ISSN 0915-633X

NATIONAL INSTITUTE FOR FUSION SCIENCE

Bootstrap Currents in Stellarators and Tokamaks

M. Okamoto and N. Nakajima

(Received – Sep. 17, 1990)

NIFS-53 Sep. 1990

RESEARCH REPORT
NIFS Series

NAGOYA, JAPAN

BOOTSTRAP CURRENTS IN STELLARATORS AND TOKAMAKS

Masao OKAMOTO and Noriyoshi NAKAJIMA

National Institute for Fusion Science, Nagoya 464-01, Japan

Abstract

The remarkable feature of the bootstrap current in stellarators is it's strong dependence on the magnetic field configuration. Neoclassical bootstrap currents in a large helical device of torsatron/heliotron type ($L = 2$, $M = 10$, $R = 4$ m, $B = 4$ T) is evaluated in the banana ($1/\nu$) and the plateau regime. Various vacuum magnetic field configurations are studied with a view to minimizing the bootstrap current. It is found that in the banana regime, shifting of the magnetic axis and shaping of magnetic surfaces have a remarkable influence on the bootstrap current ; a small outward shift of the magnetic axis and vertically elongated magnetic surfaces are favourable for a reduction of the bootstrap current. It is noted, however, that the ripple diffusion in the $1/\nu$ regime has opposite tendency to the bootatrap current ; it increases with the outward shift and increases as the plasma cross section is vertically elongated. The comparison will be made between bootstrap currents in stellarators and tokamaks.

Key words : bootstrap current, Ohkawa current, neoclassical theory, stellarator, parallel viscosity, $1/\nu$ regime, ripple diffusion, Large Helical Device.

Presented at The Joint Varenna-Lausanne International Workshop on "Theory of Fusion Plasmas" (International School of Plasma Physics < *Piero Caldirola* >), Villa Monastero, Varenna, Italy, August 27-31, 1990.

§1 Introduction

The exsistence of the bootstrap current in a tokamak was predicted theoretically by Bickerton, Conner, and Taylor in 1971 [1]. At the same time this neoclassical current was investigated by Galeev and Sagdeev [2], and by Kadomtsev and Shafranov [3]. This automatically induced current is beneficial to tokamaks which require a toroidal current to maintain the equilibrium, but it was not observed in tokamaks for a long time. This was partly due to the difficulty of obtaining low collisionality in a high poloidal beta regime. However, in 1984, neoclassical Pfirsch-Schluter and bootstrap currents have been observed in a toroidal octupole [4], and in Proto-Cleo stallarator [5] by probe measurements. Further, in 1988, it was found that the observed currents in beam heated TFTR plasmas can be explained only by models including the neoclassical bootstrap current [6]. Since then bootstrap currents have been observed in JET [7], and in JT-60 [8]. Especially, in JT-60 the experimental data are analyzed such that nearly 80 % of total toroidal current are attributed to the neoclassical bootstrap current. The neoclassical theory for tokamak plasmas are almost completed [9, 10] and the above mentioned experiments are compared to the calculations based on the neoclassical theory. However, plasma transports are governed by anomalous diffusions in tokamaks and stellaraters while the observed bootstrap current seems to be explained by neoclassical processes. The gap between the two would be filled by theoretical works for fluctuation-induced current or the effects of fluctuations on the neoclassical bootstrap current [11, 12, 13].

It is interesting that the existence of bootstrap current is being confirmed in stellarators of Heliotron-E [14], W7-A [15], ATF [16], and CHS [17]. It is possible that the bootstrap current is induced automatically (without the necessity of supplying a seed current) in stellarators, the seed current being replaced by an externally produced rotational transform [1]. For stellarator plasmas the neoclassical theory has not yet been completed, but neoclassical bootstrap currents are formulated in the limit of $1/\nu$

and plateau regimes [18, 19, 20, 21, 22]. These formulations in the $1/\nu$ regime [18, 19] show that the bootstrap current in stellarators depend strongly on the magnetic field structures. In the stellarators of heliotron/torsatron type with $L = 2$, dipole and quadrupoole components of the magnetic fields are very effective, suggesting that the bootstrap current can be controllable by externally applied poloidal fields.

In §2 we survey the parallel currents of stellarator plasmas in the $1/\nu$ regime. The bootstrap currents are calculated for the Large Helical Device and the geometrical dependences are investigated in §3. In §4 the geometrical dependences are compared between the ripple diffusion and bootstrap current. §5 is devoted to conclusion and discussion.

