

# NATIONAL INSTITUTE FOR FUSION SCIENCE

## **A Method for the High-speed Generation of Random Numbers with Arbitrary Distributions**

T. Ishikawa, P. Y. Wang, K. Wakui and T. Yabe

(Received – Oct. 15, 1991)

NIFS-121

Nov. 1991

### **RESEARCH REPORT** **NIFS Series**

This report was prepared as a preprint of work performed as a collaboration research of the National Institute for Fusion Science (NIFS) of Japan. This document is intended for information only and for future publication in a journal after some rearrangements of its contents.

Inquiries about copyright and reproduction should be addressed to the Research Information Center, National Institute for Fusion Science, Nagoya 464-01, Japan.

# **A Method for the High-speed Generation of Random Numbers with Arbitrary Distributions**

T. Ishikawa, P. Y. Wang, K. Wakui and T. Yabe

*Department of Electronic Engineering, Gunma University, Tenjin-chou 1-5-1, Kiryu, Gunma 376, Japan*

A new method for the high-speed generation of random numbers with arbitrary distributions is proposed. The method stores random numbers in two memories, and then picks out two random numbers and assembles them, resulting in a new random number. When the method is realized with Fortran, the method can generate random numbers with normal distribution about eleven times faster than the method relied on the central limit theorem, and those with Planck distribution about nine times faster than the acceptance-rejection method. The method is realized by hardware. The system generates  $5 \times 10^6$  random numbers per second with any distribution. The statistical quality of these random numbers is similar to or better than that by the conventional methods.

**Keywords :** Monte Carlo, random number, high-speed generation, hardware, Planck distribution

## 1. Introduction

The Monte Carlo method is now the powerful and commonly used technique for analyzing statistical problems. Moreover, the method can also be used for the simulation of deterministic problems. Therefore, applications of the method have been found in many fields [1]. Since random or pseudorandom numbers with given distributions are used in the Monte Carlo method, it is worth while to search for a new method to generate them with an accelerated speed.

The following three methods are well known as procedures for generating random numbers with arbitrary distributions, that is, the inverse transform method, the composition method proposed by Butler and the acceptance-rejection method proposed by von Neumann [2]. Moreover, there are special procedures for generating random numbers with a specific distribution. For example, those with normal distribution can be generated by the well-known method relied on the central limit theorem or the method proposed by Box and Müller [2]. Although these procedures can generate random numbers with sufficient quality, it is better to accelerate the speed to generate them. For this purpose, a lot of methods for the high-speed generation of random numbers have been proposed [3-9]. However, almost all the methods were limited to the generation of random numbers with a specific distribution.

In this paper, we propose a new method for the high-speed generation of random numbers with arbitrary distributions. The method prepares two tables of random numbers and stores them in two memories of a computer, and then picks out two random numbers and assembles them, resulting in a new random number. The proposed method is realized not only by software, but also by hardware for the application to a special purpose computer, which can be as fast as or faster than general-purpose commercial super computers at modest

cost [10].

## 2. Basic algorithm

In this paper, we consider the generation of random numbers with an arbitrary density function  $f(z)$  as shown by the solid line in Fig.1. The interval  $(C, AB + C)$  in which the density function  $f(z)$  is defined is divided into  $A$  regions with the small interval of  $B$ , and the function  $f(z)$  is approximated with a piece-wise constant function  $F(z)$ , the value of which in each region is equal to that of  $f(z)$  at the center of the region. It is evident that  $F(z)$  is a good approximation to  $f(z)$  when the number of regions is large. Therefore, we try to generate random numbers  $z$ , the density function of which is equal to this  $F(z)$ . Random numbers  $z$  are obtained in terms of two random numbers  $x$  and  $y$  as

$$z = z_1 + z_2 \tag{1}$$

$$z_1 = B \text{int}(Ax) + C \tag{2}$$

$$z_2 = By \tag{3}$$

where,  $\text{int}(w)$  is the integer part of  $w$ , and  $0 \leq x, y \leq 1$ . The density function of  $x$  is chosen to be equal to that of  $z$ , therefore the density function of  $z_1$  is represented by the distribution of the dots in Fig.1. The density function of  $y$  is chosen to be uniform.

