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Electrostatic Drift Mode in Toroidal Plasma with Minority Energetic Particles

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Abstract
The electrostatic drift wave theory is developed in the toroidal plasma with minority energetic particles and temperature inhomogenieties by taking into account the global toroidal gyromotion and arbitrary Larmor radius effect of particle making use of the toroidal propagator. When the toroidal eigenmode equation is approximated by the Weber type function, the dispersion relation indicates that the toroidal effect strongly destabilizes the ion drift mode even in uniform temperature plasmas. The elecrostatic toroidal drift ion mode is insensitive to the presence of energetic ions with the Maxwellian distribution except small Larmor radius and small toroidal effects cases (cylindrical fluid like plasmas), in which the mode is stabilized by the energetic ions.
Keywords: Electostatic drift wave, toroidal propagator, tokamak, toroidal resonance, finite Lamor radius effect, energetic ions, temperature inhomogeniety, growth rate, shear damping.

§1. Introduction

In high temperature fusion plasmas, there are minority energetic particles which may be produced by auxiliary heatings and fusion reactions. Whether these energetic particles stabilize or destabilize the low frequency drift modes in bulk plasma is interesting and important, because the drift waves are believed to be a cause of the anomalous transport phenomena in high temperature plasmas. In toroidal geometry, the low frequency plasma oscillations may be excited by troidal geometrical effect. However, the geometrical effect of confinement system on drift waves is not fully understood.

The purpose of this paper is to present a theory of the electrostatic drift wave in a toroidal plasma with minority energetic particles. In toroidal geometry, circulating untrapped particles make toroidal gyromotion, and induce toroidal(transit) resonance with various modes. Recently the transit resonance phenomenon recived attention in connection with the toroidal Alfven waves[1], energetic particle resonance[1)2)3)], and anomalous transport[4]. The toroidal resonance with the drift mode has also been studied in a previous paper[5]. We develop here the toroidal drift wave theory to the case with nonuniform temperatures, and energetic particles with Maxwellian distribution.

§2. Toroidal Propagator

We start with the linear perturbation of the Vlasov equation for distribution function $\tilde{f}(\mathbf{r},\mathbf{v},t)$ induced by the perturbed electric field $\tilde{\boldsymbol{E}}$:

$$\frac{\partial \tilde{f}}{\partial t}+\mathbf{v}.\nabla\tilde{f}+\frac{e}{cM}\mathbf{v}\times\boldsymbol{B}\,\frac{\partial \tilde{f}}{\partial \mathbf{v}}=-\frac{e}{M}\tilde{\boldsymbol{E}}\,\frac{\partial f_o}{\partial \mathbf{v}} \tag{1}$$

For the unperturbed distribution function f_o, we assune the form $f_o=N_o(P_\varphi)F_M(\mathbf{v})$ with N_o and F_M being the unperturbed plasma density and Maxwellian distribution:

$$F_M(\boldsymbol{v}) = (\pi v_{th})^{-\frac{3}{2}} \exp(-(\frac{v}{v_{th}})^2) \tag{2}$$

Here v_{th} is the thermal velocity, $P_\varphi=MRv_\varphi+(e/c)\psi(r)$ is the toroidal angular momentum which is assumed to be an invariant of particle motion in the usual toroidal coordinate system (r,θ,φ), $R=R_o+r\cos\theta$, $v_\varphi=v_\parallel B_\varphi/B \cong v_\parallel$, the flux ψ is defined by $\partial\psi/\partial r=-RB_\theta$ with B_θ being the polidal magnetic field. All other notation are standard. We will indicate the particle species by the subscript e and i for election and ion, respectively only when they are necessary to avoid confusion. Otherwise they will be omitted.

Since the left hand side of eq.(1) can be considered as the time derivative along the particle orbit, $d\tilde{f}/dt$, the solution of eq.(1) can be written in the form

$$\tilde{f}(\boldsymbol{r},\boldsymbol{v},t) = -\int_{-\infty}^{t} dt' \frac{e}{M}\tilde{\boldsymbol{E}}\frac{\partial f_o}{\partial \boldsymbol{v}} \quad , \tag{3}$$

where the time integral in eq.(3) should be performed along the particle orbit[6]. From eq.(2) and the definition of P_φ, the integral in eq.(3) can be written in the form

$$\frac{e}{M}\tilde{\boldsymbol{E}}\frac{\partial f_o}{\partial \boldsymbol{v}} = -\frac{ef_o}{T}(\boldsymbol{v}\tilde{\boldsymbol{E}} + v_d\tilde{\boldsymbol{E}}_\varphi) \tag{4}$$

where the diamagnetic drift velocity v_d is given by

$$v_d = \frac{eT}{eB_\theta}\{1+\eta((\frac{v}{v_{th}})^2-\frac{3}{2})\}\frac{d}{dr}\ln N_o \tag{5}$$

with $\eta = d\ln T/d\ln N_o$.

