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Multi-dimensional Cubic Interpolation for ICF Hydrodynamics Simulation

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Abstract

A new interpolation method is proposed to solve the multi-dimensional hyperbolic equations which appear in describing the hydrodynamics of ICF implosion. The advection phase of the cubic-interpolated pseudo-particle (CIP) is greatly improved, by assuming the continuities of the second and the third spatial derivatives in addition to the physical value and the first derivative. These derivatives are derived from the given physical equation. In order to evaluate the new method, Zalesak's example is tested, and we obtain successfully good results.

keywords ICF, inertial confinement fusion, multi-dimension, hydrodynamics, target implosion, cubic interpolation, hyperbolic equation, cubic-interpolated pseudo-particle, CIP

I Introduction

Non-uniform target implosion of inertial confinement fusion (ICF) has been discussed recently. The hydrodynamics of the imploding target is quite important, and many works using simulation have been devoted to this issue. Most of them used the Lagrange scheme, however, it causes mesh distortion in multi-dimensional cases, so that a kind of rezoning is commonly introduced. This means that a large artificial viscosity is included, and shock waves are not captured accurately.

As a universal solver of hyperbolic equations, Cubic-Interpolated Pseudo-Particle (CIP) method has been proposed[1–4], and successfully good results have been obtained for many one-dimensional (1-D) problems[4]. In the method, the equations are split into two phases named non-advective phase and advection phase. When the given equation has the following form,

$$\frac{\partial f}{\partial t} + \nabla \cdot f\boldsymbol{u} = q \ , \tag{1}$$

the equation for the non-advection phase is

$$\frac{\partial f}{\partial t} = q - f\nabla \cdot \boldsymbol{u} \equiv \tilde{q} \ , \tag{2}$$

where the advection velocity is denoted as $\boldsymbol{u}$. In the non-advection phase, the physical value of the previous time step f^n is advanced to f^* by a usual finite difference method. The superscripts mean the time index. In the advection phase, the equation for the physical value is

$$\frac{\partial f}{\partial t} + \boldsymbol{u} \cdot \nabla f = 0 \ , \tag{3}$$

as is proposed in the previous papers[1-4]. We also advance the spatial derivative f' in the non-advection phase, by differentiating Eq.(1) with respect to x,

$$\frac{\partial f'}{\partial t} = \tilde{q}\ ' - \boldsymbol{u}' \cdot \nabla f \ . \tag{4}$$

In the advection phase, the equation for the spatial derivative is $\partial f'/\partial t + \boldsymbol{u} \cdot \nabla f' = 0$. In order to determine f'^*, we approximated Eq.(4) without calculating $\tilde{q}\ '$. In 1-D case, we used the following difference equation,

$$\frac{f'^*_i - f'^n_i}{\Delta t} = \frac{f^*_{i+1} - f^*_{i-1}}{2\Delta x} - \frac{f^n_{i+1} - f^n_{i-1}}{2\Delta x} - \left(\frac{\partial u}{\partial x}\right)_i f'^n_i \ , \tag{5}$$

where the subscript means the index for the spatial difference, and we define $\Delta x = x_{i+1} - x_i$.

In the advection phase of 1-D case, we make use of the analytic solution of Eq.(3), that is well known as $f(x,t) = f(x - u\Delta t, t - \Delta t)$. At the time step $t - \Delta t$, the cubic interpolation function $F(x)$ within spatial grids is determined by the continuities of the physical value f^* and the first spatial derivative f'^* on the grids. For $u_i < 0$, $F(x)$ is interpolated between x_i and x_{i+1}, and is described as follows,

$$\begin{aligned} F(x) &= aX^3 + bX^2 + f_i'^* X + f_i^* , \\ a &= \frac{f_i'^* + f_{i+1}'^*}{\Delta x^2} - \frac{2f_i^* - f_{i+1}^*}{\Delta x^3} , \qquad b = \frac{3f_i^* - f_{i+1}^*}{\Delta x^2} - \frac{f_i'^* - 2f_{i+1}'^*}{\Delta x} , \end{aligned} \tag{6}$$

where we define X as $X \equiv x - x_i$. The physical value f^{n+1} and the first derivative f'^{n+1} at the time t become,

$$\begin{aligned} f_i^{n+1} &= F(x_i - u_i\Delta t) = a\xi^3 + b\xi^2 + c\xi + d , \\ f_i'^{n+1} &= \frac{\partial F}{\partial x}(x_i - u_i\Delta t) = 3a\xi^2 + 2b\xi + c , \end{aligned}$$

where $\xi = -u_i\Delta t$. For $u_i \geq 0$, $F(x)$ is interpolated between x_i and x_{i-1}, and $\xi = \Delta x - u_i\Delta t$.

