparallel field point the axial (toroidal) field almost completely vanishes. This indicates that the point concerned is a completely antiparallel reconnection point. Therefore, the current is largely peaked there, while $\boldsymbol{B} = 0$, thus indicating that $\boldsymbol{J} \cdot \boldsymbol{J}$ is considerably enhanced but $\boldsymbol{J} \cdot \boldsymbol{B}$ almost vanishes there.

Recalling Eqs. (3) and (4) we find a beautiful dispensation of Nature that the current driven instability deforms the magnetic field and generates a completely antiparallel magnetic field configuration where W is drastically decreased but K is not influenced (complete reconnection) as if the system knew its destination and the shortest path. The topology of the magnetic field thereby is globally changed.

Shown in Fig. 4 is one example of the magnetic field behavior near the reconnection point in the RFP self-reversal process[2], i.e., a self-organizing process in the RFP. As is evident here, the magnetic field again is minimized at the driven reconnection point, which indicates the tendency of K conservation.

In conclusion, the magnetohydrodynamic self-organization is governed by the nonlinearly driven reconnection whereby the magnetic helicity is conserved but the magnetic energy is stepwise decreased. This reminds us the mystery and dispensation of Nature.

In this regard we shall examine the time evolutions of the average wavenumber of the magnetic energy and the magnetic helicity. This is shown in Fig. 5 for the same example as that of Fig. 2. It is interesting to observe that the energy spectrum cascades normally, while the helicity spectrum cascades inversely. Higher mode spectrum of the energy as a matter of fact swiftly disappears, resultantly, the (magnetic) entropy in the system decreases to create a new ordered structure which has a simpler helical structure.

In the case of tokamak, on the other hand, there is a strong toroidal (potential) field which is supplied from an external coil current and a relatively peak current flows in the toroidal direction which produce a relatively weak poloidal field convertible to kinetic and thermal energies. Even though current driven instabilities occur by releasing the free (small) poloidal magnetic energy, the released (flow) energy is not sufficiently large that the massive toroidal field is folded to make a completely antiparallel field configuration. This indicates that the magnetic field would never vanish at a reconnection point, in other words, only the poloidal field components can experience reconnection (incomplete or partial reconnection). Therefore, $\boldsymbol{J}\cdot\boldsymbol{B}$ would be enhanced at a reconnection point rather than being minimized, thus the helicity could not be conserved. This is the reason why the tokamak configuration cannot be obtained by Taylor's theory[3] assuming the helicity conservation. The sawtooth oscillations in tokamaks, therefore, should be a partial self-organization process which exhibits a slight topological change due to partial reconnection.

4. MHD Time Scale Evolution

Since resistivity is intrinsic to the reconnection process, it is conceivable that self-organization should take place in the time scale dependent strongly on the resistive time scale t_R. As we will see in Figs. 2 and 5, however, the actual evolution time scale of the self-organization is much smaller than, and rather independent of, the resistive time scale. The time scale is of the order of a few tens of the Alfven transit time, t_A, where the Reynolds number $R = t_R/t_A$.

The magic of this fact time scale can be well explained in terms of a driven type reconnection[4]. A physical difference between the spontaneous reconnection and driven reconnection is depicted in Fig. 6. As we can see in the upper part of this figure, in a spontaneous reconnection process antiparallel field components must exist initially where the current is locally peaked. Subject to a finite resistivity, the current peak tends to be flattened. The time scale of this flattening is naturally governed by the resistive diffusion because the resistivity is the direct cause of reconnection. In a tearing mode instability, feedback flows leading to enhancement of the neutral point current exist, thereby, the current flattening is enhanced. Nevertheless, the process is still largely dependent on the resistivity because the flows are the effect of the resistivity induced reconnection.

On the other hand, in the case of driven reconnection which is depicted in the lower part the current peaking is locally created at a converging point of the plasma flows driven externally or induced internally by an ideal MHD instability, notably, a current-driven helical kink instability. In this case the time scale of current peaking is determined by the strength of the flow amplitude. Therefore, the flows are the cause of reconnection and the dissipation is determined by $\boldsymbol{J}^2$ rather than η. This indicates that the time scale of the self-organization is governed by the MHD (i.e., MHD flows) time scale[4].

5. Conclusion

By performing simulations for different initial and boundary conditions we come up with the following unified model for the magnetohydrodynamic self-organization [4].