§2 Parallel currents

In order to analyse the bootstrap current in JT-60, Kikuchi et al. [8] have solved directly the parallel momentum and heat balance equations for electrons, ions, impurities, and fast ions, with the friction and viscosity coefficients evaluated by Hirshmann and Sigmar [10]. Tokuda et al. [23] have also developed a computer code of equilibrium including the neoclassical parallel currents consistently. We summarize the plasma currents in the direction parallel to the magnetic field in the same way as Kikuchi et al. s' [8] for the plasma in the $1/\nu$ regime of stellarators. The parallel momentum and parallel heat balance equations are given by

$$< \vec{B} \cdot \nabla \cdot \pi_a > = < \vec{B} \cdot \vec{F}_{a1} > + < \vec{B} \cdot \vec{F}_{ab1} > + < \vec{B} \cdot n_a e_a \vec{E}^{(A)} > \quad (1)$$

$$< \vec{B} \cdot \nabla \cdot Q_a > = < \vec{B} \cdot \vec{F}_{a2} > + < \vec{B} \cdot \vec{F}_{ab2} > \quad (2)$$

where $< >$ indicates the flux surface average and subscript a refers to species of electrons or plasma ions. $< \vec{B} \cdot \nabla \cdot \pi_a >$ and $< \vec{B} \cdot \nabla \cdot Q_a >$ are parallel viscosity and parallel heat viscosity, $< \vec{B} \cdot \vec{F}_{a1} >$ is the inter-frictional force, $< \vec{B} \cdot \vec{F}_{ab1} >$ is the friction force of species a with fast beam ions (say, produced by netral beam injection), and $\vec{E}^{(A)}$ is the induction field. The forces with subscript 2 are related to heat frictions. The relations between the frictional forces ($\vec{F}_{a1}$, $\vec{F}_{a2}$, $\vec{F}_{ab1}$, $\vec{F}_{ab2}$) and flows are given by Hirshmann and Sigmar [10] with viscosity coefficients evaluated in all collisionality regimes. The evaluations in the banana regime of tokamak plasmas are correct in the $1/\nu$ regime plasma in stellarators. The differences between the frictional forces in tokamaks and stellarators appear only through the distribution of trapped particles. However, the parallel viscosities are quite different between the two, resulting in different bootstrap currents in stellarators from in tokamaks. Parallel viscosities $< \vec{B} \cdot \nabla \cdot \pi_a >$ and $< \vec{B} \cdot \nabla \cdot Q_a >$ are calculated for stellarators in the $1/\nu$ regime by Shaing et al. [18, 19].

Equations (1) and(2) can be expressed by matrix equations as follows ;

$$M\vec{X} = < G_{bs} > \vec{V} - \frac{n_b m_b}{\tau_s} < Bu_{\parallel b} > \vec{S}^{(b)} - en_e < BE_{\parallel}^{(A)} > \vec{S}^{(A)} \tag{3}$$

Here M is the matrix of 4 by 4 including frictional coefficients and viscosity coefficients. The factor $< G_{bs} >$ is the geometric factor [18] which depends strongly on the magnetic field structures. In Eq. (3), n_b, m_b, τ_s, and $u_{\parallel b}$ are the density, mass, slowing down time, and parallel flow velosity of fast ions. The vectors are given by

$$\vec{X} = [\ < Bu_{\parallel e} >,\ -\frac{2}{5}\frac{< Bq_{\parallel e} >}{p_e},\ < Bu_{\parallel i} >,\ -\frac{2}{5}\frac{< Bq_{\parallel i} >}{p_i}\ ]^T \tag{4}$$

$$\vec{V} = [\ V_1,\ V_2,\ V_3,\ V_4\ ]^T \tag{5}$$

$$\vec{S}^{(b)} = [\ 1,\ \frac{3}{2},\ 0,\ 0\ ]^T \tag{6}$$

$$\vec{S}^{(A)} = [\ -1,\ 0,\ 1,\ 0\ ]^T \tag{7}$$

where T denotes the transpose of the vectors and $u_{\parallel a}$, $q_{\parallel a}$, and p_a are the particle flow, heat flow, and pressure of a species, respectively. V_1, V_2, V_3, and V_4 are thermodynamic forces and are given by

$$V_1 = -\mu_{e1}\frac{T_e}{e}(\frac{p_e'}{p_e} - \frac{e\phi'}{T_e}) + \mu_{e2}\frac{T_e'}{e} \tag{8}$$

$$V_2 = -\mu_{e2}\frac{T_e}{e}(\frac{p_e^{'}}{p_e} - \frac{e\phi^{'}}{T_e}) + \mu_{e3}\frac{T_e^{'}}{e} \tag{9}$$

$$V_3 = \mu_{i1}\frac{T_i}{eZ}(\frac{p_i^{'}}{p_i} + \frac{eZ\phi^{'}}{T_i}) - \mu_{i2}\frac{T_i^{'}}{Ze} \tag{10}$$

$$V_4 = \mu_{i2}\frac{T_i}{eZ}(\frac{p_i^{'}}{p_i} + \frac{eZ\phi^{'}}{T_i}) - \mu_{i3}\frac{T_i^{'}}{eZ} \tag{11}$$