II Two-dimensional case

In two-dimensional (2-D) case, the non-advective phase is easily extended from the 1-D case. For simplicity, we construct the formulation in the Cartesian coordinate, and the advection velocity is $\boldsymbol{u} = (u,\ v)$, and it is assumed to be constant $(u < 0,\ v < 0)$. We define the first derivatives as $\alpha \equiv \partial f/\partial x$ and $\beta \equiv \partial f/\partial y$, and these quantities are advanced by the following approximate equations,

$$\frac{\alpha_{i,j}^* - \alpha_{i,j}^n}{\Delta t} = \frac{f_{i+1,j}^* - f_{i-1,j}^*}{2\Delta x} - \frac{f_{i+1,j}^n - f_{i-1,j}^n}{2\Delta x} - \left(\frac{\partial u}{\partial x}\right)_{i,j}\alpha_{i,j}^n - \left(\frac{\partial v}{\partial x}\right)_{i,j}\beta_{i,j}^n , \tag{7}$$

$$\frac{\beta_{i,j}^* - \beta_{i,j}^n}{\Delta t} = \frac{f_{i,j+1}^* - f_{i,j-1}^*}{2\Delta y} - \frac{f_{i,j+1}^n - f_{i,j-1}^n}{2\Delta y} - \left(\frac{\partial u}{\partial y}\right)_{i,j}\alpha_{i,j}^n - \left(\frac{\partial v}{\partial y}\right)_{i,j}\beta_{i,j}^n . \tag{8}$$

The advection phase uses the analytic solution $f(x,t) = f(x - u\Delta t, y - v\Delta t, t - \Delta t)$ for the equation, $\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} = 0$, so that we have to determine a two dimensional cubic interpolation function $F(x,y)$ within four grids $(x_i,\ y_j) - (x_{i+1},\ y_j) - (x_{i+1},\ y_{j+1}) - (x_i,\ y_{j+1})$ at the time step $t - \Delta t$. If we use an analogy of the 1-D case, the interpolation function is thought to be

$$F(x,y) = \sum_{n=0}^{3}\sum_{m=0}^{3} C_{n,m}x^n y^m , \tag{9}$$

and the 16 unknown coefficients $C_{n,m}$ must be determined by the conditions at the four surrounding grids. In the previous paper [5], it was used that f^*, α^*, and β^* were continuous at the 3 grids, $(x_i,\ y_j)$, $(x_{i+1},\ y_j)$, and $(x_i,\ y_{j+1})$. In addition, f^* was continuous at $(x_{i+1},\ y_{j+1})$. The interpolation function had the following form,

$$\begin{aligned} F(x,y) &= C_{3,0}X^3 + C_{2,0}X^2 + \alpha_{i,j}X + f_{i,j} + C_{0,3}Y^3 + C_{0,2}XY + \beta_{i,j}Y \\ &+ C_{2,1}X^2Y + C_{1,1}XY + C_{1,2}XY^2 \ , \end{aligned} \tag{10}$$

$$\begin{aligned} C_{3,0} &= [-2f_{i+1,j} + 2f_{i,j} + (\alpha_{i,j+1} + \alpha_{i,j})\Delta x]/\Delta x^3 \ , \\ C_{2,1} &= [A - (\alpha_{i+1,j} - \alpha_{i,j})\Delta x]/\Delta x^2\Delta \ , \\ C_{2,0} &= [3f_{i+1,j} - 3f_{i,j} - (\alpha_{i+1,j} + 2\alpha_{i,j})\Delta x]/\Delta x^2 \ , \\ C_{1,1} &= [-A + (\alpha_{i,j+1} + \alpha_{i,j})\Delta x + (\beta_{i+1,j} + \beta_{i,j})\Delta y]/\Delta x\Delta y \ , \\ C_{0,3} &= [-2f_{i,j+1} + 2f_{i,j} + (\beta_{i,j+1} + \beta_{i,j})\Delta y]/\Delta y^3 \ , \\ C_{1,2} &= [A - (\beta_{i+1,j} + \beta_{i,j})\Delta y]/\Delta x\Delta y^2 \ , \\ C_{0,2} &= [3f_{i,j+1} - 3f_{i,j} - (\beta_{i,j+1} + 2\beta_{i,j})\Delta y]/\Delta y^2 \ , \end{aligned}$$

where $A = f_{i,j} - f_{i+1,j} - f_{i,j+1} + f_{i+1,j+1}$, $X \equiv x - x_i$, and $Y \equiv y - y_j$. The coefficients $C_{3,3}$, $C_{3,2}$, $C_{2,3}$, $C_{3,1}$, $C_{1,3}$, and $C_{2,2}$ remain unknown, and we set them to be zero. The quantities at time index $n+1$ were given as $f_i^{n+1} = F(x_i - u\Delta t,\ y_j - v\Delta t)$, $\alpha_i^{n+1} = \frac{\partial F}{\partial x}\Big|_{x=x_i-u\Delta t,\ y=y_j-v\Delta t}$, and $\beta_i^{n+1} = \frac{\partial F}{\partial x}\Big|_{x=x_i-u\Delta t,\ y=y_j-v\Delta t}$. We call this interpolation 'TYPE-A' here, and it gives sufficient results for many problems, however, it becomes diffusive in applying to the case of $u\Delta t/\Delta x > 0.5$ and $v\Delta t/\Delta y > 0.5$.