Expanding the perturbations in Fourier series of the form

$$\tilde{f}(\boldsymbol{r},\boldsymbol{v},t) = \sum_{mn} f_m(\boldsymbol{r},\boldsymbol{v})\exp(-i\omega t+i\xi_{mn}(t)) \tag{6}$$

with

$$\xi_{mn}(t) = m\theta(t) - n\varphi(t) \quad , \tag{7}$$

we have , from eq.(3),

$$f_m(\mathbf{r},\mathbf{v}) = -\frac{e}{T}[1+ig_m(\omega)(\omega-\omega_d)]\Phi_m f_o \tag{8}$$

where $\omega_d = \omega_* q_o (n/m)\{1+\eta((v/v_{th})^2-3/2)\}$ with $\omega_* = cTk_\theta/(eB)\ (d/dr)\ln N_o$, $k_\theta = m/r$, Φ_m is the scalar potential, and g_m is the toroidal propagator defined by

$$g_m(\omega) = \int_{-\infty}^{0} d\tau \exp(-i\omega\tau + i\xi_{mn}(t+\tau) - i\xi_{mn}(\tau)) \quad . \tag{9}$$

For the sake of simplicity, we assume the circular cross section of the magnetic surface, and neglect time variation of the radial coordinate $r(t)$.

We now determine the toroidal propagator $g_m(\omega)$ by calulating the coordinates $\theta(t)$ and $\varphi(t)$ of particle orbit making use of the guiding center velocity,

$$\mathbf{v}_g = v_\parallel \mathbf{b} + \frac{c\mathbf{E}\times\mathbf{b}}{B} + \mathbf{v}_B \tag{10}$$

where $\mathbf{b}=\mathbf{B}/B$ and $\mathbf{v}_B = \mathbf{b}\times(v_\perp^2/2\nabla\ln B + v_\parallel^2\kappa)/\Omega$ is the precessional drift velocity with $\kappa = (\mathbf{b},\nabla)\mathbf{b}$ being the curvalure of the magnetic field lines. In the low-β plasmas, $\beta = 8\pi p/B^2 \ll 1$, since $\kappa \cong \nabla \ln B$, we have $\mathbf{v}_B = (v_\parallel^2 + v_\perp^2/2)\nabla \ln B/\Omega$. The unperturbed magnetic field is assumed to be given by $B_\theta = B_{\theta o}(1+\varepsilon\Lambda\cos\theta)$[7], and $B_\varphi = B_o(1-\varepsilon\cos\theta)$ where $\varepsilon = r/R_o$ and $\Lambda = 1+\beta_\theta - li/2$ with β_θ and li being the poloidal beta $\beta_\theta = 8\pi p_o/B^2$ and the inductance of plasma ring, respectively. In the tokamak ordering, $B \cong B_\varphi$, the magnetic drift velocity becomes $\mathbf{v}_B = v_o(-\cos\theta, \sin\theta, o)$ with $v_o = (v_\parallel^2 + v_\perp^2/2)/(\Omega R_o)$. If we assume that the unperturbed electric field has only radial component: $\mathbf{E} = (E_r, 0, 0)$, we have the system of equations from eq.(10)

$$\frac{dr}{dt} = -v_o\cos\theta \quad , \tag{11}$$

$$\frac{d\theta}{dt} = \frac{v_\parallel}{r} b_\theta - \frac{E_r b_\varphi}{crB} + \frac{v_o}{r}\sin\theta , \qquad (12)$$

$$\frac{d\varphi}{dt} = \frac{v_\parallel}{R} b_\varphi + \frac{E_r b_\varphi}{cRB} \quad . \qquad (13)$$

Making use of the invariants of motion: $E=M(v_\parallel{}^2+v_\perp{}^2)/2+e\Phi_0$ (energy) and $\mu=Mv_\perp{}^2/(2B)$ (angular momentum), the parallel velocity is expressed by

$$v_\parallel = \left(\frac{2}{M}(E - \mu B - e\Phi_o)\right)^{\frac{1}{2}} \quad , \qquad (14)$$

where $E_r=-d\Phi_0/dr$. It can also be written in term of $\lambda=\mu B/E=(v_\perp/v)^2$:

$$v_\parallel = v_{\parallel o}(1 - \lambda_o(1 - \varepsilon\cos\theta) - e\Phi_o/E)^{1/2} \qquad (15)$$

with $v_{\parallel o}=(2E/M)^{1/2}$. As a function of poloidal angle θ, $v_\parallel$ shows a closed orbit when $k_0=(1-\lambda_0-e\Phi_0/E-\varepsilon\lambda_0)/(2\varepsilon\lambda_0)>1$, i.e., the particle orbit is trapped inside the separatrix at $k_0=1$, while for $k_0<1$, it is outside the separatrix representing passing particle orbit.

For the passing particles, $v_\parallel$ can be given by the Jacobian elliptic function dn[8]. Away from the separatrix, however, it can be approximated by a constant. In this case, from eqs.(12) and (13), the eikonal function defined by eq.(7) can, to the first order of ε, be written in the form

$$\xi_{mn}(t) = (k_{\parallel m} v_\parallel - \omega_E)t + a_m\sin\theta(t) - a_d\cos\theta(t) \quad , \qquad (16)$$

where $k_{\parallel m} = (m-nq_0)/(R_0 q_0)$, $q_0=rB_0/(R_0 B_{\theta 0})$ is the safety factor, $\omega_E=k_\theta E_r/(eB)$ is the poloidal circulation frequency due to the radial electric field, $a_m=\varepsilon(m(1+\Lambda)+nq_0)$, $a_d=v_0/(r\omega_t)$ and $\omega_t=v_\parallel/(R_0 q_0)$ is the transit frequency. The first term in eq.(16) is the usual secular term shifted by ω_E. The second term represents the global toroidal gyromotion, and the third term comes from the precessional drift motion.