When the magnetic free energy is sufficient, the current driven instability deforms the magnetic field so largely that completely antiparallel field points are created somewhere inside the system whereby the magnetic topology is globally changed and a minimum energy topology of magnetic field is realized. It is noted that the magnetic helicity is conserved during this proccess, while the magnetic energy is drastically decreased. The reversed field pinch (RFP) relaxation is a typical example of this type [2].

When the free energy is not so large, only partially antiparallel field configuration is formed where only partial reconnection occurs. Partial reconnection can not globally change the magnetic topology. The magnetic helicity is not conserved in this case because $\boldsymbol{J}\cdot\boldsymbol{B} \neq 0$ at the reconnection point. A typical example of the latter is the tokamak sawtooth relaxation[5]. Our conclusion on the magnetohydrodynamic self-organization process is summarized in Fig. 7.

References

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Figure Captions

Figure 1 : The snapshots of a self-organizing magnetohydrodynamic plasma.

Figure 2 : Time evolutions of the magnetic energy W and the magnetic helicity K.

Figure 3 : The structure of the poloidal and toroidal magnetic field arround the poloidally antiparallel field point.

Figure 4 : The magnetic field behavior near the reconnection point in the RFP self-reversal process.

Figure 5 : The time evolutions of the average wavenumber of the magnetic energy and the magnetic helicity.

Figure 6 : A physical difference between the spontaneous reconnection and driven reconnection.

Figure 7 : The summary of the magnetohydrodynamic self-organization process.