We consider to use the continuities of α and β at $(x_{i+1},\ y_{j+1})$ in addition to the above conditions. The interpolation function is described as follows,

$$\begin{aligned} F(x,y) &= C_{3,0}X^3 + C_{2,0}X^2 + \alpha_{i,j}X + f_{i,j} + C_{0,3}Y^3 + C_{0,2}Y^2 + \beta_{i,j}Y \\ &+ C_{3,1}X^3Y + C_{2,1}X^2Y + C_{1,2}XY^2 + C_{1,3}XY^3 + C_{1,1}XY \ , \end{aligned} \tag{11}$$

$$\begin{aligned} C_{3,0} &= 2(f_{i,j} - f_{i+1,j})/\Delta x^3 + (\alpha_{i,j} + \alpha_{i+1,j})/\Delta x^2 \ , \\ C_{2,0} &= 3(-f_{i,j} + f_{i+1,j})/\Delta x^2 - (\alpha_{i,j} + \alpha_{i+1,j})/\Delta x \ , \\ C_{0,3} &= 2(f_{i,j} - f_{i,j+1})/\Delta y^3 + (\beta_{i,j} + \beta_{i,j+1})/\Delta y^2 \ , \\ C_{0,2} &= 3(-f_{i,j} + f_{i,j+1})/\Delta y^2 - (\beta_{i,j} + \beta_{i,j+1})/\Delta y \ , \\ C_{3,1} &= -2A/\Delta x^3\Delta y + (-\alpha_{i,j} - \alpha_{i+1,j} + \alpha_{i,j+1} + \alpha_{i+1,j+1})/\Delta x^2\Delta y \ , \\ C_{2,1} &= 3A/\Delta x^2\Delta y + (2\alpha_{i,j} + \alpha_{i+1,j} - 2\alpha_{i,j+1} - \alpha_{i+1,j+1})/\Delta x\Delta y \ , \end{aligned}$$

$$C_{1,1} = -A/\Delta x\Delta y + (-\alpha_{i,j} + \alpha_{i,j+1})/\Delta y + (-\beta_{i,j} + \beta_{i+1,j})/\Delta x \ ,$$
$$C_{1,2} = 3A/\Delta x\Delta y^2 + (2\beta_{i,j} - \beta_{i+1,j} + 2\beta_{i,j+1} - \beta_{i+1,j+1})/\Delta x\Delta y \ ,$$
$$C_{1,3} = -2A/\Delta x\Delta y^3 + (-\beta_{i,j} + \beta_{i+1,j} - \beta_{i,j+1} + \beta_{i+1,j+1})/\Delta x\Delta y^2 \ .$$

In comparison with Eq.(10), the symmetry of Eq.(11) increases with respect to X and Y for $0 \le X \le \Delta x$ and $0 \le Y \le \Delta y$. The unknown coefficients $C_{3,3}$, $C_{3,2}$, $C_{2,3}$, and $C_{2,2}$ are assumed to be zero. This interpolation is named 'TYPE-B', and the accuracy of interpolation is increased more than Eq.(10), especially for the case of $u\Delta t/\Delta x > 0.5$ and $v\Delta t/\Delta y > 0.5$.

In this paper, we consider the interpolation of the advection phase in a different way. It is essential to interpolate the quantities at the point $(\xi,\ \eta)$ on the $(x,\ y)$-plane, not to determine the functional form $F(x-u\Delta t, y-v\Delta t)$, where $\xi = x_i - u\Delta t$ and $\eta = y_j - v\Delta t$. We propose to use the continuity of the second mutual derivative $\delta \equiv \frac{\partial^2 f}{\partial x \partial y}$, in addition to the continuities of f^*, α^*, and β^*. We illustrate the process of the interpolation from $\mathrm{S}(x_i,\ y_j)$ to $\mathrm{T}(\xi,\ \eta)$ in Fig. 1. Starting from the point S, we have two ways to arrive at the point B, that is, $\mathrm{S} \to \mathrm{A}(\xi, y_i) \to \mathrm{T}$ and $\mathrm{S} \to \mathrm{B}(x_i,\ \eta) \to \mathrm{T}$. For instance, we show the way of S → A → T. It is necessary that the physical values and the spatial derivatives at all the grids have been advanced in the non-advection phase. To advance the second mutual derivative from δ^n to δ^*, we use

$$\frac{\delta^*_{i,j} - \delta^n_{i,j}}{\Delta t} = \frac{f^*_{i+1,j+1} - f^*_{i-1,j+1} - f^*_{i+1,j-1} + f^*_{i-1,j-1}}{4\Delta x\Delta y} - \frac{f^n_{i+1,j+1} - f^n_{i-1,j+1} - f^n_{i+1,j-1} + f^n_{i-1,j-1}}{4\Delta x\Delta y} - \left(\frac{\partial^2 u}{\partial x\partial y}\right)_{i,j}\alpha^n_{i,j} - \left(\frac{\partial^2 v}{\partial x\partial y}\right)_{i,j}\beta^n_{i,j} \ . \quad (12)$$

In the advection phase, first of all, f^*, α^*, β^*, and δ^* at the point A are interpolated between S and $(x_{i+1},\ y_j)$ in the x-direction. This interpolation is similar with Eq.(1), and we obtain

$$f_{(A)} = a_1\xi^3 + b_1\xi^2 + \alpha_{i,j}\xi + f_{i,j} \ ,$$
$$\alpha_{(A)} = 3a_1\xi^2 + 2b_1\xi + \alpha_{i,j} \ ,$$
$$\beta_{(A)} = a_2\xi^3 + b_2\xi^2 + \delta_{i,j}\xi + \beta_{i,j} \ ,$$
$$\delta_{(A)} = 3a_2\xi^2 + 2b_2\xi + \delta_{i,j} \ ,$$

$$a_1 = \frac{\alpha_{i,j} + \alpha_{i+1,j}}{\Delta x^2} - \frac{2f_{i,j} - f_{i+1,j}}{\Delta x^3}, \quad b_1 = \frac{3f_{i,j} - f_{i+1,j}}{\Delta x^2} - \frac{\alpha_{i,j} - 2\alpha_{i+1,j}}{\Delta x} \ ,$$
$$a_2 = \frac{\delta_{i,j} + \delta_{i+1,j}}{\Delta x^2} - \frac{2\beta_{i,j} - \beta_{i+1,j}}{\Delta x^3}, \quad b_2 = \frac{3\beta_{i,j} - \beta_{i+1,j}}{\Delta x^2} - \frac{\delta_{i,j} - 2\delta_{i+1,j}}{\Delta x} \ .$$

