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Fluid Description of Ponderomotive Force Compatible with the Kinetic One in a Warm Plasma

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The ponderomotive effects of electromagnetic wave fields are practically of great importance in such cases as RF stabilization, RF-induced transport in magnetically confined plasmas, and so on. Theoretically, however, there are still discrepancies among expressions derived by a number of authors.

In this short note we restrict our attention to the discrepancy between fluid[1,2] and kinetic[3] expressions of the ponderomotive force often found in literatures dealing a warm plasma and discusss where it stems from and how it is resolved.

The ponderomotive force density of electromagnetic wave fields based on the Vlasov theory has been obtained as[3]

$$\underline{F} = q\langle \tilde{n}\, \underline{\tilde{E}} \rangle + \frac{q}{c}\langle \underline{\tilde{\Gamma}} \times \underline{\tilde{B}} \rangle - \nabla \cdot \underline{\underline{\delta \Pi}} \ , \tag{1}$$

$$\underline{\underline{\delta \Pi}} = m \int \underline{v}\, \underline{v}\, \delta f d\underline{v} \ , \tag{2}$$

where $\underline{\Gamma} = n\underline{V}$, n is the density, $\underline{V}$ the flow velocity, $\underline{E}$ the electric field, $\underline{B}$ the magnetic field, q the charge, m the mass and c the light speed. δf is the nonlinear wave-induced component of a distribution funtion and is calculated from the quasi-linear theory. Here the physical quantity X is divided into rapidly oscillating and slowly varying parts as $X = \langle X \rangle + \tilde{X} = \bar{X} + \tilde{X}$, where $\langle \cdots \rangle$ denotes time averaging over the rapid oscillation period. By definition, $\langle \tilde{X} \rangle = 0$. On the other hand, the fluid ponderomotive force density is given by[1,2]

$$\underline{F} = q\langle \tilde{n}\, \underline{\tilde{E}} \rangle + \frac{q}{c}\langle \underline{\tilde{\Gamma}} \times \underline{\tilde{B}} \rangle - m\nabla \cdot \left(\frac{1}{\bar{n}}\langle \underline{\tilde{\Gamma}\, \tilde{\Gamma}} \rangle\right) \ , \tag{3}$$

where the effect of the thermal pressure is taken into account through the rapid oscillating quantities obtainable from the linearized equations of continuity and motion in a warm plasma[1,2].

Hereafter, we consider the case of electrostatic high-frequency waves in a unmagnetized plasma, for simplicity. Therefore, we put $\bar{\underline{B}} = \tilde{\underline{B}} = 0$. A general case of electromagnetic waves with an external magnetic field will be reported elsewhere. To make the discrepancy between the both descriptions clear, we need to compare $\underline{\underline{\delta \Pi}}$ in eq. (1) with $m\langle \underline{\tilde{\Gamma}\tilde{\Gamma}} \rangle / \bar{n}$ in eq. (3).

We first calculate $\underline{\underline{\delta \Pi}}$. In the quasi-linear theory, the slow time evolution of an averaged distribution function is determined by[4]

$$\frac{\partial}{\partial t} f = \left(\frac{q}{m}\right)^2 \int d\tau \frac{|\tilde{\underline{E}}_{k\omega}(\underline{r},t)|^2}{k^2} \underline{k} \cdot \frac{\partial}{\partial \underline{v}} \frac{\gamma}{(\omega - \underline{k}\cdot\underline{v})^2 + \gamma^2} \underline{k} \cdot \frac{\partial}{\partial \underline{v}} f \ , \tag{4}$$

where $|\tilde{\underline{E}}_{k\omega}|^2 = |\tilde{\underline{E}}_k|^2 2\pi \, \delta[\omega - \omega(\underline{k})]$, $\int d\tau \equiv \int d\underline{k} d\omega / (2\pi)^4$, $\omega(\underline{k})$ is the wave frequency, $\gamma(\underline{k})$ the wave growth rate and $\underline{k}$ the wavenumber. The wave amplitude $\tilde{\underline{E}}_{k\omega}$ is slowly varying in space and time and eq. (4) is still valid to such a case. When we assume that f is composed of the Maxwellian distribution function f_0 and the wave-induced correction δf as $f = f_0 + \delta f$, we can solve eq. (4) by the perturvation analysis. Using $\partial |\tilde{\underline{E}}_{k\omega}|^2 / \partial t = 2\gamma |\tilde{\underline{E}}_{k\omega}|^2$ and replacing f in the right-hand side of eq. (4) by f_0, we obtain δf as

$$\delta f = \left(\frac{q}{m}\right)^2 \int d\tau \frac{|\tilde{\underline{E}}_{k\omega}(\underline{r},t)|^2}{2k^2} \underline{k} \cdot \frac{\partial}{\partial \underline{v}} \frac{1}{(\omega - \underline{k}\cdot\underline{v})^2} \underline{k} \cdot \frac{\partial}{\partial \underline{v}} f_0 \ , \tag{5}$$

where only adiabatic interactions($|\omega - \underline{k}\cdot\underline{v}|^2 \gg \gamma^2$) are retained since we concentrate the discussion to the comparison with the fluid description. Substituting eq. (5) into eq. (2) and performing the integral in $\underline{v}$ after the expansion of $1/(\omega - \underline{k}\cdot\underline{v})^2$ in powers of $\underline{k}\cdot\underline{v}/\omega$, we obtain to the lowest order in $Tk^2/m\omega^2$,

$$\underline{\underline{\delta \Pi}} = \int d\tau \frac{\bar{n} q^2}{m\omega^2} \frac{\underline{k}\,\underline{k}}{k^2} |\tilde{\underline{E}}_{k\omega}(\underline{r},t)|^2 \left(1 + \frac{9Tk^2}{m\omega^2}\right) \ , \tag{6}$$

where we assumed $Tk^2/m\omega^2 \ll 1$. The second term in eq. (6) represents the warm plasma effect.