In addition to the parallel motion along the magnetic field lines and slow drift motion, particles suffer fast Larmor gyromotion which is determined by the equation of motion[9]: $dv/dt=\Omega \mathbf{v}\times\mathbf{b}$ or

$$\frac{d^2v_r}{dt^2}+\Omega^2 v_r=0 \quad \text{and} \quad \frac{d^2v_\theta}{at^2}+\Omega^2 v_\theta=0$$

which yield $v_r=v_\perp\cos\chi$ and $v_\theta=v_\perp\sin\chi$ with $\chi=\chi_0+\Omega t$. If we add this Larmor gyromotion to eq.(16), and the second and third terms are coupled together, we have

$$\xi_{mn}(t)=(k_{\parallel m}v_\parallel-\omega_E)t+a_t\sin(\theta(t)-\delta)+a_L\cos\chi(t)\ , \tag{17}$$

where $a_t=(a_m{}^2+a_d{}^2)^{1/2}$, $\delta=\tan^{-1}(a_t/a_m)$ and $a_L=k_\theta v_\perp/\Omega$.

Introducing eq.(17) into eq.(9), making use of the formula, $\exp(ia\sin\theta)=\sum_{l=-\infty}^{\infty}J_l(a)e^{il\theta}$, we have the propagator in the form

$$g_m(\omega)=i\sum_{p,p',l,l'}\frac{J_l(a_L)J_{l'}(a_L)J_p(a_t)J_{p'}(a_t)}{\omega-\omega_E-k_{\parallel m}v_\parallel-p\omega_t-l\Omega}e^{i(l-l')\chi}e^{i(p-p')\theta} \tag{18}$$

where J_l is the Bessel function. In what follows, we consider the case of $\omega_E=0$ and $\omega\ll\Omega$. We introduce the velocity moment of the propagator which is also averaged over the magnetic surface:

$$G_p=\frac{i\omega}{2\pi}\oint d\theta\int d^3v g_m(\omega)\left(\frac{v}{v_{th}}\right)^p F_M(\boldsymbol{v})\ . \tag{19}$$

Integrating both side of eq.(8), we have the perturbed density in terms of G_p:

$$N_m=-\frac{eN_o}{T}[1+G_o(1-\frac{\omega_*}{\omega}(1-\frac{3}{2}\eta))+\frac{\omega_*}{\omega}\eta G_2]\Phi_m\ . \tag{20}$$

Substitution of eq.(18) into eq.(19), making use of the integral formulae

$$\int_0^\infty dv_\perp v_\perp J_o^2(a_L)\exp(-(\frac{v}{v_{th}})^2)=\frac{v_{th}^2}{2}\Gamma_o(b)\ , \tag{21}$$

$$\int_0^\infty dv_\perp v_\perp{}^3 J_o^2(a_L)\exp(-(\frac{v}{v_{th}})^2)=\frac{v_{th}^4}{2}\{\Gamma_o(b)+b(\Gamma_1(b)-\Gamma_o(b))\} \tag{22}$$

we obtain

$$G_o = \Gamma_o(b)\,\hat{Z}_o(\omega)\,, \tag{23}$$

$$G_2 = \Gamma_o(b)\,\hat{Z}_2(\omega) + (\Gamma_o + b(\Gamma_1 - \Gamma_o))\,\hat{Z}_o(\omega)\ , \tag{24}$$

where $\Gamma_0(b) = e^{-b} I_0(b)$, $b = (k_\perp \rho)^2$ with I_0 being the modified Bessel function, and the toroidal plasma dispersion function $Z_p(\omega)$ is defined by

$$\hat{Z}_p(\omega) = \sum_{l=-\infty}^{\infty} \frac{\omega}{|k_{m-l}| v_{th}} J_l^2(a_t)\, \bar{Z}_p(\zeta_{m-l})\ , \tag{25}$$

$\zeta_{m-l} = \omega/(k_{m-l} v_{th})$ and $\bar{Z}_p$ has been defined by

$$\bar{Z}_p(\zeta) = Z_p(\zeta) - \varepsilon_T^{-\frac{p}{2}} Z_p(\bar{\zeta}) \tag{26}$$

with $\bar{\zeta} = \zeta \varepsilon_T^{-1/2}$, $\varepsilon_T = \varepsilon/(1+\varepsilon)$, and Z_p is the p-th moment of the plasma dispersion function:

$$Z_p(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{ds}{s-\zeta} s^p e^{-s^2} \tag{27}$$