Similarly, we can interpolate these quantities at the point $A'(\xi, y_{j+1})$ between (x_i, y_{j+1}) and (x_{i+1}, y_{j+1}) as follows,

$$
\begin{aligned}
f_{(A')} &= a_1\xi^3 + b_1\xi^2 + \alpha_{i,j+1}\xi + f_{i,j+1} \ , \\
\alpha_{(A')} &= 3a_1\xi^2 + 2b_1\xi + \alpha_{i,j+1} \ , \\
\beta_{(A')} &= a_2\xi^3 + b_2\xi^2 + \delta_{i,j+1}\xi + \beta_{i,j+1} \ , \\
\delta_{(A')} &= 3a_2\xi^2 + 2b_2\xi + \delta_{i,j+1} \ ,
\end{aligned}
$$

$$
\begin{aligned}
a_1 &= \frac{\alpha_{i,j+1} + \alpha_{i+1,j+1}}{\Delta x^2} - \frac{2f_{i,j+1} - f_{i+1,j+1}}{\Delta x^3} \, , b_1 = \frac{3f_{i,j+1} - f_{i+1,j+1}}{\Delta x^2} - \frac{\alpha_{i,j+1} - 2\alpha_{i+1,j+1}}{\Delta x} \, , \\
a_2 &= \frac{\delta_{i,j+1} + \delta_{i+1,j+1}}{\Delta x^2} - \frac{2\beta_{i,j+1} - \beta_{i+1,j+1}}{\Delta x^3} \, , b_2 = \frac{3\beta_{i,j+1} - \beta_{i+1,j+1}}{\Delta x^2} - \frac{\delta_{i,j+1} - 2\delta_{i+1,j+1}}{\Delta x} \, .
\end{aligned}
$$

The quantities at the destination $T(\xi, \ \eta)$ are interpolated between A and A′ in the y-direction, and we have

$$
\begin{aligned}
f_i^{n+1} &= f_{(B)} = a_1\eta^3 + b_1\eta^2 + \beta_{(A)}\eta + f_{(A)} \ , && (13) \\
\alpha_i^{n+1} &= \alpha_{(B)} = a_2\eta^3 + b_2\eta^2 + \delta_{(A)}\eta + \alpha_{(A)} \ , && (14) \\
\beta_i^{n+1} &= \beta_{(B)} = 3a_1\eta^2 + 2b_1\eta + \beta_{(A)} \ , && (15) \\
\delta_i^{n+1} &= \delta_{(B)} = 3a_2\eta^2 + 2b_2\eta + \delta_{(A)} \ , && (16)
\end{aligned}
$$

$$
\begin{aligned}
a_1 &= \frac{\beta_{(A)} + \beta_{(A')}}{\Delta y^2} - \frac{2f_{(A)} - f_{(A')}}{\Delta y^3} \, , \quad b_1 = \frac{3f_{(A)} - f_{(A')}}{\Delta y^2} - \frac{\beta_{(A)} - 2\beta_{(A')}}{\Delta y} \, , \\
a_2 &= \frac{\delta_{(A)} + \delta_{(A')}}{\Delta y^2} - \frac{2\alpha_{(A)} - \alpha_{(A')}}{\Delta y^3} \, , \quad b_2 = \frac{3\alpha_{(A)} - \alpha_{(A')}}{\Delta y^2} - \frac{\delta_{(A)} - 2\delta_{(A')}}{\Delta y} \, .
\end{aligned}
$$

The above interpolation is named 'TYPE C'. The second mutual derivative $\frac{\partial^2 f}{\partial x \partial y}$ was used to interpolate $\frac{\partial f}{\partial y}$ in the x-direction and $\frac{\partial f}{\partial x}$ in the y-direction. If we write down the above interpolation such as Eq.(9), there is no unknown coefficient, because the continuities of f, α, β, and δ are used at 4 grids. In the case that we choose the other way, S $\rightarrow$ B$(x_i, \ \eta)$ $\rightarrow$ T, in Fig.1, it is easily shown that the same interpolation result is obtained.

