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Two- and three-dimensional behavior of Rayleigh-Taylor and Kelvin-Helmholtz instabilities

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Abstract

Two- and three-dimensional behavior of the R-T and K-H instabilities is examined with a newly developed hydrodynamic code CIP. The mushroom structure owing to the K-H instability is quite different in two and three dimensions. The simulation without gravity show a similar behavior and hence this difference between two and three dimensions does not originate from the R-T instability. This difference cannot be explained by a linear analysis on the K-H instability also.

The Rayleigh-Taylor(R-T) instability has been a subject of primary interest for many years in many fields of physics. For example, it may be an origin of fuel-pusher mixing during implosion process in inertial confinement fusion (ICF).[1,2] Recently, the interest in this instability has grown in astrophysics because it may cause a mixing of materials in Super Nova 1987A as suggested from the observation data.[3,4] Although the R-T instability is well known and has been studied for many years, direct comparison between two- and three-dimensional simulations did not appear yet in open literatures.

We have developed a new general hyperbolic solver CIP (Cubic Interpolated Pseudoparticle) method and applied it to a number of test problems.[5-8] It has been proved that the CIP can give a less-diffusive and quite accurate result[7] without any flux limiting procedure frequently used in most of modern schemes.

In this paper, we apply the CIP method to the classical R-T instability in two and three dimensions. We will show here that a mushroom structure owing to the Kelvin-Helmholtz (K-H) instability is quite different in two and three dimensions. This difference cannot be explained by a linear analysis.

Let us first describe a configuration used in the simulation. Initially two fluids are placed at rest contacting with each other. The density of those fluids are $\rho = 1.0$ for $0 \leq x \leq 0.3$ and $\rho = 0.3$ for $0.3 \leq x$. The gravity g is imposed in the x-direction and its magnitude is 1.0. Pressure is obtained from a static force balance $\partial p/\partial x = \rho g$ starting from $p = 0.1$ at $x = 0$. The mesh is rectangular in every directions, number of meshes is $60(x) \times 15(y) \times 15(z)$ and its spacing is $\Delta x = 0.01$. The spacing in other directions is changed case by case because of the reason discussed below. Boundary is free in the x-direction and mirror in the y- and z-directions. In order to select the instability mode, velocity perturbation of imcompressible mode $(\nabla \cdot \vec{v})$ is imposed around the interface. Its wavelength in the y- and z-directions is chosen so that the system size in the y- and z-directions is equal to a half wavelength.

Figures 1(a) and 1(b) show the time sequence of density contours in two- and three-dimensional simulations, respectively. In both cases, initial perturbation velocity is set to 0.8. It is clear that the mushroom structure in the three-dimensional case is much smaller than that in the two-dimensional case. If the wavenumber in y and z directions is denoted by k and l, respectively, the growth rate of the R-T instability is proportional to $(gk)^{1/2}$ and $[g(k^2+l^2)^{1/2}]^{1/2}$ in two and three dimensions. Thus, in three dimensions we must use $2^{1/2}$ times smaller wavenumbers that make the growth rate equal to that in two dimensions. In reality, wavenumbers in Fig.1(b) are $2^{1/2}$ times smaller than that in Fig.1(a). This has been done by using $\Delta y = 0.01$ in two dimensions and $\Delta y = \Delta z = 0.01 \times 2^{1/2}$ in three dimensions. If we compare the case having the same wavelength, the difference is even larger because the growth rate of the R-T instability is $2^{1/2}$ larger than that in two dimensions and the K-H instability does not have enough time to grow.

It is widely recognized that the mushroom structure originates from the K-H instability. In order to seperate this effect from the R-T instability, we set $g = 0$ to eliminate the R-T instability. In this case, the interface moves with the speed intially given. The relative motion of two fluids at the interface induces the K-H instability. The numerical results are shown in Fig.2, where 2(a) and 2(b) are again the two- and three-dimensional results. This result is quite similar to that in Fig.1. As is easily understood, however, it takes much longer time to reach the final state at $t = 0.71$ in Fig.1 because the motion is not accelerated by the R-T instability.