The matrix equation corresponding to Eq. (3) derived by Kikuchi et al. [8] is 7 by 7, since they included the bootstrap current due to impurity ions and fast ions (the heat flow of fast ions is not taken into account). We can solve Eqs. (3) to (11) for $u_{\|e}$ and $u_{\|i}$ to obtain the parallel currents ;

$$< BJ_{\|} >= en_e < B(u_{\|i} - u_{\|e}) >=< BJ_{\|} >_E + < BJ_{\|} >_R + < BJ_{\|} >_B \tag{12}$$

where we have decomposed the parallel currents into the Spitzer current, the electron return current due to frictions with fast ions and the bootstrap current. If we add the fast ion beam current $< BJ_{\|} >_F= eZ_b n_b < Bu_{\|b} >$ to $< BJ_{\|} >_R$, we obtain the beam-driven current (Ohkawa current). The results for parallel currents are

$$< BJ_{\|} >_E= \sigma_{NC} < BE_{\|}^{(A)} > \tag{13}$$

$$< BJ_{\|} >_O= F_{NC} < BJ_{\|} >_F \tag{14}$$

$$< BJ_{\|} >_B= - < G_{bs} > \{L_{31}(p_e^{'} + p_i^{'}) + L_{32}^{e} n_e T_e^{'} + L_{32}^{i} n_i T_i^{'}\} \tag{15}$$

with

$$\sigma_{NC} = \frac{e^2 n_e \tau_{ei}}{m_e}\Lambda_{NC}^0 \tag{16}$$

$$\Lambda_{NC}^0 = \frac{Z(l_{22}^{ee} + \mu_{e3})}{D} \tag{17}$$

$$L_{31} = \frac{\mu_{e1}(l_{22}^{ee} + \mu_{e3}) - \mu_{e2}(l_{12}^{ee} + \mu_{e2})}{D} \tag{18}$$

$$L_{32}^{e} = \frac{\mu_{e3} l_{12}^{ee} - \mu_{e2} l_{22}^{ee}}{D} \tag{19}$$

$$L_{32}^{i} = -L_{31}\frac{\mu_{i2} l_{22}^{ii}}{\mu_{i1}(l_{22}^{ii} + \mu_{i3}) - \mu_{i2}^2} \tag{20}$$

$$F_{NC} = 1 - \frac{Z_b}{Z}\{\Lambda_{NC}^0 + \frac{3}{2}\Lambda_{NC}^1\} \tag{21}$$

$$\Lambda_{NC}^{1} = -\frac{Z(l_{12}^{ee} + \mu_{e2})}{D} \quad (22)$$

$$D = (l_{11}^{ee} + \mu_{e1})(l_{22}^{ee} + \mu_{e3}) - (l_{12}^{ee} + \mu_{e2})^2 \quad (23)$$

Here we have neglected $O(\sqrt{m_e/m_i})$. As noted already the bootstrap currents depend strongly on the field geometry which may quite different for tokamaks and stellarators, while the expressions for neoclassical conductivity and beam driven current remain the same. It is noted, however, that the frictional coefficients ($l_{11}^{ee}, \cdots$) in Eqs. (16) to (23) are constants (including the ionic charge number Z), while viscosity coefficients ($\mu_{e1}, \cdots$) are proportional to $\frac{f_t}{f_c}$ where f_t is the fraction of trapped particles and $f_c = 1 - f_t$. Thus the differences in σ_{NC} and F_{NC} between in tokamaks and stellarators appear through the distribution of trapped particles.

§3 Boostrap current in a Large Helical Device

In stallarators the equilibrium are created only by external currents, not requiring the inner plasma current. The stellarators are then often characterized "currentless", but actually there may exist bootstrap current, beam-driven current, FCT current and others. Among these currents the bootstrap current could be enhanced in a stellarator, since the bootstrap current in stellarators depends strongly on the equilibrium configuration and the plasma profiles and the externally produced rotational transform plays a role of the seed current, which is necessary to drive a bootstrap current [1]. The neoclassical bootstrap currents in both Pfirsch-Schluter and banana regimes in non-axisymmetric toroidal systems have been studied by Shaing and Callen [18]. In the banana regime, their formulation is restricted to the $1/\nu$ regime, and it is found that the bootstrap current in the banana regime depends strongly on the magnetic field structure and can be reduced with increasing toroidal bumpiness. The paper by Shaing and Callen has been revised by Shaing et al. [19] to study the role of helically trapped particles on the bootstrap current in the banana regime. The viscosity coefficients for neoclassical transport in the plateau regime of non-axisymmetric toroidal plasmas have been calculated by Coronado and Wobig [21], and the bootstrap current has been obtained by Shaing et al. [20]. It is shown that the helical modulation contributes significantly to the parallel viscosities and that it changes the direction of the bootstrap current and the Ware pinch flux in the plateau regime. In Ref. [24], it is pointed out that the bootstrap current in the banana regime in stellarators would be similar to that in tokamaks at comparable beta values and safety factors since stellarators are intrinsically high aspect ratio devices.