In order to check the new interpolation, we have tested Zalesak's example, that is, the 2-D solid body revolution[6,7]. The given equation is $\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} = 0$, and the equations for the non-advection phase reduce to

$$
f_i^* = f_i^n \ , \quad \frac{\alpha_{i,j}^* - \alpha_{i,j}^n}{\Delta t} = -\left(\frac{\partial v}{\partial x}\right)_{i,j}\beta_{i,j}^n \ , \quad \frac{\beta_{i,j}^* - \beta_{i,j}^n}{\Delta t} = -\left(\frac{\partial u}{\partial y}\right)_{i,j}\alpha_{i,j}^n \ , \quad \delta_{i,j}^* = \delta_{i,j}^n \ . \quad (17)
$$

Figure 2(a) shows the schematic view of the test. We show the true result and the computational results after one complete revolution in different interpolation ways in Fig.2(b). The contours are drawn from $f = 0.1$ to $f = 1.0$ with intervals of 0.1. It is shown that 'TYPE C' gives the least diffusive result among them.

III Three-dimensional case

In the three-dimensional (3-D) case, the interpolation technique is almost same as the 2-D one. However, the advection velocity has three elements $\boldsymbol{u} = (u, v, w)$, and we assume $u < 0$, $v < 0$, and $w < 0$. Here, we propose to use the continuities of the set of the quantities $\Gamma = \left\{f,\ \frac{\partial f}{\partial x},\ \frac{\partial f}{\partial y},\ \frac{\partial f}{\partial z},\ \frac{\partial^2 f}{\partial x \partial y},\ \frac{\partial^2 f}{\partial y \partial z},\ \frac{\partial^2 f}{\partial z \partial x},\ \frac{\partial^3 f}{\partial x \partial y \partial z}\right\} \equiv \{f,\ \alpha,\ \delta,\ \gamma,\ \delta,\ \lambda,\ \mu,\ \omega\}$. All of these elements must be advanced in the non-advection phase. We use the similar approximation with 2-D case, so that we can advance $\omega \equiv \frac{\partial^3 f}{\partial x \partial y \partial z}$, from ω^n to ω^*,

$$\begin{aligned}
\frac{\omega^*_{i,j,k} - \omega^n_{i,j,k}}{\Delta t} &= (f^*_{i+1,j+1,k+1} - f^*_{i-1,j+1,k+1} - f^*_{i+1,j-1,k+1} + f^*_{i-1,j-1,k+1} \\
&\quad - f^*_{i+1,j+1,k+1} + f^*_{i-1,j+1,k+1} + f^*_{i+1,j-1,k+1} - f^*_{i-1,j-1,k+1})/8\Delta x \Delta y \Delta z \\
&- (f^n_{i+1,j+1,k+1} - f^n_{i-1,j+1,k+1} - f^n_{i+1,j-1,k+1} + f^n_{i-1,j-1,k+1} \\
&\quad - f^n_{i+1,j+1,k+1} + f^n_{i-1,j+1,k+1} + f^n_{i+1,j-1,k+1} - f^n_{i-1,j-1,k+1})/8\Delta x \Delta y \Delta z \\
&- \left(\frac{\partial^3 u}{\partial x \partial y \partial z}\right)_{i,j,k} \alpha^n_{i,j,k} - \left(\frac{\partial^3 v}{\partial x \partial y \partial z}\right)_{i,j,k} \beta^n_{i,j,k} - \left(\frac{\partial^3 w}{\partial x \partial y \partial z}\right)_{i,j,k} \gamma^n_{i,j,k}\ .
\end{aligned} \tag{18}$$

After obtaining the set $\Gamma^* = \{f^*,\ \alpha^*,\ \beta^*,\ \gamma^*,\ \delta^*,\ \lambda^*,\ \mu^*,\ \omega^*\}$, we advance all the elements to $\Gamma^{n+1} = \{f^{n+1},\ \alpha^{n+1},\ \beta^{n+1},\ \gamma^{n+1},\ \delta^{n+1},\ \lambda^{n+1},\ \mu^{n+1},\ \omega^{n+1}\}$, using 3-D version of 'TYPE C' interpolation. In Fig.3, we show the 3-D interpolation process to have Γ^* at the point T(ξ, η, ζ), where $\zeta = z_k - w\Delta t$. First, we interpolate Γ^* at the point B on the (x, y)-plane ($z = z_k$), using the same process with the previous section. Next, Γ^* is similarly interpolated at the point B$'$ on the ($z = z_{k+1}$)-plane. The set Γ^* at the destination T is interpolated in the z-direction between B and B$'$. This is equal to Γ^{n+1} at the grid S. The third derivative $\frac{\partial^3 f^*}{\partial x \partial y \partial z}$ is used to interpolate $\frac{\partial^2 f^*}{\partial y \partial z}$ in the x-direction, $\frac{\partial^2 f^*}{\partial z \partial x}$ in the y-direction, and $\frac{\partial^2 f^*}{\partial x \partial y}$ in the z-direction. We showed the process of the interpolation tracing the way S $\rightarrow$ A $\rightarrow$ B $\rightarrow$ T. Although there are 5 ways from S to T besides this, all of them give the same result. If we write down the 3-D interpolation function, it will be the following form,

$$F(x,y,z) = \sum_{n=0}^{3}\sum_{m=0}^{3}\sum_{l=0}^{3} C_{n,m,l}x^n y^m z^l \ , \tag{19}$$

and there are 64 unknown coefficients $C_{n,m,l}$. The above interpolation used the continuities of 8 elements of the set Γ at the 8 grid points, so that the 64 conditions were used to determine $C_{n,m,l}$.