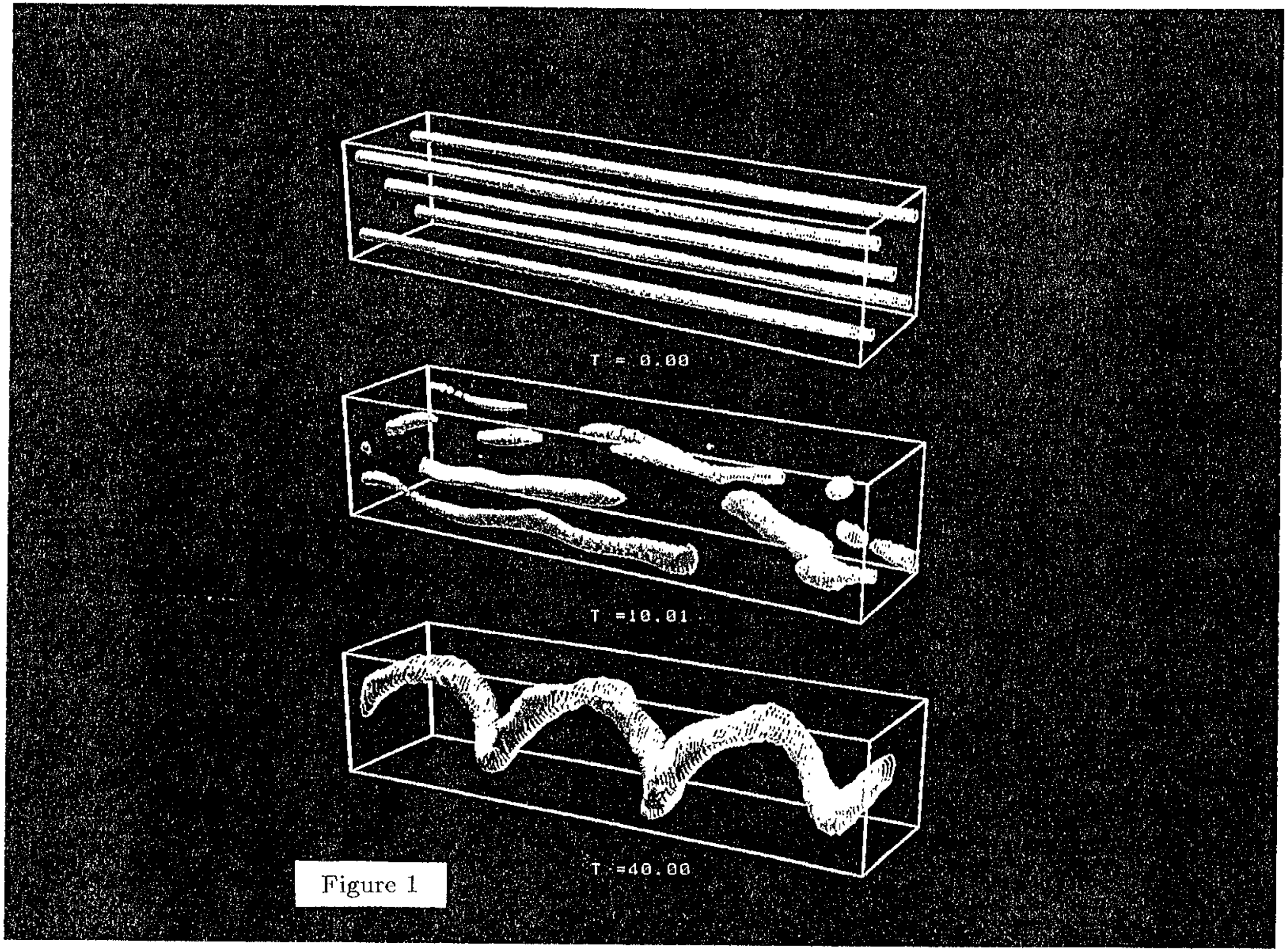
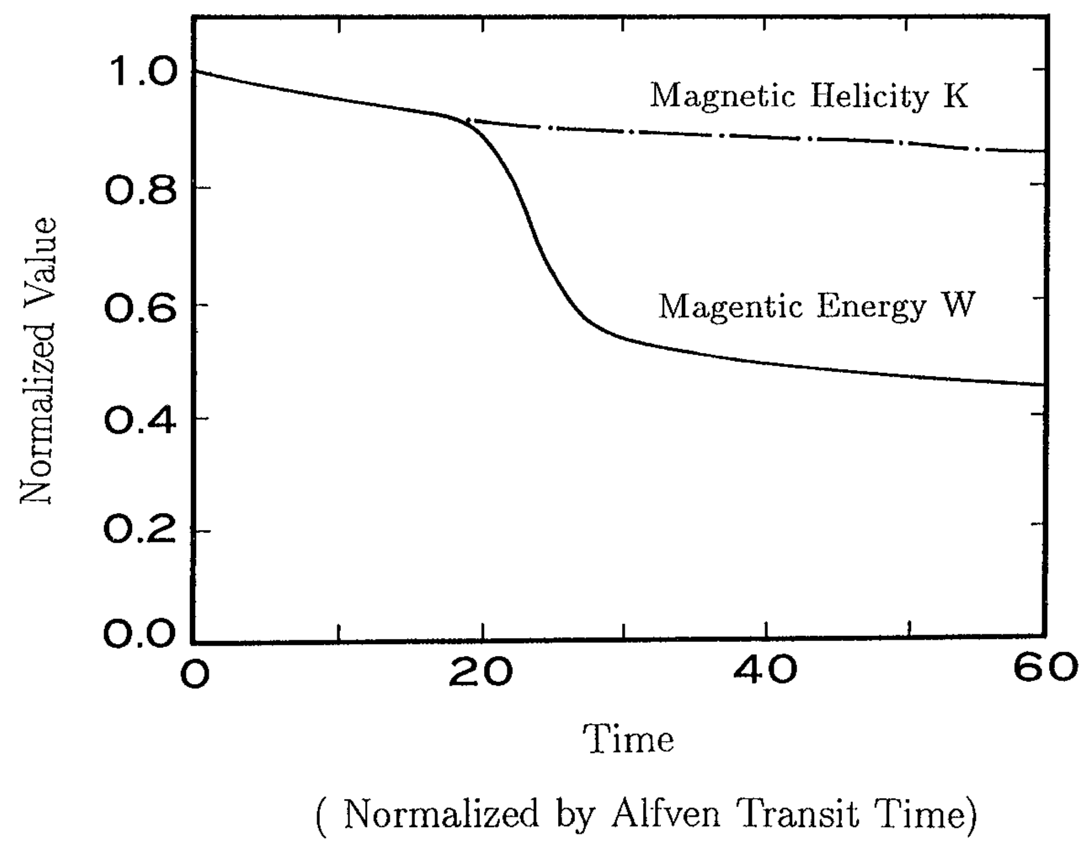