In this section we review the results of bootstrap currents in the Large Helical Device (LHD) to be constructed in The National Institute for Fusion Science (NIFS). The basic machine parameters for LHD are as follows [25] ; $L = 2$, $M = 10$, $R_0 = 4$ m, $B_0 = 3 \sim 4$ T, $\gamma_c = 1.20 \sim 1.25$, $P_t = 0 \sim 0.1$, $P_{heat} = 20$ MW, where L and M are the poloidal and toroidal pitch numbers, respectively, R_0 is the major radius, B_0 is the magnetic field strength, $\gamma_c = \frac{M}{L}\frac{a_c}{R_0}$ is the pitch parameter for the helical winding coils, P_t is the coil pitch modulation parameter, and P_{heat} is the heating power.

The bootstrap current in the $1/\nu$ regime in non-axisymmetric toroidal systems is given by Eq. (15) or by Ref. [18] ;

$$< J_{bs}B >= -2.95\frac{f_t}{f_c}G_{bs}\{a_1(T_e+T_i)\frac{dn}{dV} + a_2 n\frac{dT_i}{dV} + a_3 n\frac{dT_e}{dV}\} \tag{24}$$

where J_{bs} is the bootstrap current density, B is the magnetic field strength, $< >$ indicates the flux surface average, n is the density, T_e and T_i are the electron and ion temperatures, V is the volume, and the numerical factors are $a_1 = 0.554$, $a_2 = -0.0941$ and $a_3 = 0.1404$. In Eq.(24), f_t and f_c are fractions of trapped and circulating particles, respectively, and $G_{bs} =< \tilde{G}_b >$ is the flux averaged geometrical factor, with $\tilde{G}_b$ given by Eq.(75b) of Ref. [18]. Equation (24) has been solved numerically for the LHD parameters [26]. In Eq.(24), the geometric factor $G_{bs}f_t/f_c$ is determined only by the magnetic field configuration and the rest part is determined by plasma profiles of density and temperatures. The behaviours of the geometric factor $G_{bs}f_t/f_c$ are investigated for various vacuum magnetic fields by changing the dipole and quadrupole components, B_V, B_Q, of poloidal fields. It is found that $G_{bs}f_t/f_c$ changes little for different values of M, but when B_V is changed so as to shift the plasma outward (or inward), $G_{bs}f_t/f_c$ becomes smaller (or larger). A reduction by a factor of 2 to 4 is possible through the control of B_V. The polidal quadrupole component B_Q can alter the elipticity of the plasma cross section. If the toroidally averaged plasma cross sections are horizontally elongated, the geometrical factor increases and if the cross sections are vertically elongated elipse, the geometrical factor decreases drastically. Control of B_Q can lead to variation of $G_{bs}f_t/f_c$ from 600 to several tens or even nagative values. Thus the bootstrap current in the $1/\nu$ regime can be controllable by the poloidal fields.

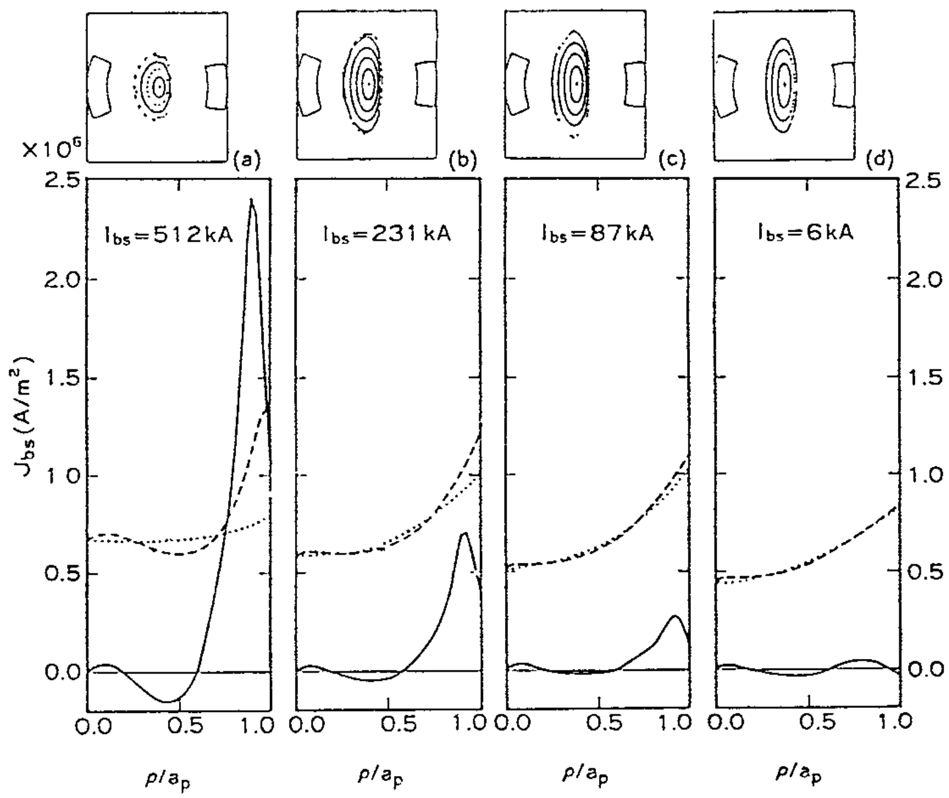

Fig. 1 Bootstrap currents in the banana regime for the high density case.
(a) $B_Q = 150\%$, (b) $B_Q = 0\%$, (c) $B_Q = -50\%$, (d) $B_Q = -100\%$.
Solid lines : bootstrap current density, Dotted lines : vacuum rotational transform, Dashed lines : total rotational transform