In eq.(26), the second term represents the trapped particle loss cone effect which vanishes without the toroidal effect $\varepsilon \to 0$. Equation (25) also reduces to the usual form $\hat{Z}_p = \zeta_m Z_p(\zeta_m)$ when $\varepsilon \to 0$. The moments can be obtained from the usual plasma dispersion function $Z_0(\zeta)$ by the relation[8]: $Z_1(\zeta) = 1 + \zeta Z_0(\zeta)$, $Z_2 = \zeta Z_1(\zeta)$, $Z_3(\zeta) = 1/2 + \zeta Z_2(\zeta)$. Introducing eqs.(23) and (24) into eq.(20), we have

$$N_m = -\frac{eN_o}{T}[1 + \{(1 - \frac{\omega_*}{\omega}(1-\eta))\Gamma_o + \frac{\omega_*}{\omega}\eta b(\Gamma_1 - \Gamma_o)\}\hat{Z}_o + \frac{\omega_*}{\omega}\eta\Gamma_o \hat{Z}_2]\Phi_m\,. \tag{28}$$

For trapped particles, if we neglect the magnetic drift effect, the propagator may be approximated by

$$g_m = i\sum_{l,l'} \frac{J_l(k_{\perp m} v_\perp/\omega_b)\, J_{l'}(k_{\perp m} v_\perp/\omega_b)}{\omega - l\omega_b} e^{i(l-l')\theta}\,, \tag{29}$$

where $\omega_b=\omega_t\pi\sqrt{\varepsilon}\frac{v_\parallel}{v_\perp}2^{\frac{3}{2}}K(k)$ is the frequency and v_1 is the velocity of bounce motion.

§3. Dispersion Relation

We proceed to the derivation of the electrostatic eigen mode equation for Φ_m. For electrons, we assume $b_e=(k_\perp\rho_e)^2/2=0$, and then $\Gamma_0(b_e)=1$ and $\Gamma_1(b_e)=0$. In this case, eq.(28) can be written as

$$N_m^e=-\frac{eN_o}{T}[1+(1-\frac{\omega_*}{\omega}(1-\eta_e))\hat{Z}_o+\frac{\omega_*}{\omega}\eta_e\hat{Z}_2]\Phi_m \quad , \tag{31}$$

where ω_* is the electron diamagnetic drift frequency. The ion diamagnetic drift frequency becomes $\omega_{*i}=-\omega_*\tau$ with $\tau=T_i/T_e$.

In order to introduce the radial variation of $\Phi_m(r)$, we separate $k_\perp^2$ in b as $k_\perp^2=k_\theta^2-(\partial/\partial r)^2$ and write $\bar{b}=b-(\rho\partial/\partial r)^2$ with $b=(k_\theta\rho)^2/2$. Then, the function of $\bar{b}$ may be expanded in the form : $\Gamma_0(\bar{b})=\Gamma_0(b)(1+\alpha(\partial/\partial x)^2)$, where $\alpha=-d\ln\Gamma_0(b)/db$, and $x=(r-r_s)/\rho_i$ with r_s being the radial position of rational surfce $k_m(r_s)=0$. The same expansion will be applied for $\bar{b}(\Gamma_1-\Gamma_0)=(1+\alpha_1(\partial/\partial x)^2)$ with $\alpha_1=-d\ln b(\Gamma_1-\Gamma_0)/db$. Introducing these expansions for $\Gamma_0(\bar{b})$ and $\bar{b}(\Gamma_1-\Gamma_0)$ into eq.(28) both for ions and energetic ions, and substituting resultant expressions and eq.(30) into the quasineutrality condition, $-eN_{me}+eN_{mi}+eN_{mh}=0$, we have the eigen mode equation:

$$\frac{d^2\Phi_m(x)}{dx^2}+P_m(x)\Phi_m(x)=0 \quad , \tag{32}$$

where the potential coefficient has been given by

$$P_m(x)=\frac{P_N(x)}{P_D(x)} \quad , \tag{33}$$

and the numerator P_N and denominator P_D are, respectively, given as follows:

$$P_N(x) = 1+\tau+ (1-\frac{\omega_*}{\omega}(1-\eta_e))\hat{Z}_o(\zeta_e) - \frac{\omega_*}{\omega}\eta_e \hat{Z}_2(\zeta_e) + \{(1+\tau\frac{\omega_*}{\omega}(1-\eta_i))\Gamma_o(b)$$

$$+\eta_i\tau\frac{\omega_*}{\omega}b_i(\Gamma_1-\Gamma_o)\}\hat{Z}_o(\zeta_i) + \tau\frac{\omega_*}{\omega}\eta_i \hat{Z}_2(\zeta_i)$$

$$+c_h[\{(1-\frac{\omega_{*h}}{\omega}(1-\eta_h))\Gamma_o(b_h) - \eta_h\frac{\omega_{*h}}{\omega}b_h(\Gamma_1-\Gamma_o)\}\hat{Z}_o(\zeta_h) - \frac{\omega_{*h}}{\omega}\eta_h \hat{Z}_2(\zeta_h)] \quad (34)$$

$$P_D(x) = \alpha_i[\{(1+\frac{\omega_*}{\omega}(1-\eta_i)\Gamma_o(b_i) + \eta_i\frac{\omega_*}{\omega}.\frac{\alpha_i}{\alpha}b_i(\Gamma_1-\Gamma_o)\}\hat{Z}_o(\zeta_i)$$

$$+\frac{\omega_*}{\omega}\eta_i \hat{Z}_2(\zeta_i) + c_h\frac{\rho_h}{\rho_i}\alpha_h[\{1-(1-\frac{\omega_{*h}}{\omega}(1-\eta_h))\Gamma_o(b_h)$$

$$-\eta_h\frac{\omega_{*h}}{\omega}.\frac{\alpha_i}{\alpha}b_h(\Gamma_1-\Gamma_o)\}\hat{Z}_o(\zeta_n) - \frac{\omega_{*h}}{\omega}\eta_h \hat{Z}_2(\zeta_n)]] \quad (35)$$