In order to evaluate the developed 3-D interpolation processing, the problem of 3-D solid body transfer[5,8] is checked. The non-advection phase disappears, because the given equation is the linear advection equation. We set $u = v = w = 1.0$ everywhere, and the spatial division and the time step are employed to be $\Delta x = \Delta y = \Delta z = 1.0$ and $\Delta t = 0.5$. In Fig.4(a), the schematic picture of this test is shown. The detail profile of the initial state is drawn in the left-hand side picture of Fig.4(a). At the boundary and inside the shaded region, $f = 1.0$ elsewhere $f = 0.0$, and this region is located at the extent $0 \le x \le 10$, $0 \le y \le 10$, and $0 \le z \le 10.0$. Idealy, the profile after transfer should be the same as the initial one. Figure 2(b) shows the profiles on $z = k$ plane ($k = 20, \cdots, 31$) after 40-time-steps computation. The contours are drawn from 0.1 to 1.0 with each interval of 0.1. The computational result shows that the initial shape is well maintained.

IV Summary

In order to solve the hydrodynamic equations, we proposed new interpolation processing named 'TYPE C' in the advection phase of the multi-dimensional CIP scheme. We introduced the idea to use the continuities of the second and third spatial derivatives $\frac{\partial^2 f}{\partial x \partial y}$, $\frac{\partial^3 f}{\partial x \partial y \partial z}$, and so on. The procedure did not become so much cumbersome, and the computation results were quite less diffusive. This interpolation processing is seemed to be the true extension of the 1-D CIP scheme. In the hydrodynamics equation[8] which is used to simulate the ICF implosion, only the non-advection phase changes, and the proposed interpolation processing is available in the advection phase.

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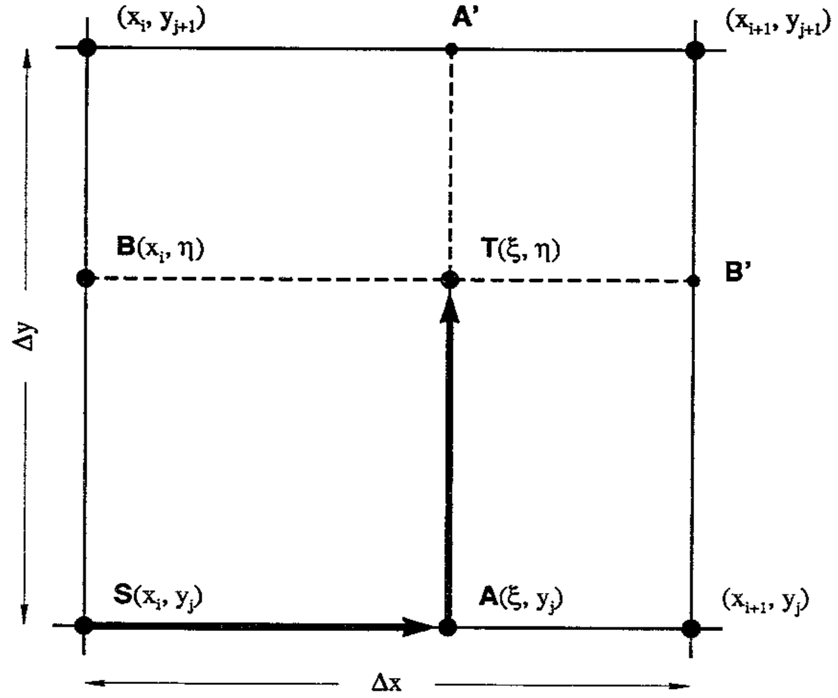

Fig. 1 Interpolation process of 'TYPE C' in 2-D CIP scheme

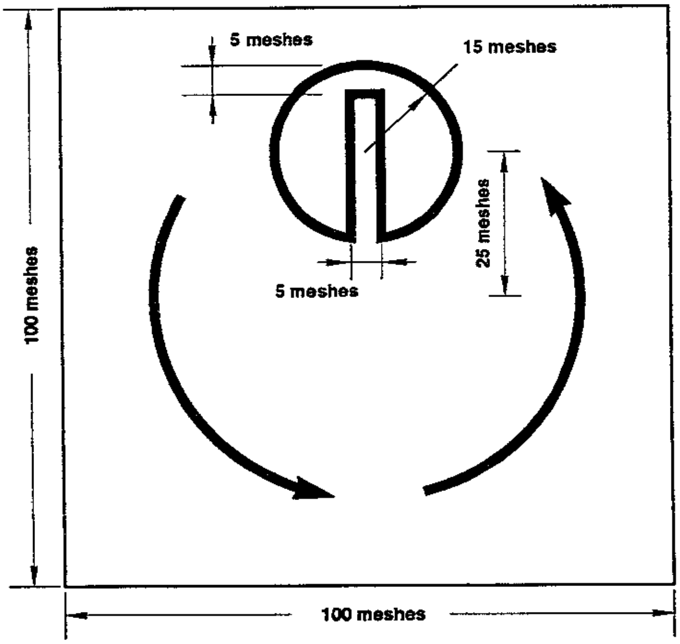

Fig.2(a) Schematic view of two dimensional solid body revolution as in ref[?]. The value of f inside the cut-out cylinder is 1.0, while outside f=0.0. The rotational speed is such that one complete revolution is performed in 628 steps.