One of our originalities can be found in the method to produce  $z_1$  and  $z_2$ . At first, two tables are prepared for random numbers  $z_1$  and  $z_2$ . The table is recorded in two dimensions, and the size of that is  $NX_m \times NY_m$ . Therefore, the number  $N_m$  of random numbers on the table is  $NX_m \times NY_m$ . We call the table "mother assembly" hereafter. Secondly,  $N_c$  random numbers shaded in Fig.2(a) are picked out from the mother assembly, where  $N_c$  is obviously less than  $N_m$ . We call the set of  $N_c$  random numbers "child assembly" hereafter. The mother

assembly is assumed to be cyclic in two dimensions. Therefore, by setting a left-upper position  $(i_0, j_0)$  of the child assembly, we can pick out an arbitrary child assembly from the mother assembly as shown in Fig.2(b). If the number of random numbers larger than  $N_c$  is needed, another position is chosen to pick out another child assembly.

Let us introduce the method to produce two mother assemblies. The mother assembly for random numbers  $z_1$  is produced as follows. If  $N_m$  random numbers are generated to produce the mother assembly by the well-known previous methods, for example, by the method given by Lehmer when the density function of  $z_1$  is uniform, by the method based on the central limit theorem when the function is normal, and by the acceptance-rejection method for the other functions, it is known that the random numbers have the error  $O(N_m^{-1/2})$ , because the number of random numbers on mother assembly is  $N_m$ . The error approaches 0 according to the increase of  $N_m$ , that is, the number of random numbers. However, since the proposed method picks out one or more child assemblies from one mother assembly, each child assembly has the error corresponding to that of the mother assembly. Therefore, when  $JN_c$  random numbers are generated by picking out  $J$  child assemblies, the error is not  $O((JN_c)^{-1/2})$  but  $O(N_m^{-1/2})$ . In other words, the error does not approach 0 even if the number of generated random numbers increases. In the paper, to overcome this demerit, we generate the random numbers on the mother assembly not by the previous methods, but by an alternative method. The mother assembly for random numbers  $z_1$  is made by randomly storing  $N_m$  numbers the distribution of which is exactly equal to that of the value of  $F$  shown by the dots in Fig.1. To put it in the concrete, we prepare a number of digits whose value is  $C$  with population  $F(C)$  in number and value  $B+C$  with population  $F(B+C)$ ,  $\dots$ , value  $AB+C$  with  $F(AB+C)$ . Then, we store these numbers, the total number of

which is  $N_m = F(C) + F(B + C) + \dots + F(AB + C)$ , randomly in the memory. In contrast, the mother assembly for random numbers  $z_2$  can be made fast by storing sequentially the  $N_m$  random numbers produced by the previous methods, because random numbers  $z_2$  have little influence on the statistical quality of random numbers  $z$ . It is clear that the distribution of the random numbers generated by picking out a lot of child assemblies from these two mother assemblies should approach to the profile of  $F(z)$  in Fig.1.

### 3. Realization by software

The generated random numbers can be either real numbers or integral numbers, according to the values of  $A$ ,  $B$  and  $C$  in Eqs. (1),(2) and (3). For example, let

$$A = B = 2^8 = 256$$

$$C = 0$$

then the value of the random numbers is distributed from 0 to 65535( $= 2^{16} - 1$ ). The sizes of mother assembly and child assembly are chosen as follows:

$$N_m = 2^7 \times 2^7 = 128 \times 128$$

$$N_c = 100 \times 100$$

Figs. 3, 4 and 5 show the distribution of random numbers generated by the proposed method with the following three density functions:

- 1) uniformly distributed random numbers.
- 2) normally distributed random numbers.

$$f(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \quad (4)$$

Here  $\mu$  is the mean and  $\sigma^2$  is the variance.

3) random numbers with Planck distribution [11].

$$f(z) = \frac{z^3}{e^z - 1} \quad (5)$$

The solid lines in these figures are the exact distributions. These figures also show the distribution of random numbers generated by the previous methods, that is, by the library of Pro Fortran-77 for uniformly distributed random numbers, by the method based on the central limit theorem

$$z = x_1 + x_2 + \dots + x_{12} - 6 \quad (6)$$

for normally distributed random numbers, and by the acceptance-rejection method for the Planck distribution. Fig.3 shows that the distribution of random numbers by the proposed method is similar to or less diffusive than that by the library of Pro Fortran-77. For the normally distributed random numbers, it is well known that the distribution of random numbers generated by Eq.(6) is different from the profile of Eq.(4). It can be certified by Fig.4(d) that the top of the distribution of random numbers generated by Eq.(6) is crushed slightly. In contrast, there is no crush for that by the proposed method. For the Planck distribution, Fig.5 also shows that the proposed method can give a good statistical quality.