The coefficient c_h is given by $c_h = n_h T/(N_o T_h)$ and $\tau = T_i/T_e$.

We derive the dispersion relation assuming the Pearstein-Berk mode[10] approximation for the eigenfunction of eq.(32). Since $P_m(x)$ is an even function with respect to x, it can be expanded in the form

$$P_m(x) = P_m(0) + (\sigma x)^2. \quad (36)$$

where $\sigma = (1/2 d^2 P_m/dx^2)^{1/2}|_{x=0}$. For $\Phi_m = H_n(z)\exp(-i\sigma z^2)$ with H_n being the Hermite function and $z^2 = i\sigma x^2$, the eigen value condition becomes

$$P_m(0) = (2n+1)i\sigma \quad (37)$$

with n=0,1,2,....

Uniform Temperature Plasmas

First we consider a simple case of uniform temperature plasma $T_i = T_e$ and $\eta_e = \eta_i = \eta_h = 0$. In this case, from eqs.(34) and (35), P_N and P_D are reduced to

$$P_N(x) = 2+ (1-\frac{\omega_*}{\omega})\hat{Z}_o(\zeta_e) + (1+\frac{\omega_*}{\omega})\Gamma_o(b)\hat{Z}_o(\zeta_i) + c_h(1-\frac{\omega_{*h}}{\omega})\Gamma_o(b_h)\hat{Z}_o(\zeta_n) \quad (38)$$

$$P_D(x) = \alpha_i[(1+\frac{\omega_*}{\omega})\Gamma_o(b_i)\hat{Z}_o(\zeta_i) + c_h\frac{\rho_h}{\rho_i}\frac{\alpha_h}{\alpha_i}(1-\frac{\omega_{*h}}{\omega})\Gamma_o(b_h)\hat{Z}_o(\zeta_n) \quad (39)$$

we must examine the characteristics of the potential function P_m near the rational surface x=0. Making use of the usual expression $k_m(x)=k_m(0)+ k_\theta\rho_i x/L_s$, from the definition, we have $\zeta_{m+1}=\omega/(\omega_s x+l\omega_t)$ where $\omega_s=k_\theta\rho_i v_{th}/L_s$ and L_s is the shear length. Near the rational surface, $\omega_s x\ll\omega_t$, for l=0, $\zeta_m=\omega/(\omega_s x)$ which can be much larger than unity. While for l=0, $\zeta_{m+1}\cong\omega/l\omega_t$ which is independent of x.

Recalling the approximation $\zeta Z_o(\zeta)\cong-1-1/(2\zeta^2)$ for $\zeta\gg1$, we have the toroial plasma dispersion function in the form

$$\hat{Z}_o=-G_o-\frac{1}{2}J_o^2(a_t)\left(\frac{\omega_s}{\omega}x\right)^2+i\delta \tag{40}$$

where $\quad G_o=J_o^2(a_t)(1-\sqrt{\varepsilon_T})+\sqrt{\varepsilon_T}\quad$ and

$$\delta=J_o^2(a_t)\frac{\omega}{\omega_s|x|}\mathrm{Im}\bar{Z}_o\left(\frac{\omega}{\omega_s x}\right)+2\sum_{l=1}^{\infty}J_l^2(a_t)\frac{\omega}{l\omega_t}\mathrm{Im}\bar{Z}_o\left(\frac{\omega}{l\omega_t}\right)\ , \tag{41}$$

with $\mathrm{Im}\bar{Z}_o(\zeta)=\sqrt{\pi}[\exp(-\zeta^2)-\exp(-\varepsilon_T^{-1}\zeta^2)]$. In the slab geometry, $\varepsilon=0$, the second term (toroidal resonance) in eq.(41) vanishes and the first term reduces to the usual one which vanishes rapidly near the rational surface x=0.