Figure 1 shows an example for a set of plasma profiles of n, T_e and T_i which are given by a transport code calculation. The average density is $\sim 10^{20}\mathrm{m}^{-3}$, the profile of which is slightly hollow. The average electron and ion temperatures are around 3 keV. The position of the plasma magnetic axis is shifted by the application of B_V by 5 cm. The poloidal quadrupole fields are varied ; (a) $B_Q = 150\%$, (b) $B_Q = 0\%$, (c) $B_Q = -50\%$, (d) $B_Q = -100\%$, where $B_Q = 150\%$ means that the 150 % of the axisymmetric quadrupole components created by helical coils are cancelled by the quadrupole components of the poloidal coils. In the case of $B_Q = -100\%$, the quadrupole fields by helical coils are doubled. In the case of (a) the plasma cross sections are slightly horizontally elongated and the case of (d) is vertically elongated elipse. Negative bootstrap current are seen in Fig.1, but this is attributed to the hollow density profile. Total bootstrap currents I_{bs} are : (a) $I_{bs} = 512$ kA, (b) $I_{bs} = 231$ kA, (c) $I_{bs} = 87$ kA, (d) $I_{bs} = 6$ kA. These values show that the bootstrap current in the $1/\nu$ regime is controllable by changing the poloidal fields.

We employ the formula given by Shaing et al. [20] for the bootstrap current in the plateau regime.

$$< J_{bs}B >= -H_{bs} \times 1.9351\sqrt{\pi}\frac{v_{Te}}{\nu_{ei}} \times \{1.16(P_e^{'} + P_i^{'}) + 1.5449nT_e^{'} + 0.58nT_i^{'}\} \quad (25)$$

The geometric factor H_{bs} is expressed in the Hanada cooodinate system [27]. As described in Appendix, many Fourier modes of the magnetic field B are required when H_{bs} is calculated in the Hamada coodinate system. We derive H_{bs} expressed in the Boozer coodinate system [28] by the transformation of the Hamada coodinates to the Boozer coodinates to calculate H_{bs} using less Fourier mode numbers. The calculation results of the bootstrap current in the plateau regime is shown in Ref. [26]. When the plasma axis is shifted outward by 5 cm and the quadrupole fields by poloidal coils and the axisymmetric qudrupole fields produced by helical coils are just cancelled (the plasma shape is nearly circular in this case), the bootstrap current amounts to $I_{bs} = -26$ kA for the plateau plasma with parabolic profiles. It is shown that the bootstrap current increases as M increases (or ϵ_h increases), while the bootstrap current does not depend on B_V, or B_Q appreciablly, which are different tendency from the case of the $1/\nu$ regime.

§4 Ripple diffusion and bootstrap current in the $1/\nu$ regime

In stellarators, the magnetic field strength varies along a line of magnetic field due to helical ripples as well as toroidal ripples. Particles with a large pitch angle are trapped in the helical ripples and bounce in the trough back and forth. On the other hand such particles drift across the magnetic field lines. If $\omega_b \gg \nu_{eff} \gg \omega_p$, where ω_b is the bounce frequency of helically trapped particles, ν_{eff} is the effective collision frequency, and ω_p is the characteristic frequency of poloidal drift motion, particles suffer from a large movement across the magnetic fields. The resultant diffusion coefficient is inversely proportional to ν_{eff}. Such collisional regime is called the $1/\nu$ regime. In the $1/\nu$ regime the ripple diffusions are formulated by solving the drift kinetic equation [29]. The results are given by

$$< \vec{\Gamma}_a \cdot \nabla\psi >= -\frac{G_D}{\nu_a^*}\frac{T_a^2}{q_a^2}\{L_{11a}\frac{dn_a}{d\psi} + L_{12a}\frac{n_a}{T_a}\frac{dT_a}{d\psi} + L_{13a}\frac{n_a q_a}{T_a}\frac{d\phi}{d\psi}\} \tag{26}$$

$$< \vec{Q}_a \cdot \nabla\psi >= -\frac{G_D}{\nu_a^*}\frac{T_a^2}{q_a^2}\{L_{21a}T_a\frac{dn_a}{d\psi} + L_{22a}n_a\frac{dT_a}{d\psi} + L_{23a}n_a q_a\frac{d\phi}{d\psi}\} \tag{27}$$

where a designate the particle species. $\vec{\Gamma}_a$ and $\vec{Q}_a$ are particle and heat fluxes, ψ is the flux function, q_a is the charge number, L_{ij} are numerical coefficients, and ϕ is the electric potential. In Eqs. (26) and (27), G_D is the geometrical factor which is determined only by the magnetic field structures and is separated from other terms obtained by the plasma transports in the same way as in Eq. (24) for the bootstrap current.