For a more detailed discussion, Fig.6 shows the chi-square goodness-of-fit tests. In the tests, the interval is divided into 10 nonoverlapping subintervals, and we have

$$\chi^2 = \frac{10}{N} \sum_{j=1}^{10} (N_j - \frac{N}{10})^2 \quad (7)$$

where  $N$  is the total number of random numbers, and  $N_j$  is the observed number of trial outcomes. Ten samples with different initial

seed are chosen for each cases. Fig.6 shows that the  $\chi^2$  by the proposed method is smaller than that by the library on the whole, and that while one sample of random numbers by the library exceeds 5 % of significance level, no sample by the proposed method exceeds it. Fig.7 shows the relative error  $\varepsilon$  of uniformly distributed random numbers, where  $\varepsilon$  is written as

$$\varepsilon = \sum_{j=1}^{50} \left| \frac{N_j - N/50}{N/50} \right| \quad (8)$$

When these results are approximated as a linear line by least-squares methods, the slope is  $-0.495$  for the results by the library and  $-0.482$  by the proposed method. This demonstrates that the error of the random numbers generated by the proposed method decreases in proportion to  $O(N^{-1/2})$ , which is not limited by the size of the mother assembly.

Fig.8 shows the computational time required to generate the random numbers on the personal computer PC-286LS(12MHz 80286 + 80287). In the figure, not the library but the Lehmer method is used. The proposed method can run with the same computational time to generate random numbers being in any distributions such as uniform distribution, normal distribution and Plank distribution, as shown by the solid line in Fig.8. This computational time includes the time required for generating two mother assemblies. The figure shows that the proposed method even with the time to make mother assembly being included can generate  $10^6$  random numbers with normal distribution about eleven times faster than the method relied on the central limit theorem, and those with Planck distribution about nine times faster than the acceptance-rejection method.



#### 4. Realization by hardware

In order for the proposed method to be realized by hardware, it is convenient that the following conditions are added, that is,

$$A = 2^a$$

$$B = 2^b$$

$$C = 0$$

and Eq.(3) is changed to be

$$z_2 = \text{int}(By) \tag{9}$$

Then, random numbers  $z$  are expressed as integer numbers of  $(a + b)$  bits. However, we can regard the random numbers as real numbers with fixed point. The  $a$  upper bits correspond to random numbers  $z_1$ , and the  $b$  lower bits correspond to  $z_2$ . Therefore, random numbers  $z$  of  $(a + b)$  bits can be generated by connecting random numbers  $z_1$  of  $a$  bits and random numbers  $z_2$  of  $b$  bits in series. Fig.9 shows the block diagram of the system for generating random numbers. ROMX is the read only memory (ROM) in which the mother assembly of random numbers  $z_1$  is stored, and ROMY is the ROM in which that of random numbers  $z_2$  is stored. Counters  $i_x, j_x, i_y$  and  $j_y$  count one hundred from  $i_0$  and  $j_0$ , respectively, to specify the address of child assembly. Header ROMs  $H_x$  and  $H_y$ , in which the values from 0 to 127 are randomly stored, give the left-upper position  $(i_0, j_0)$  of child assembly, and the position  $(i_0, j_0)$  is read out from two header ROMs  $H_x, H_y$  by counting counter  $h$  up. As ROMs 27256 with 32kbytes are used as header ROMs, the left-upper position  $(i_0, j_0)$  can be set  $32 \times 10^3/2$  times. Control circuits give the control signals  $\phi, E_h, E_i, E_j, L_i$  and  $L_j$  to operate these ROMs and counters sequentially, where  $\phi$  is a clock signal,  $L_i$  and  $L_j$  are load signals for counters,