Applying eq.(40) to eq.(38), we have

$$P_N(x)=2-\left(1-\frac{\omega_*}{\omega}\right)(G_o+G_{1e}x^2-i\delta_e)-\left(1+\frac{\omega_*}{\omega}\right)\Gamma_o(b_i)(G_o+G_{1i}x^2-i\delta_i)$$
$$+C_h\{1-\left(1-\frac{\omega_*}{\omega}\right)(G_o+G_{1h}x^2-i\delta_h)\Gamma_o(b_h)\}\ , \tag{42}$$

where $G_1=J_o^2(a_t)(\omega_s/\omega)^2/2$. From eq.(37), the zeroth order dispersion relation may be written by $P_m(0)=0$, which can be rewritten, from eqs.(33) and (42), in the form

$$P_N(0)=2-\left(1-\frac{\omega_*}{\omega}\right)(G_o-i\delta_e)-\left(1+\frac{\omega_*}{\omega}\right)\Gamma_o(b_i)(G_o-i\delta_i)$$
$$+C_h\{1-\left(1-\frac{\omega_{*h}}{\omega}\right)(G_o-i\delta_h)\Gamma_o(b_h)\}=0\ . \tag{43}$$

From eq.(43), we obtain the normalized real frequency

$$\frac{\omega}{\omega_*}=-\frac{G_o\{1-\Gamma_o(b_i)+C_h\frac{\omega_{*h}}{\omega_*}\Gamma_o(b_h)\}}{D_n}\ , \tag{44}$$

where $D_n=2-G_o(1+\Gamma_o)+C_h(1-G_o\Gamma_o(b_h))$. By the same manner, the normalized growth rate is obtained in the form

$$\frac{\gamma}{\omega_*} = -\frac{(1-\frac{\omega}{\omega_*})\delta_e + (1+\frac{\omega}{\omega_*})\Gamma_o(b_i)\delta_i + C_h(\frac{\omega}{\omega_*}-\frac{\omega_{*h}}{\omega_*})\Gamma_o(b_h)\delta_h}{D_n} \tag{45}$$

Variations of ω/ω_* as a function of b_i for various values of a_t calculated from eq.(44) are presented in Fig.1. The energetic ion contribution to ω is small, because $C_h\omega_{*h}/\omega_* \cong N_h/N_o \simeq 10^{-2}$. It can be dominant in eq.(44) only when $b_i \to 0$. As seen in Fig.1a, ω is negative and close to the ion drift frequency, $\omega \cong -\omega_*$. the electron contribution may be negligible in eq.(45) because $\omega/\omega_{te} << 1$ and $|\delta_e| << |\delta_i|$. The electron contribution is important only when $\omega = -\omega_*$ at which the ion term in eq.(45) vanishes. Although the coefficient of energetic ion term is small, of the order of N_h/N_o, since $\omega/\omega_{th} \simeq 1$, δ_h can be large as compared with δ_i and the energetic ion term may have some stabilizing contribution. Variation of the normalized growth rate as a function of b_i are presented in Fig.1b for various values of the toroidal effect a_t.

We now consider the shear damping effect σ. Since $P_N(x)$ and $P_D(x)$ are even function near x=0, we have $P'_N(0)=P'_D(0)=0$. Making use of the zeroth order relation $P_N(0)=0$, we obtain

$$P_m''(0) = \frac{P_N''(0)}{P_D(0)} \quad . \tag{46}$$

From eqs.(39) and (42), we have

$$P_N''(0) = -2G_{1e}(1-\frac{\omega_*}{\omega}) - 2G_{1i}(1+\frac{\omega_*}{\omega})\Gamma_o(b_i) - 2G_{1h}C_h(1-\frac{\omega_{*h}}{\omega})\Gamma_o(b_h)] , \tag{47}$$

$$P_D(0) = -\alpha_i[(1+\frac{\omega_*}{\omega})\Gamma_o(b_i)(G_o-i\delta_i) + C'_h(1-\frac{\omega_{*h}}{\omega})\Gamma_o(b_h)(G_o-i\delta_h) , \tag{48}$$

The energetic contribution in eq.(40) is small, $C_h'\omega_{*h}/\omega \simeq N_h/N_o$, and may be neglected.

Since the scale of the eigenmode equation is normalized by the ion Larmor radius, if we take the ion term in eqs.(47) and (48), from eq.(46), we have

$$P_m''(0) = \frac{2G_{1i}}{\alpha_i(G_o - i\delta_i)} \quad . \tag{49}$$

Introducing eqs.(43),(48) and (49) into eq.(37), the normalized shear damping correction to the growth rate γ is obtained in the form

$$\frac{\gamma_s}{\omega_*} = -\left|\frac{J_c}{D_n}\frac{\omega_{si}}{\omega}\right| \left(\frac{\alpha_i}{2}\right)^{\frac{1}{2}} \left(1+\frac{\omega_*}{\omega}\right) \Gamma_o(b_i) \left(G_o^2+\delta_i^2\right)^{\frac{1}{4}} \cos\phi \tag{50}$$

where $\varphi=(1/2)\tan^{-1}(\delta_i/G_o)$. In eq.(50), the energetic particle effect is in D_n in the denominator, which is small. The shear damping becomes very large when $b_i \to 0$, because $\omega \to 0$ as seen in Fig.1a and $\gamma_s \to -\infty$ as seen in eq.(50). The total growth rate $\bar{\gamma}/\omega_* = (\gamma+\gamma_s)/\omega_*$ as a function of b_i is plotted in Fig.1c in the case of no energetic ions for various values of the toroidal effect a_t. As seen in Fig.1c, the ion drift mode is completely stabilized by the shear damping when the toroidal effect a_t is small.