In this section we investigate the dependence of the geometric factor $G_D(d\psi/d\rho)^{-2}$ on the magnetic field structures using the vacuum magnetic field configurations for

LHD as in §2. Figures 2-(a) and (b) show the behaviours of $G_D(d\psi/d\rho)^{-2}$ versus the average radius ρ. The average radius ρ is defined by $\psi = B_0/2\rho^2$. The equilibrium quantities are expressed as a function of ρ, resulting in the appearance of $(d\psi/d\rho)^{-2}$ in the geometric factor. Figure 2-(a) shows the effect of the magnetic axis D_a (effect of the vertical fields). $D_a < 0$ is the inward shift and $D_a > 0$ the outward shift.

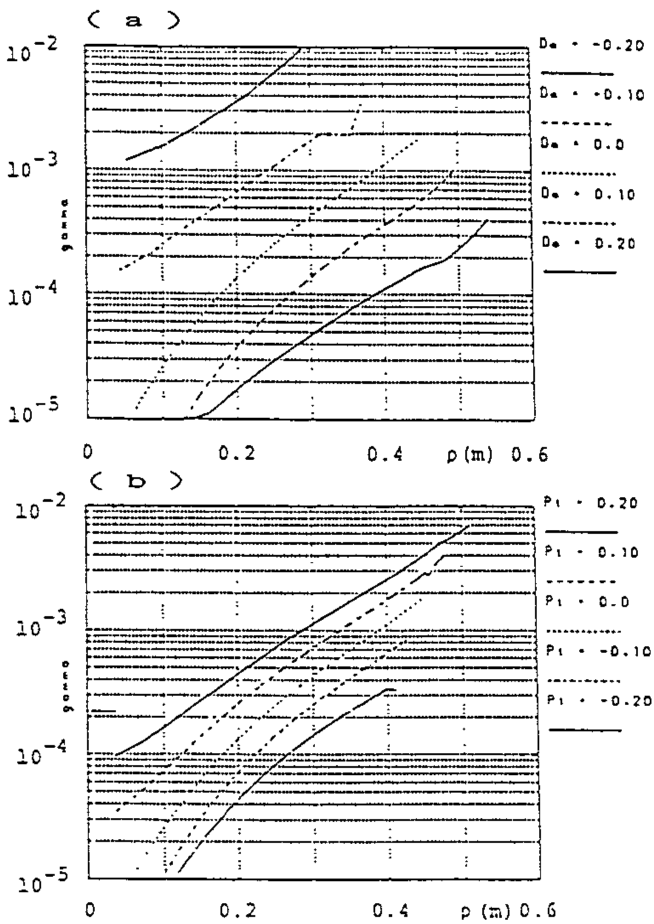

Fig. 2 Geometric factor $G_D(d\psi/d\rho)^{-2}$(= *gamma*) vs. averaged radius ρ.
(a) Effect of the magnetic axis shift D_a, (b) Effect of the pitch modulation P_t.

The geometric factor decreases as the plasma is shifted inward. It is reduced by one order from $D_a = +0.1$ m to $D_a = -0.1$ m. Figure 2-(b) indicates the influence of the pitch modulation parameter P_t. The negative pitch modulations ($P_t < 0$) contribute to a reduction of ripple diffusion. The effect of the quadrupole field is also investigated (not shown). The case of $B_Q = 100\%$ (net quadrupole fields vanishes and the plasma shape is nearly circular) seems most preferable, but B_Q effect is less important when compared to the effects of D_a or P_t. As is well known [30], the geometrical factor is small for the case when helical ripples are localized on the inside of the torus (strong field side), while for the case of ripples localized on the outside the geometric factor becomes large hence the neoclassical ripple diffusion is greatly enhanced.

Finally we compare the dependences of the geometric factors for the bootstrap current and ripple diffusion on the magnetic field configurations. Table 1 shows the summary of both field dependences of geometric factors. The characteristics of a single particle confinement are also shown. For the improvement of particle orbits or the ripple diffusion, the inward shift of the plasma position is most effective. Nearly circular plasma cross sections are most favourable to reduce the ripple diffusion, but in this case the bootstrap current can not be minimized. It is then important to evaluate the tolerable amounts of bootstrap current.

Table 1 Magnetic field dependence of geometric factors

		G_{bs}	G_D	orbit
shift D_a	inward	large	small	good
			effective	
	outward	small	large	bad
pitch $\alpha^* = P_t$	$\alpha^* < 0$	large	small	good
			effective	
	$\alpha^* > 0$	small	large	bad
elongation B_Q	horizontally	large	large	bad
	near circular	*effective*	small	good
	vertically	small	small or large	bad

§5 Conclusion and discussion

Characteristics of the bootstrap current in the stellarator of heliotron/torsatron type with $L = 2$ have been clarified. In the banana regime the bootstrap current can be changeable by the control of externally applied poloidal fields. Recently the bootstrap currents have been observed in ECH plasmas of ATF [16]. Dependences of the observed currents on the control of poloidal coil system have been confirmed. Other dependences of $I_{bs} \propto W_p$ (plasma stored energy), and $I_{bs} \propto B^{-1}$ have also been confirmed. In CHS, the bootatrap currents have been measured in both ECH and NBI plasmas [17]. The NBI plasmas are in the plateau regime and the observed toroidal current reverses it's direction in the high density or low field operations. Thus the existence of the bootstrap current is being verified in stellarators. In stellarators direct measurements of the bootstrap currents are easier than in tokamaks and the bootstrap currents are controllable by external poloidal fields. The research on the bootstrap current in stallarators can be a complementary subject to tokamaks.