Figure 1



Magnetic Helicity K
Magentic Energy W
Normalized Value
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
Time
(Normalized by Alfven Transit Time)

Figure 2

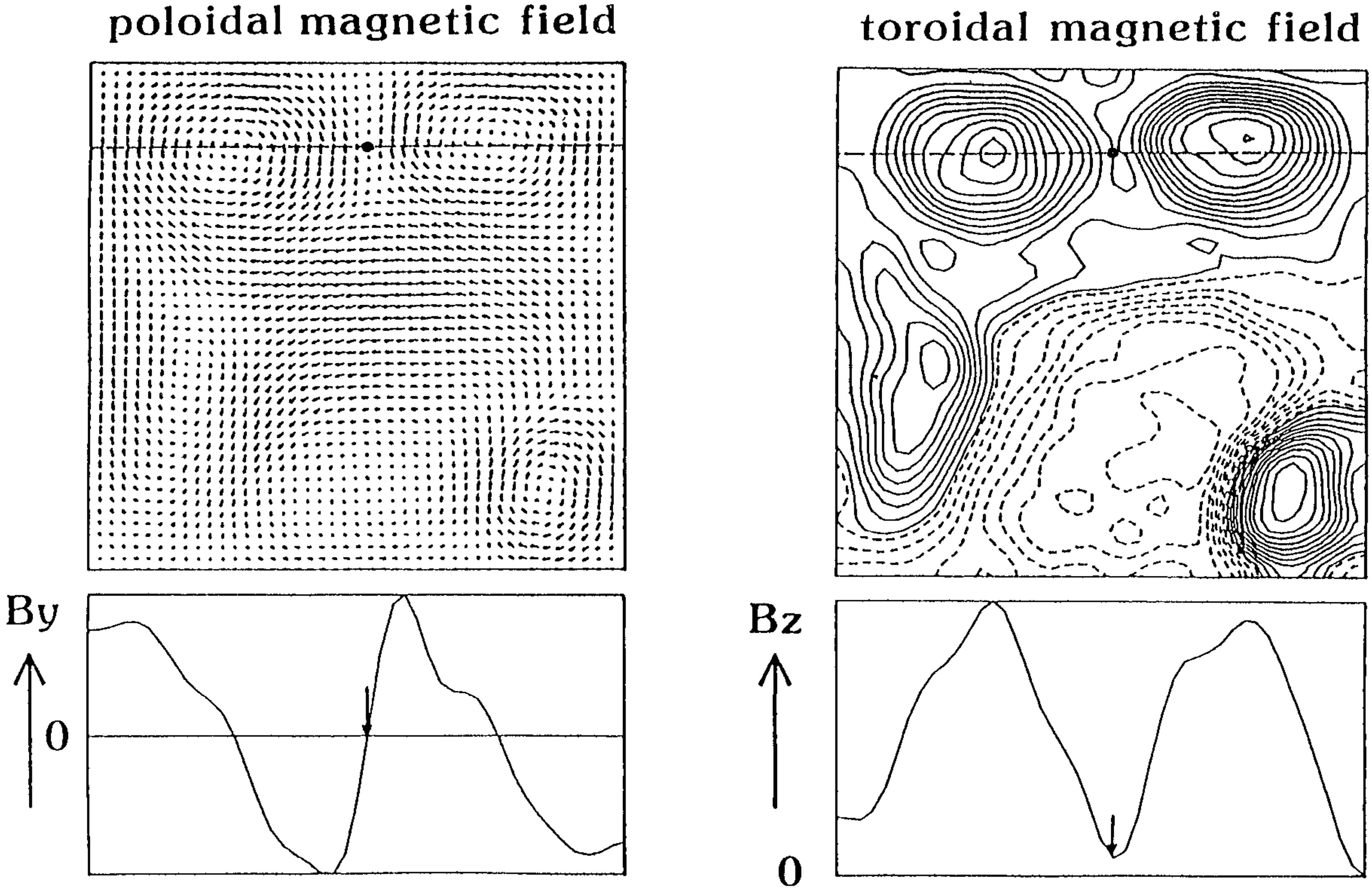


Figure 3

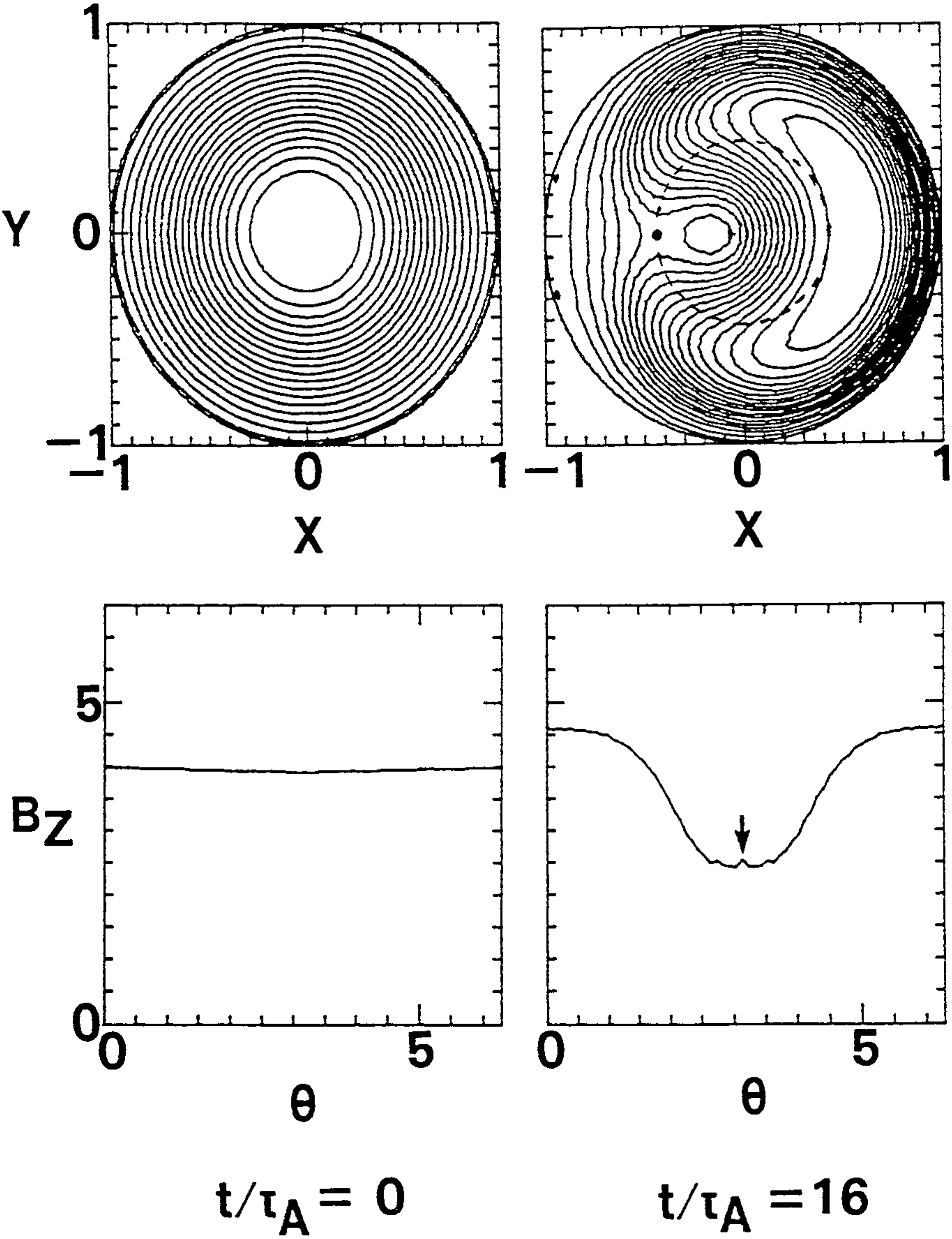


Figure 4

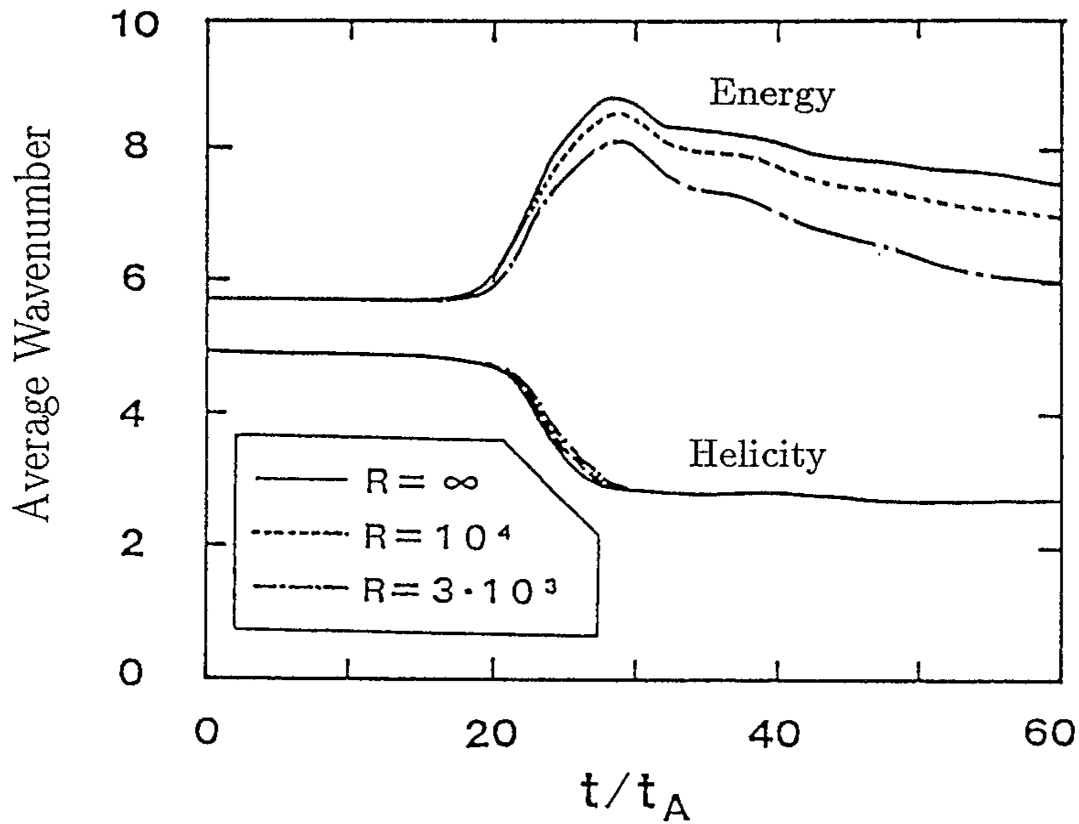
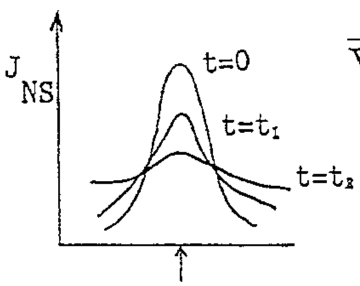