Numerical results for the case with energetic ions are plotted in Fig.2 for various values of the toroidal effect a_t. As seen in Fig.2a, the oscillation frequency change sign in the region of small b_i. In this region, the drift mode is completely stabilized as seen in Fig.2c. The energetic ions weakly stabilize the drift mode in the uniform temperature plasmas.

§4. Summary and Discussion

We have derived the eigen mode equation for the electrostatic drift waves in toroidal plasmas with minority energetic particles and temperature inhomogenieties. The toroidal propagator used in the theory has been generalized including the precessional drift motion of particles, which has however minor effect on the analysis except in the high energy region.

Assuming the Weber type potential function, the dispersion relation is derived, which is evaluated in the simple case of uniform temperature plasma. The toroidal effect strongly destabilizes the ion drift mode by the ion transit resonance. The elecrostatic drift mode was insensitive to the presence of energetic ions except for the small Larmor radius and small toroidal effect region, $b_i \to 0$ and $a_t \to 0$, where the drift mode is stabilized by energetic ions.

Although the uniform temperature drift mode is not sensitive to the energetic ions, if we introduce the temperature inhomogeniety, the dispersion relation may change significantly, which is remained to be studied.

In the derivation of the shear damping in eq.(50), the second ion term in eq.(47) was taken into account to have the same ion scale with the eigenmode equation, and eq.(49) was reduced. However, in eq.(47), the first electron term is dominant because G_{1e} is much larger than G_{1i}. If we take into account this electron term, $P_m''(0)$ becomes entirely different form as compared with eq.(49), and the sign of $P_m''(0)$ may change. This means that the potential function $P_m(x)$ near x=0 behaves differently in the electron and ion scales, i.e., when $P_m''(0) > 0$ in the lager ion scale, the opposite sign could happen in the small electron scale. If we take into account the both electron and ion scales, the eigen function may need some correction to the Hermite function, or more advanced eigenfunction should be introduced, and the dispersion relation (37) may suffer some change. This problem is also remained to be investigated.

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Figures Captions

Fig.1a: Normalized oscillation frequency ω/ω_* versus $b_i=k_\perp^2\rho_i^2/2$ for various values of $a_t=2m(1+\Lambda)\varepsilon$ in case of no energetic particles $c_h=0$.

Fig.1b: Normalized growth rate γ/ω_* versus b_i for verious values of a_t in the same case of Fig.1a.

Fig.1c: Normalized growth rate with shear damping $\bar{\gamma}/\omega_*=(\gamma+\gamma_s)/\omega_*$ versus b_i for various values of a_t in the same case as Fig.1a.

Fig.2a: Normalized oscillation frequency ω/ω_* versus b_i for various values of a_t with energetic ions, $c_h=0.005$, $\omega_{*h}/\omega_*=-20$,

Fig.2b: Normalized growth rate γ/ω_* versus b_i for verious values of a_t in the same case of Fig.2a.

Fig.2c: Normalized growth rate with shear damping $\bar{\gamma}/\omega_*=(\gamma+\gamma_s)/\omega_*$ versus b_i for various values of a_t in the same case of Fig.2a.