In order to explain this difference, we derive a linear dispersion relation for the K-H instability. In the configuration shown in Fig.2(a), the K-H instability occurs on the surface of a plane whose thickness is $2a$, whereas in Fig.2(b) the K-H instability occurs on the surface of a cylinder whose radius is a. Therefore the dispersion relation is written as[9,10]

$$\frac{\omega}{\kappa V} = \frac{1}{1+F}, \tag{1}$$

where ω is the complex frequency ,V the relative velocity of two fluids at the interface and κ is the wavenumber. Here, F is given by

$$F = [\frac{\rho_1}{\rho_2}\frac{1+exp(2\kappa a)}{1-exp(2\kappa a)}]^{1/2}, \tag{2}$$

in two dimensions and

$$F = [-\frac{\rho_1}{\rho_2}\frac{K_1(\kappa a)I_0(\kappa a)}{K_0(\kappa a)I_1(\kappa a)}]^{1/2}, \tag{2'}$$

in three dimensions. In Eq.(2'), K_0, K_1, I_0, I_1 are the zeroth- and first-order modified Bessel functions of the first and the second kind, respectively. In Eqs.(2) and (2'), F is imaginary, and hence this wave propagates on the surface and grows. The growth rate is depicted in Fig.3, for $\rho_2/\rho_1 = 0.3$.

In the configuration shown in Fig.2, the seed of the K-H instability is given at the leading edge of the heavier fluid and hence $\kappa a \sim 1$. At this wavenumber, the growth rate in three dimensions is about 72% of that in two dimensions. It seems that this difference in the growth rate may explain the difference between Fig.2(a) and (b). We can confirm this reducing the initial velocity of the perturbation by 72% in two dimensions which corresponds to the reduction of V in the dispersion relation Eq.(1). Since ω is proportional to V, this reduction will make the growth rate equal both in two and three dimensions. In Fig.2(a) this reduction has already been done. If we use the velocity $V = 0.8$ without reduction in two dimensions, the mushroom structure develops even wider in the y-direction. We should note that we have used $\Delta y = \Delta z = 0.01$ in Fig.2(b) in contrast to Fig.1(b). By this choice, we can compare the two- and three-dimensional result with the same a in Eqs.(2) and (2').

From this comparison, we may conlclude that the difference in the mushroom structure between two and three dimensions cannot be explained by the linear analysis. Probably, it is attributed to the nonlinear process. It is natural to imagine that the rolling up of the fluid stays within a plane of two dimensions in two-dimensional case, whereas the rolling up in three-dimensional case can escape in other direction and may not grow so large. In this paper, we will not treat the process in detail but will be discussed in future.

All the simulations have been done on a UNIX workstation Data General AV300.

Refereces

1. Yu.F.Afanas'ev *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 617 (1976) [JETP Lett. **23**, 566 (1976)].
2. J.R.Freeman, M.J.Clauser and S.L.Thompson, Nucl. Fusion **17**, 223 (1977).
3. M.Itoh, S.Kumagai, T.Shigeyama, K.Nomoto and J.Nishimura, Nature **330**, 233 (1987).
4. E.F.Erickson, M.R.Haas, S.W.J.Colgan, S.D.Lord, M.G.Burton, J.Wolf, D.J.Hollenbach and M.Werner, Ap. J. (Letter) **330**, L39 (1988).
5. H.Takewaki and T.Yabe, J.Comput.Phys. **70**, 355 (1987).
6. T.Yabe and E.Takei, J.Phys.Soc.Japan **57**, 2598 (1988).
7. T.Yabe, T.Ishikawa, Y.Kadota and F.Ikeda **59**, 2301 (1990).
8. T.Yabe, T.Aoki, T.Ishikawa and P.Y.Wang, Computers & Fluids in print.
9. K.Niu, T.Yabe and M.Hori, J.Phys.Soc.Japan **38**, 1141 (1975).
10. S.Chandrasekhar, *Hydrodynamic and Hydrostatic Stability* (Oxford : Clarendon Press, 1961).