Average Wavenumber
10
8
6
4
2
0
0
20
40
60
t/t_A
Energy
Helicity
R= ∞
R=10⁴
R=3·10³

Figure 5

● Spontaneous Reconnection (Linear Reconnection)

$$E_R = \eta \dot{J}_{NS}$$



Neutral Point
(X point)

$$\frac{E_R}{V_A B_0} = \frac{\eta \dot{J}_{NS}}{V_A B_0} = \frac{\eta}{\mu_0 L V_A} = S^{-1}$$

Influence η

Cause → Effect

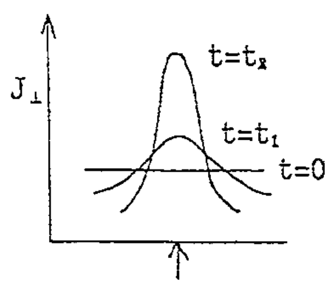
|| ||

Current — Electric Field — Diffusion process

J_{NS} — E_R — $\tau \sim \tau_{Res}$

● Flow Driven Reconnection (Nonlinear Reconnection)

$$E_D = V_D B_0$$



Neutral Point
(X point)

$$\frac{E_R}{V_A B_0} = \frac{E_D}{V_A B_0} = \frac{V}{V_A} \gg S^{-1}$$

Influence η

Cause → Effect

|| ||

Electric Field — Current — Concentration process (Increase of Energy Dissipation)