Acknowledgement

The autors are grateful to Professors A. Iiyoshi, T. Sato, and M. Fujiwara for continuous encouragement and stimulating the problem. They also thank Drs. J. Todoroki, M. Azumi, and S. Tokuda for fruitful discussions.

Appendix Remarks on the numerical calculation associated with B-spectrum

For the calculation to grasp the characteristics of the neoclassical theory, single particle orbit and other phenomena in stellarators, we often use a model magnetic field

$$B(r,\ \theta,\ \varphi) = B_0\{1 - \epsilon_t(r)\cos\theta - \epsilon_h(r)\cos(L\theta + M\varphi)\} \tag{28}$$

However, actual magnetic field structures are so complicated compared to Eq. (28) that the equations expressed in a magnetic coordinate system are solved numerically using B-spectrum. The Hamada coodinate system [26] is one of the magnetic coordinate systems and adequate for analytical treatment. In the Hamada coordinate system, the Jacobian is a magnetic surface quantity and the lines of magnetic field, current density, and the first order plasma flow are all straight. As one of other magnetic coordinate systems Boozer coordinate system is well known and is often used for numerical calculations associated with B-spectrum in stellarators. In the Boozer coordinate system, the Jacobian is inversely proportional to B^2 and the magnetic field lines are straight on the magnetic surface.

Mathematically any magnetic coordinate system gives an identical result, but a careful treatment should be paied to numerical calculations related to the B-spectrum. Figure 3 shows the distribution of B-spectrum in the Hamada and Boozer coordinate systems. The magnetic data used are a vacuum field produced by helical coils with $L = 2, M = 10, B_0 = 4$ T, and $R_0 = 5$ m. Figure 3-(a) and (b) are B-spectrum distributions near the magnetic axis in Hamada and Boozer coordinates, respectively. Figure 3-(c) and (d) are B-spectrum distributions near the outermost magnetic surface. As seen from the Figures there is no significant differences between the B-spectra in both Hamada and Boozer coordinates near the axis. However, on the magnetic surface going away from the axis the B-spectrum in the Hamada coordinates becomes broad more and nore compared to that of Boozer coordinates. Figure 3-(d) for the Boozer coordinates shows a similar spectrum to the model field, Eq.(28), while Fig.3-(c) for the Hamada coordinates indicates that a large spectrum space is required when the Hamadsa coordinates are used for calculations associated with B-spectrum. Much attension should be paid to the numerical analysis of magnetic field structures expressed in the Hamada coordinate system.