Figure Captions

Fig.1 : Time sequence of density contours in the R-T instability when $g = 1.0$. Initial velocity perturbation V is 0.8. (a) Two-dimensional result with $\Delta y = 0.01$. (b) Three-dimensional result with $\Delta y = \Delta z = 0.01 \times 2^{1/2}$.

Fig.2 : Time sequence of density contours without gravity. (a) Two-dimensional result with $\Delta y = 0.01$ and $V = 0.8 \times 0.72$. (b) Three-dimensional result with $\Delta y = \Delta z = 0.01$ and $V = 0.8$.

Fig.3 : The growth rate of the K-H instability. (a) For a surface wave on a moving plane. (b) For a surface wave on a moving cylinder.

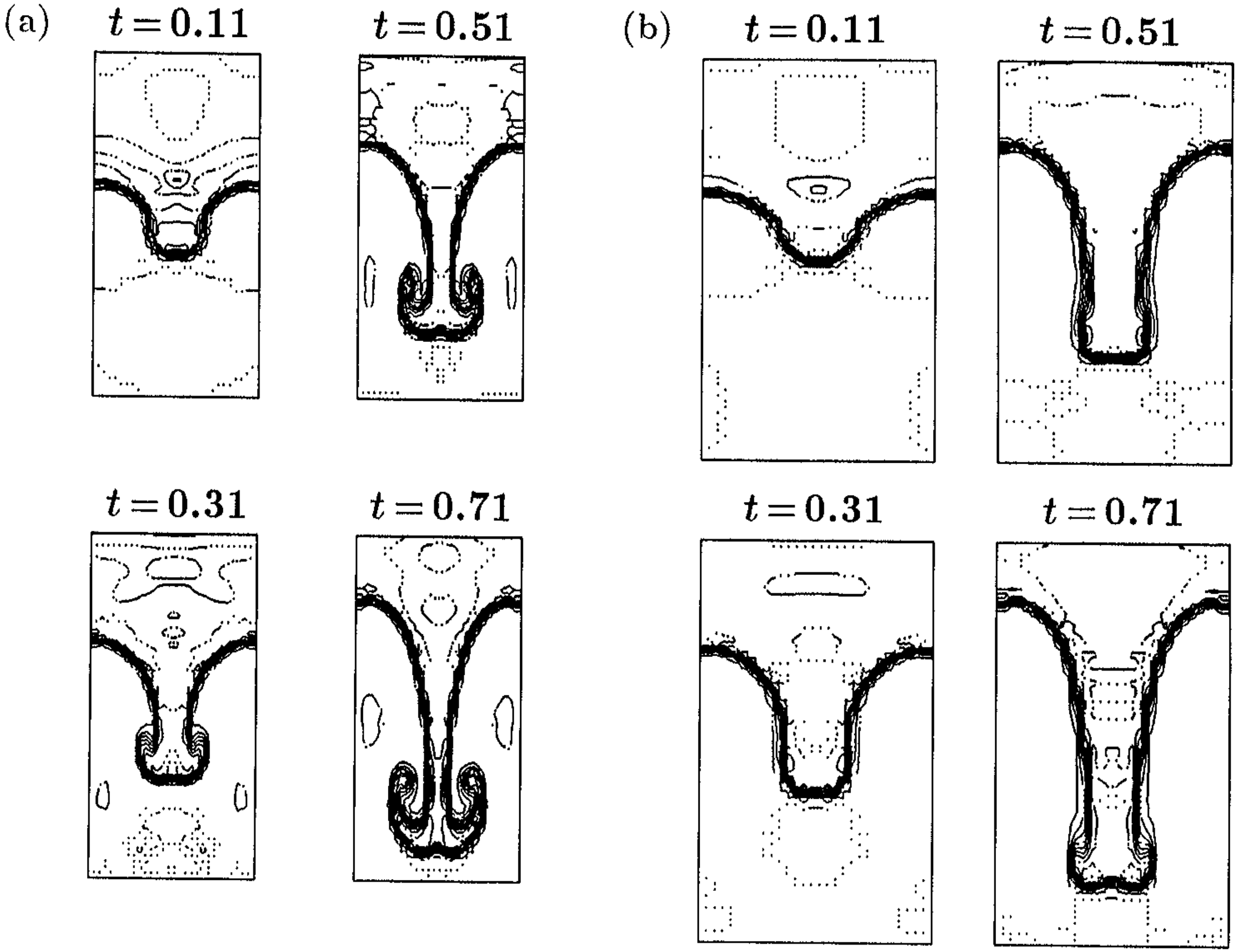

Fig.1

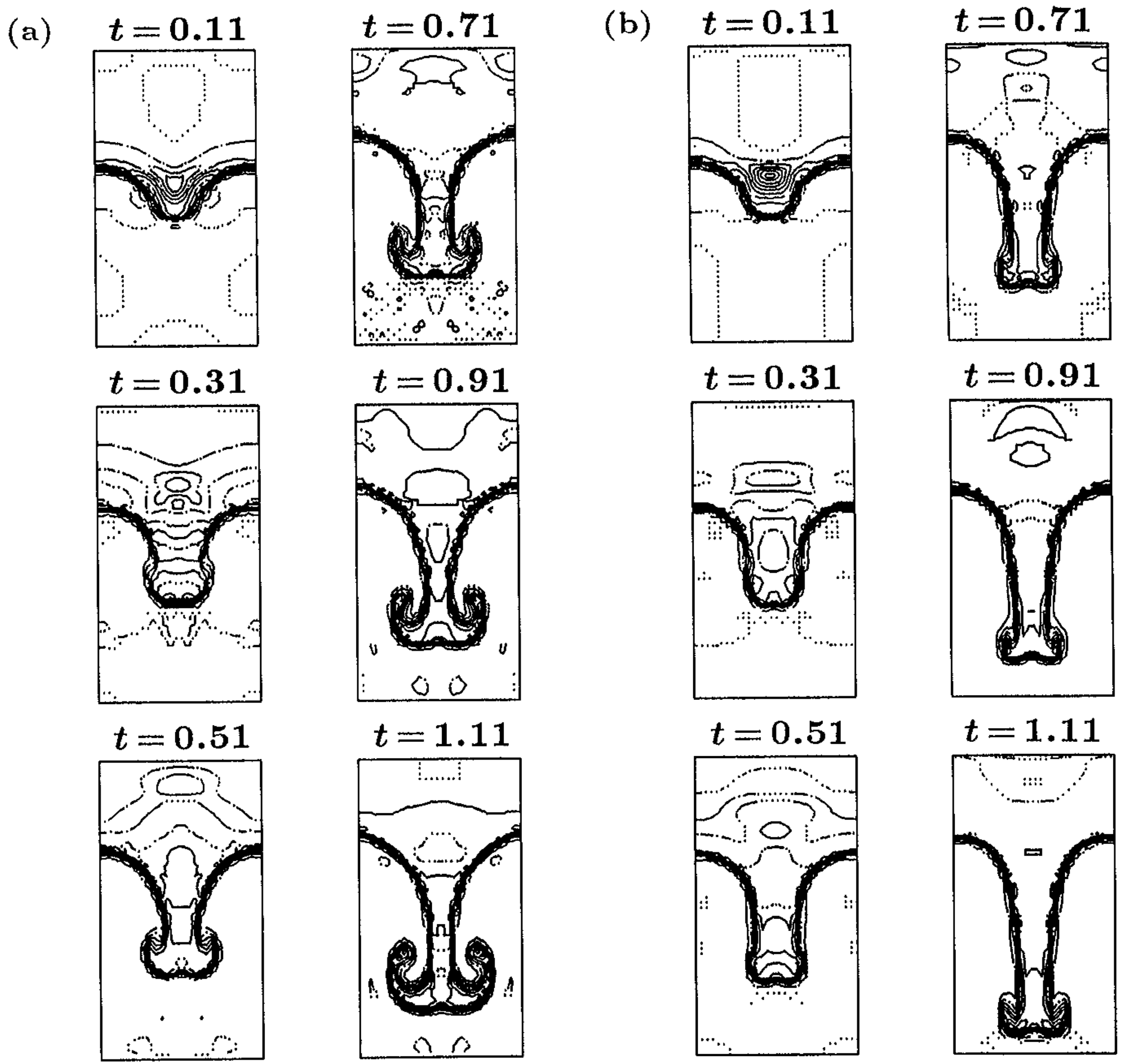
(a)
t = 0.11
t = 0.71
t = 0.31
t = 0.91
t = 0.51
t = 1.11
(b)
t = 0.11
t = 0.71
t = 0.31
t = 0.91
t = 0.51
t = 1.11