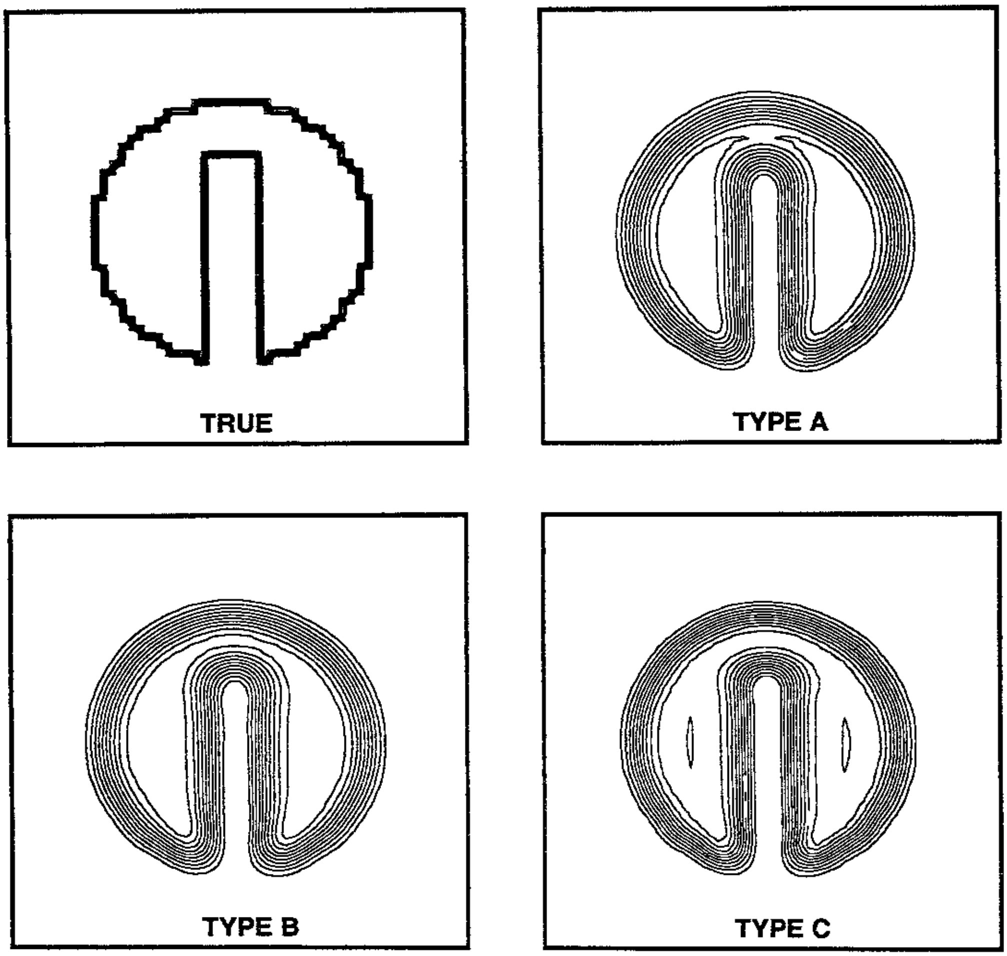

Fig.2(b) Computational results for Zalesak's example after one complete revolution.