Next we turn to $m<\underline{\tilde{\Gamma}}\,\underline{\tilde{\Gamma}}>/\bar{n}$. From the linearized equation of motion with the thermal pressure in a adiabatic process, we obtain

$$\underline{\tilde{V}}_{k\omega}=i(q/m\omega)[\underline{\underline{I}}+3T\underline{kk}/m\omega^2]\cdot\underline{\tilde{E}}_{k\omega}(\underline{r},t)\ , \tag{7}$$

where T is the temperature and $\underline{\underline{I}}$ is the 3×3 unit matrix. Using eq. (7) and $\tilde{\Gamma}=\bar{n}\tilde{V}$, we can obtain to the same order as eq. (6)

$$m<\underline{\tilde{\Gamma}}\,\underline{\tilde{\Gamma}}>/\bar{n}=\int d\tau\,\frac{\bar{n}q^2}{m\omega^2}\,\frac{\underline{k}\,\underline{k}}{k^2}|\underline{\tilde{E}}_{k\omega}(\underline{r},t)|^2(1+\frac{6Tk^2}{m\omega^2})\ , \tag{8}$$

Comparing eq. (8) with eq. (6), we find the discrepancy between $\underline{\underline{\delta\Pi}}$ and $m<\underline{\tilde{\Gamma}}\,\underline{\tilde{\Gamma}}>/\bar{n}$ in the order of $Tk^2/m\omega^2$. For a cold plasma, the both expressions coincide with each other. We can see that this discrepancy in a warm plasma is due to the lacking of a wave-induced contribution to the thermal pressure in the fluid description of the ponderomotive force density given by eq. (3). The wave-induced contribution $\underline{\underline{\delta P}}$ to the thermal pressure is obtained from the equation of energy given by

$$\frac{\partial}{\partial t}[m\bar{n}<\underline{\tilde{V}}\,\underline{\tilde{V}}>+\underline{\underline{\delta P}}]=q\bar{n}<\underline{\tilde{E}}\,\underline{\tilde{V}}+\underline{\tilde{V}}\,\underline{\tilde{E}}>\ , \tag{9}$$

where the energy flow term are neglected since they are of higher order. Because of the slow time variation of the wave amplitude, in place of eq. (7) the velocity fluctuation is obtained by replacing ω by $\omega+i\partial/\partial t$:

$$\underline{\tilde{V}}_{k\omega}=i\frac{q}{m\omega}[\underline{\underline{I}}\,(1-\frac{i}{\omega}\,\frac{\partial}{\partial t})+\frac{3T\underline{k}\,\underline{k}}{m\omega^2}(1-\frac{3i}{\omega}\,\frac{\partial}{\partial t})]\cdot\underline{\tilde{E}}_{k\omega}(\underline{r},t)\ , \tag{10}$$

where $|\omega^{-1}\partial/\partial t|\ll 1$. Using Eq. (10), we obtain

$$q\bar{n}<\underline{\tilde{E}}\,\underline{\tilde{V}}+\underline{\tilde{V}}\,\underline{\tilde{E}}>=\frac{\partial}{\partial t}\int d\tau\,\frac{\bar{n}q^2}{m\omega^2}\,\frac{\underline{k}\,\underline{k}}{k^2}|\underline{\tilde{E}}_{k\omega}(\underline{r},t)|^2(1+\frac{9Tk^2}{m\omega^2})\ . \tag{11}$$

Substituting eqs. (8), (11) into eq. (9), we obtain $\underline{\underline{\delta P}}$ as

$$\underline{\underline{\delta P}} = \int d\tau \frac{\bar{n} q^2}{m\omega^2} \frac{\underline{k}\,\underline{k}}{k^2} |\tilde{\underline{E}}_{k\omega}(\underline{r}, t)|^2 \frac{3Tk^2}{m\omega^2} \quad , \tag{12}$$

where $\underline{\underline{\delta P}}$ vanishes when $T=0$. From eqs.(6),(8) and (12), We find to the second order in $\tilde{\underline{E}}_{k\omega}$ and lowest order in $Tk^2/m\omega^2$,

$$\underline{\underline{\delta \Pi}} = m\langle \underline{\tilde{\Gamma}\,\tilde{\Gamma}} \rangle / \bar{n} + \underline{\underline{\delta P}} \; . \tag{13}$$

The difference between $\underline{\underline{\delta \Pi}}$ in eq.(1) and $m\langle \underline{\tilde{\Gamma}\,\tilde{\Gamma}} \rangle / \bar{n}$ in eq.(3) is just equal to the wave-induced contribution to the thermal pressure. Equation (13) shows that the contribution from the wave-induced pressure coupled with the thermal motion $-\nabla \cdot \underline{\underline{\delta P}}$ should be added to eq.(3) in order for the fluid description of the ponderomotive force compatible with the kinetic one in the warm plasma.

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