Fig.2

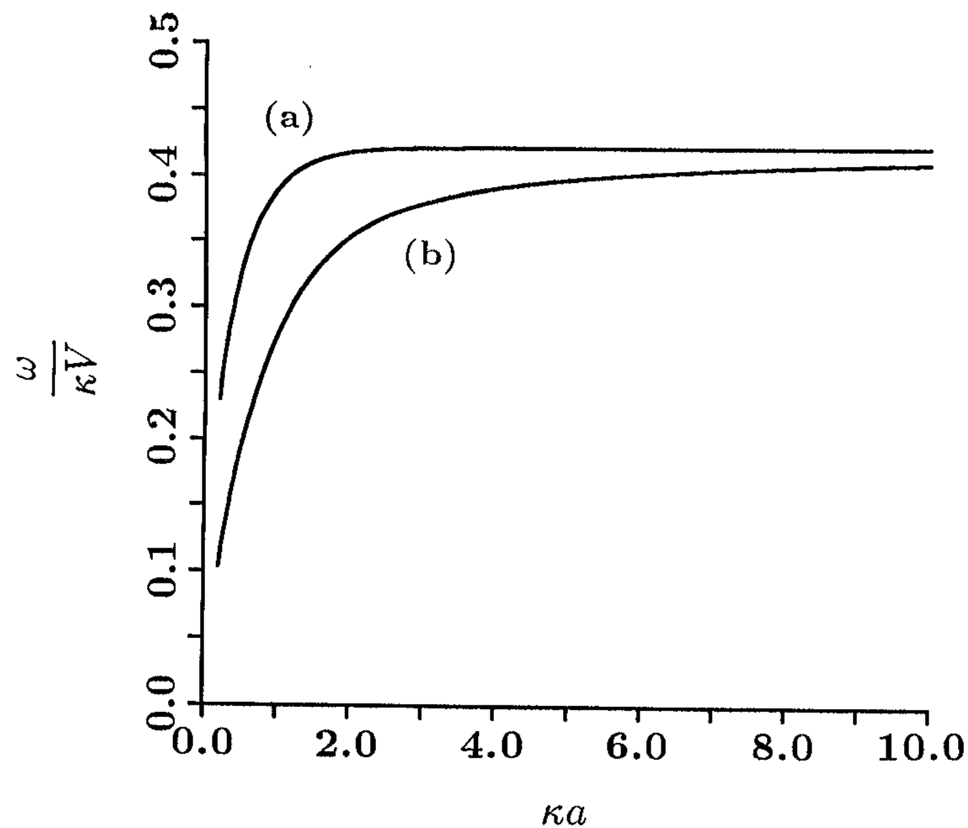

Fig.3