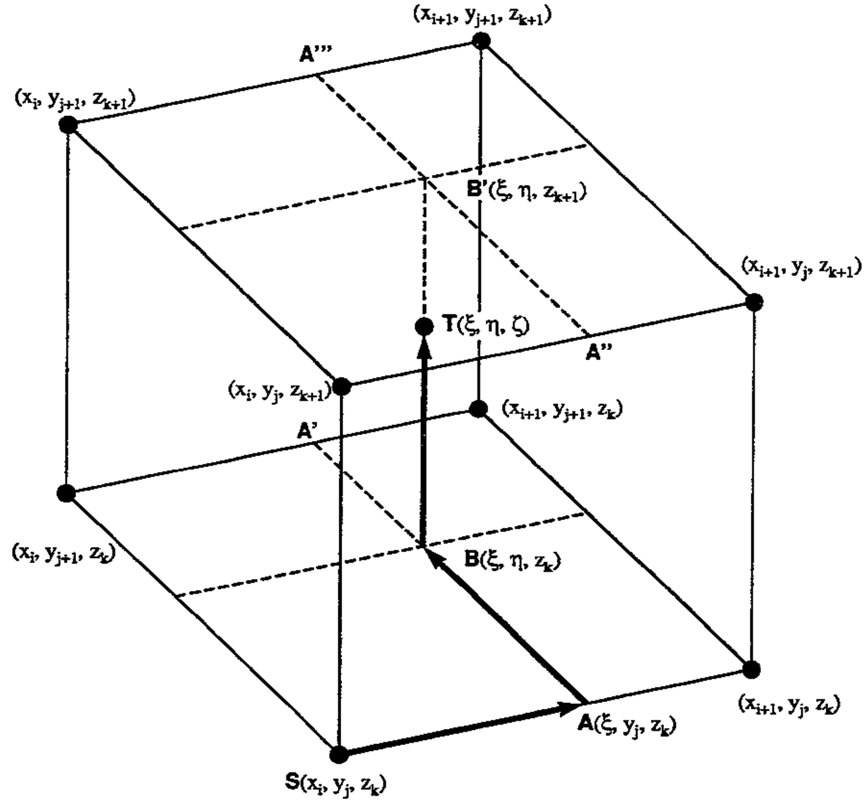

Fig. 3   Interpolation process in 3-D CIP scheme

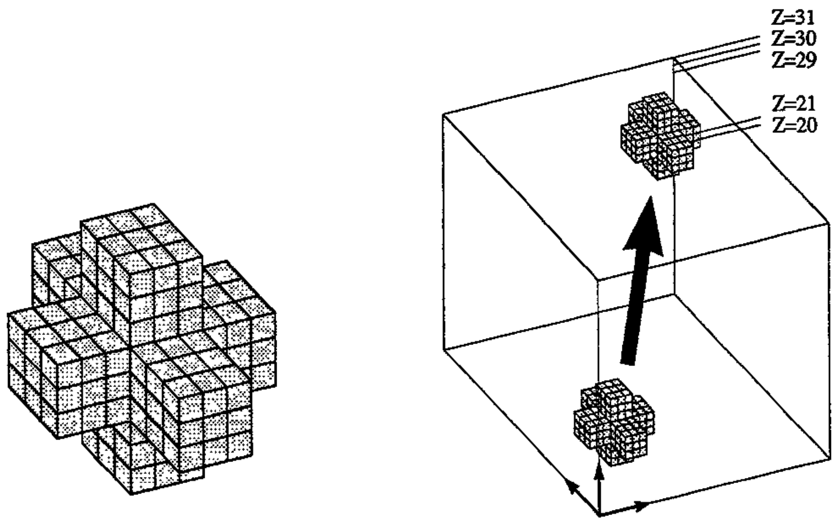

Fig.4(a) Schematic view of the 3-D solid body transfer.
At the boundary and inside the shaded region, f=1, elsewhere f=0.

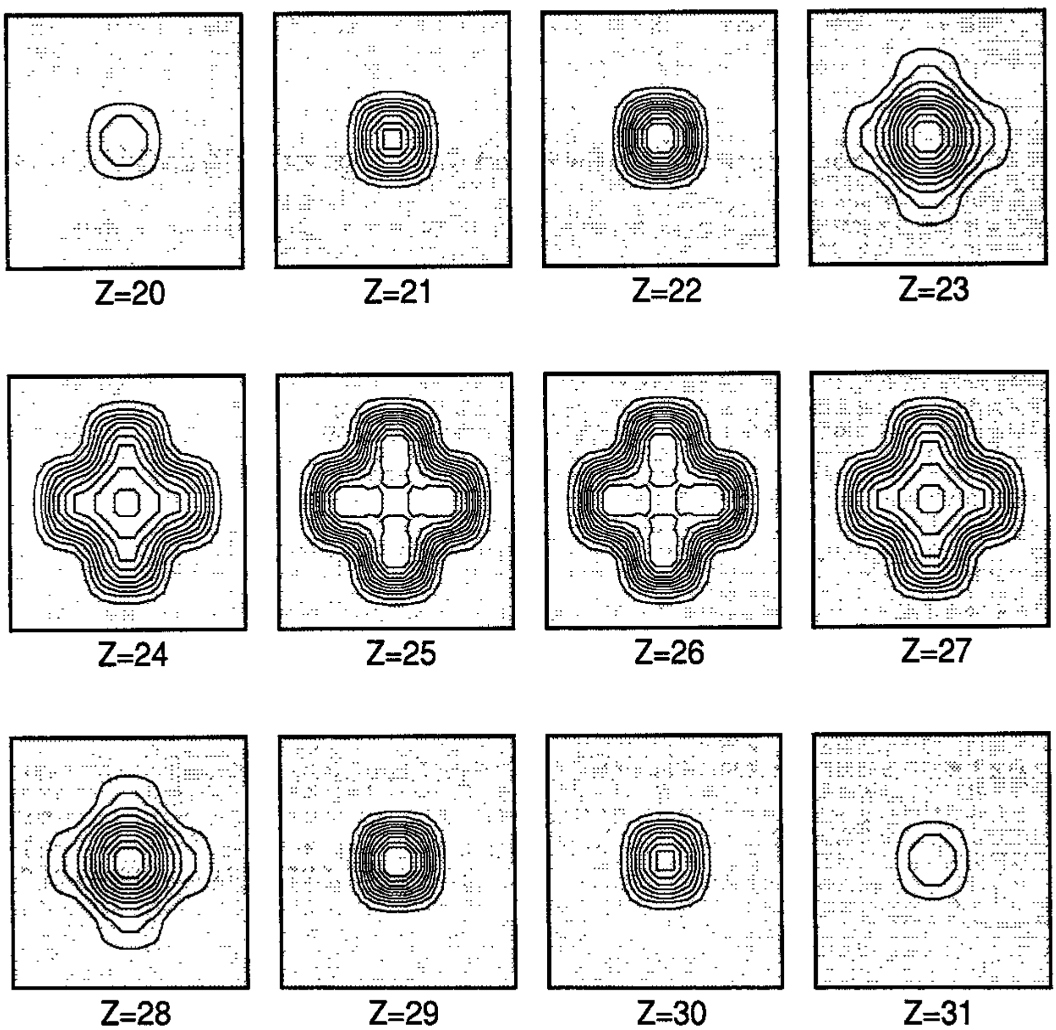

Fig.4(b) Computational results after 40 time steps. The contours show the profiles on the (z=k)-plane, k=20 - 31.

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