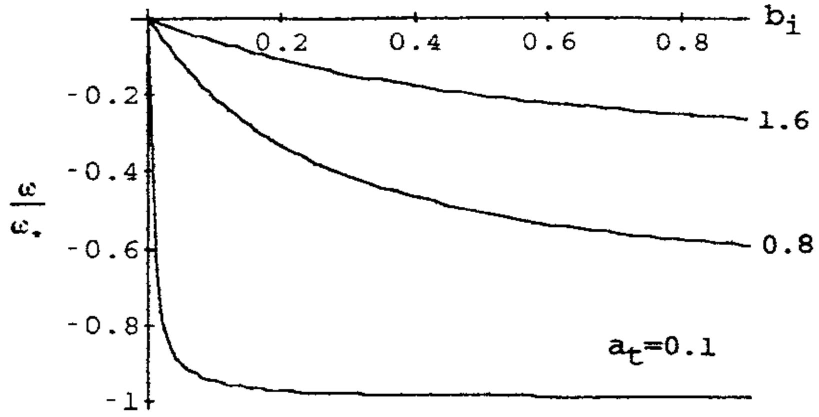

Fig.1a

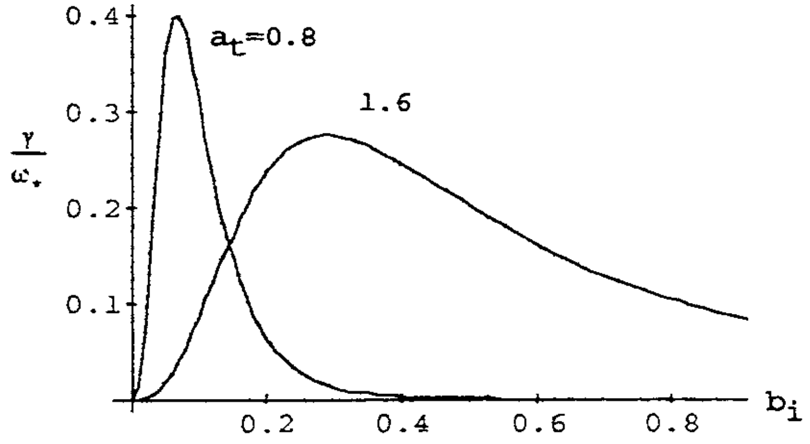

Fig.1b

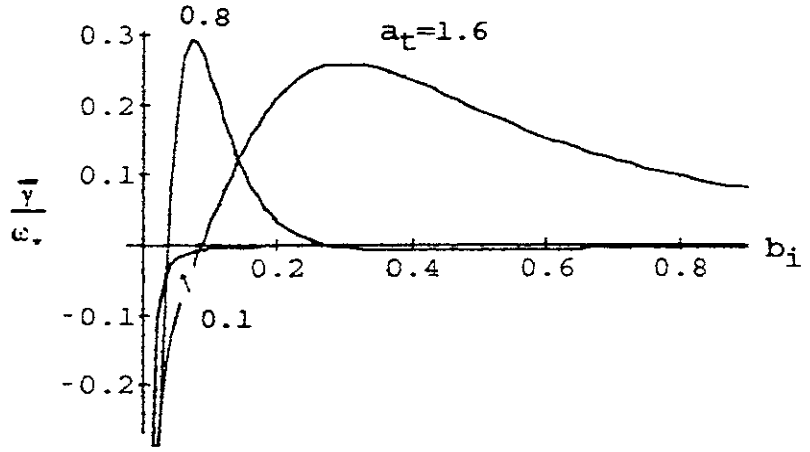

Fig.1c

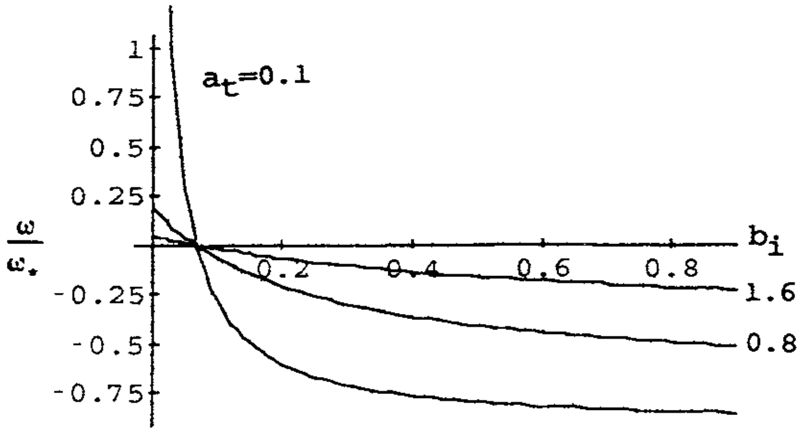

Fig.2a

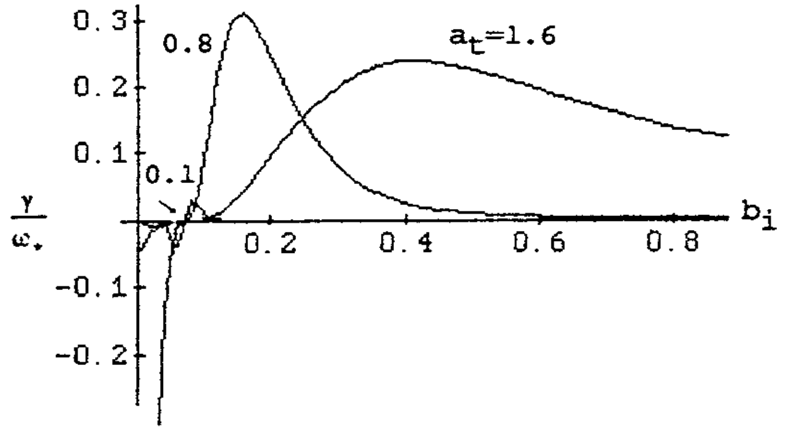

Fig.2b

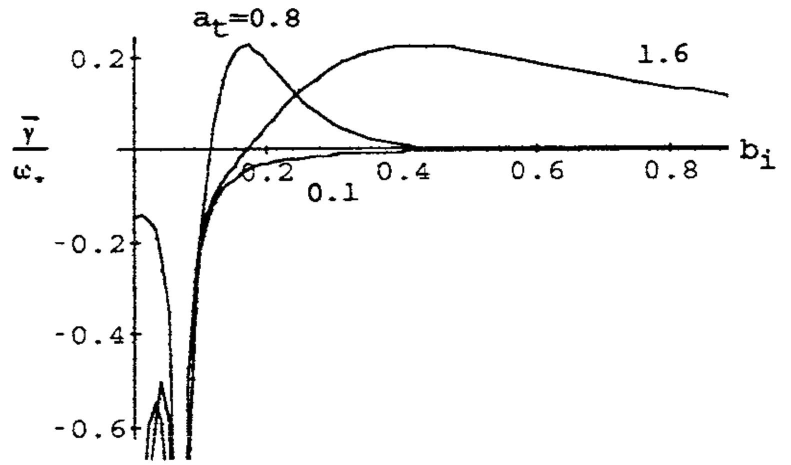

Fig.2c

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