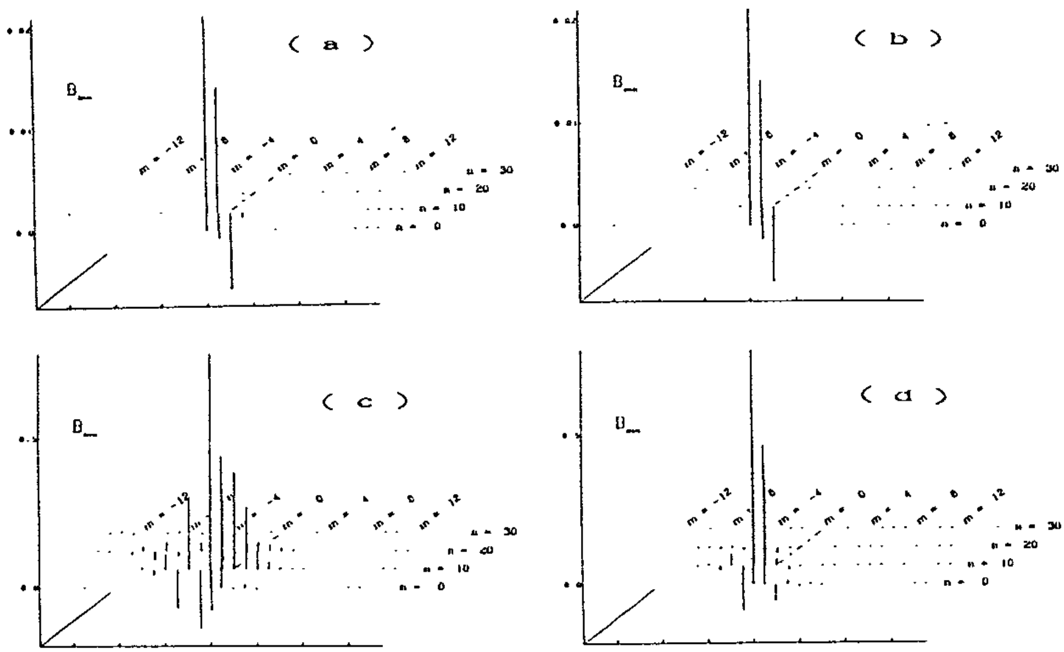

Fig. 3 Fourier spectra $B_{mn} : B = \sum B_{mn} \cos(m\chi + n\varphi)$

(a) Hamada coordinates, near the axis,

(b) Boozer coordinates, near the axis,

(c) Hamada coordinates, near the outermost surface,

(d) Boozer coordinates, near the outermost surface

References

(1) R. J. Bickerton, J. W. Conner, and J. B. Taylor : Nature Phys. Sci. **229** (1971) 110.

(2) A. A. Galeev and R. Z. Sagdeev : *in Plasma Physics and Controlled Nuclear Fusion Research* (Proc. 4th Int. Conf. Madison, 1971), Vol. 1, IAEA Vienna (1972) p.481.

(3) B. B. Kadomtsev and V. D. Shafranov : *ibid* p.479.

(4) M. C. Zarnstorff and S. C. Prager : Phys. Rev. Letters **53** (1984) 454.

(5) J. D. Treffert, J. L. Shohet, and H. L. Berk : Phys. Rev. Letters **53** (1984) 2409.

(6) M. C. Zarnstorff, M. G. Bell, and M. Bitter et al. : Phys. Rev. Letters **60** (1988) 1306.

(7) C. D. Challis J. D. Cordey, H. Hamnen et al. : Nucl. Fusion **29** (1989) 563.

(8) M. Kikuchi, M. Azumi, S. Tsuji, K. Tani, and H. Kubo : Nucl. Fusion **30** (1990) 343.

(9) F. L. Hinton and R. D. Hazeltine : Rev. Mod. Physics **48** (1976) 239.

(10) S. P. Hirshman and D. J. Sigmar : Nucl. Fusion **21** (1981) 1079.

(11) J. W. Conner and J. B. Taylor : Comments Plasma Phys. Controlled Fusion **11** (1987) 37.

(12) S. -I. Itoh and K. Itoh : Phys. Letters **A127** (1988) 267.

(13) K. C. Shaing : Phys. Fluids **31**(1988)8, and *ibid* **31** (1988) 2249.

(14) S. Besshou, O. Motojima, M. Sano, et al. : Plasma Phys. Contr. Fusion **26** (1984) 565.

(15) U. Gasparino, H. Massberg, M. Tutter, and WVII-A team : Plasma Phys. Contr. Fusion **30** (1988) 283.

(16) M. Murakami : *"Status of The ATF Experimental Program - Overview of Recent Results"*, 1st Int. TOKI Conf. Plasma Phys. Contr. Fusion, Toki, Japan, 4-7 Dec. 1989.

(17) O. Kaneko, Y. Takeiri, H. Yamada, et al. *"Driven Currents in Neutral Beam Heated CHS Plasma"*, 1st Int. TOKI Conf. Plasma Phys. Contr. Fusion, Toki, Japan, 4-7 Dec. 1989.

(18) K. C. Shaing and J. D. Callen : Phys. Fluids **26** (1983) 3315.

(19) K. C. Shaing, B. A. Carrerass, N. Dominguez, et al. : Phys. Fluids **B1** (8) (1989) 1663.

(20) K. C. Shaing, S. P. Hirshman, and J. D. Callen : Phys. Fluids **29** (1986) 521.

(21) M. Coronado and H. Wobig : Phys. Fluids **29** (1986) 527.

(22) H. Wobig : *" On Bootstrap Currents in Toroidal Systems"*, Res. Rep. Max-Planck-Institute for Plasma Physics, IPP 2/297, October 1988.

(23) S. Tokuda, T. Takeda, and M. Okamoto : J. Phys. Soc. Jpn. **58** (1989) 871.

(24) T. Ohkawa and M. S. Chu : in *International Stellarator/Heliotron Workshop* (IAEA Tech. Comm. Meeting, Kyoto, 1986), Vol.1, Plasma Physics Laboratory, Kyoto University (1987) 305.

(25) A. Iiyoshi, M. Fujiwara, O. Motojima, J. Todoroki, N. Ohyabu, and K. Yamazaki : Fusion Technol., **17** (1990) 169.

(26) N. Nakajima, M. Okamoto, J. Todoroki, Y. Nakamura, and M. Wakatani : Nucl. Fusion **29** (1989) 605.

(27) S. Hamada : Nucl. Fusion **2** (1962) 23.

(28) A. H. Boozer : Phys. Fluids **23** (1980) 904.

(29) N. Nakajima, M. Okamoto, and T. Amano : in preparation for publication.

(30) H. E. Mynick, T. K. Chu, and A. H. Boozer : Phys. Rev., Letters **48** (1982) 322.