E_D — $J_\perp = E_D/\eta$ — $\tau \sim \tau_{MHD}$

Figure 6

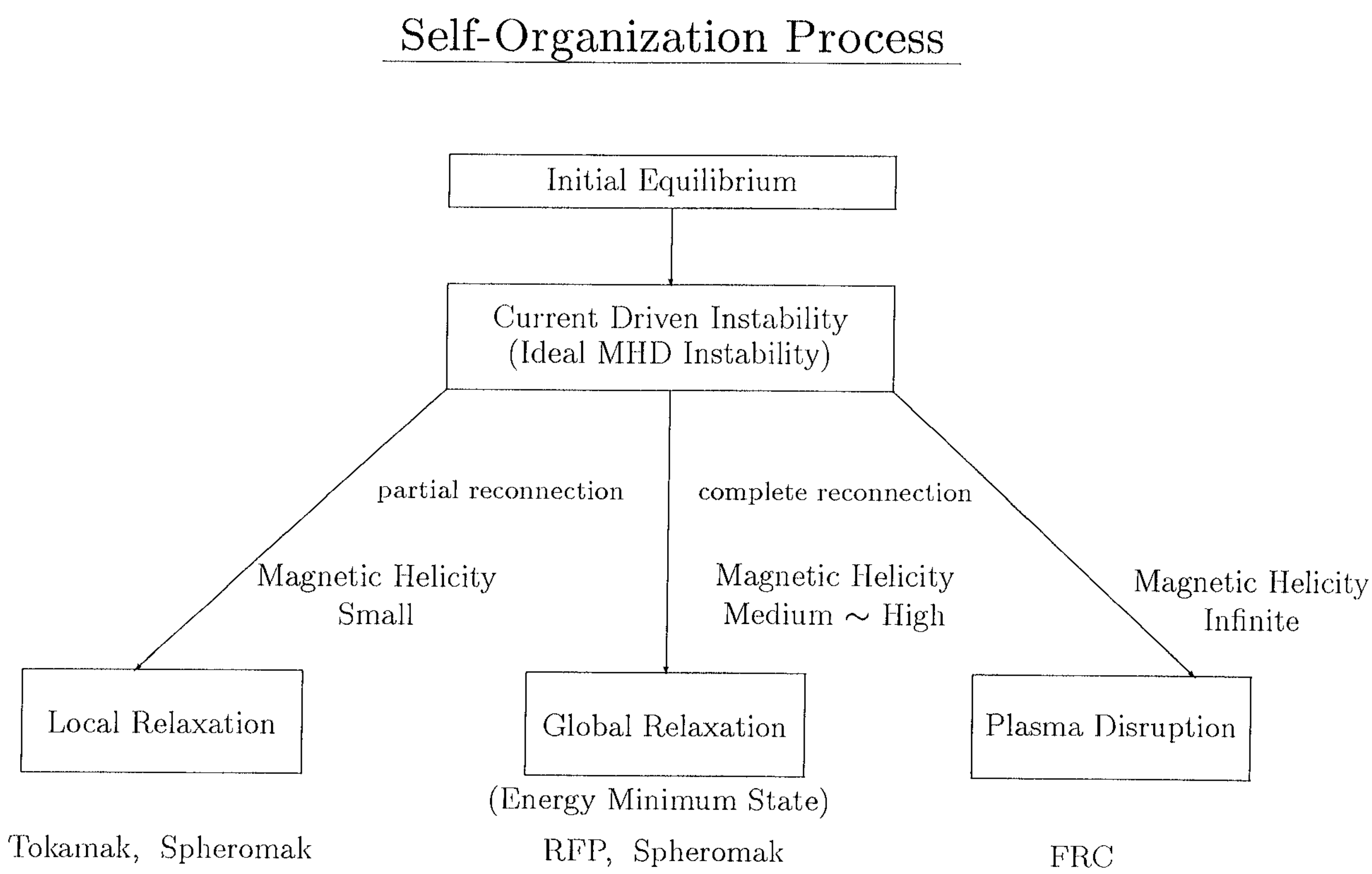

Self-Organization Process
Initial Equilibrium
Current Driven Instability
(Ideal MHD Instability)
partial reconnection
complete reconnection
Magnetic Helicity
Small
Magnetic Helicity
Medium ∼ High
Magnetic Helicity
Infinite
Local Relaxation
Global Relaxation
Plasma Disruption
(Energy Minimum State)
Tokamak, Spheromak
RFP, Spheromak
FRC

Figure 7