and  $E_i$ ,  $E_j$  and  $E_h$  are enable signals for counters. The basic time charts are shown in Fig.10. Four bits Johnson counter is used to generate the clock signal  $\phi$  and the strobe signal  $\overline{stb}$ , and four states from (1) to (4) shown in this figure correspond to one clock. In the state (1), the clock signal is made, and then counters  $i_x$ ,  $j_x$ ,  $i_y$  and  $j_y$  are operated by using this clock signal. The outputs of ROMX and ROMY become valid at  $T_{acc}$  after the settlement of the output of counters  $i_x$ ,  $j_x$ ,  $i_y$  and  $j_y$ . At this time, Johnson counter is in the state (3). Johnson counter does not become the state (4) until the request signal  $\overline{req}$  arrives. In the state (4), the control circuits generate the strobe signal  $\overline{stb}$  to inform that a random number has been set. A computer connected outside reads the random number after getting the strobe signal  $\overline{stb}$ . A random number can be generated in the cycle. If the computer connected outside can read fast, the time of the cycle is  $200ns$ . Moreover, this control circuits make  $E_i$  valid every cycle,  $L_i$  and  $E_j$  valid every 100 cycles, and  $L_j$  and  $E_h$  valid every  $100 \times 100$  cycles.

Let us investigate the statistical quality of random numbers generated by the above system, whose top and bottom views are given in Fig.11. We can say that good pseudorandom numbers should satisfy the following requirements,

- 1) A lot of random numbers can be generated fast.
- 2) Generated random numbers have good statistical quality.
- 3) A period is long enough.
- 4) There is a reproducibility.

On term 1), as the time of one cycle is  $200ns$ , this system can generate  $5 \times 10^6$  random numbers per second. On term 2), the statistic quality of random numbers generated by this system is equal to that by software in section 3. Therefore, the quality by this system is equal to or better than that by the conventional methods. On term

3), as this system can generate  $16 \times 10^3$  pairs of  $(i_0, j_0)$ , the period of random numbers is  $1.6 \times 10^8$ . On term 4), this system can reproduce the random numbers by initializing the counter  $h$ .

## 5. Conclusions

A new method for the high-speed generation of random numbers with arbitrary distributions has been proposed. When the method is realized with Fortran, the method can generate random numbers with normal distribution about eleven times faster than the method relied on the central limit theorem, and those with Planck distribution about nine times faster than the acceptance-rejection method. When the method is realized by hardware, the system can generate  $5 \times 10^6$  random numbers per second with arbitrary distributions. The statistic quality of these random numbers is similar to or better than that by the conventional methods.

A special purpose computer for the Monte Carlo simulation using the proposed system will be built in future.

## Acknowledgment

This work was carried out under the collaborating research program at the National Institute for Fusion Science. Part of this work was supported by the Grant-in-Aid for Encouragement of Young Scientists from The Ministry of Education, Science and Culture.

## References

- [1] B. Alder, S. Fernbach and M. Rotenberg, *Methods in Computational Physics*, Academic Press (1963).
- [2] R.Y.Rubinstein, *Simulation and the Monte Carlo Method*, John Wiley & Sons (1981).
- [3] J. J. Komo and A. Aridgides, *Proceeding of the IEEE* 68 (1980) 1450.
- [4] T.Saito, M.Fushimi and T.Imai, *Trans. of Information Processing Society of Japan* 26 (1985) 148.
- [5] J. F. Monahan, *ACM Trans. on Mathematical Software* 13 (1987) 168.
- [6] K.Ohnuma and R.Sato, *Trans. of the Institute of Electronics, Information and Communication Engineers J* 70-A (1987) 1681.
- [7] V. Kachitvichyanukul and B. W. Schmeiser, *ACM Trans. on Mathematical Software* 15 (1989) 394.
- [8] D. G. Carta, *Communications of the ACM* 33 (1990) 87.
- [9] N.Bralić, R.Espinosa and C.Saavedra, *Jounal of Computational Physics* 88 (1990) 484.
- [10] H.J.Hilhorst, et al, *Journal of Statistical Physics* 34 (1984) 987.
- [11] Y.B.Zel'dovich and Y.P.Raizer, *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena-Volume I*, Academic Press (1963) 116.

## Figure Captions

- Fig.1. An arbitrary density function  $f(z)$  and the approximated piece-wise constant function  $F(z)$ .
- Fig.2. Mother assembly and child assembly.  
(a) A child assembly is selected as a shaded block from a mother assembly. (b) A mother assembly is assumed to be cyclic.
- Fig.3. The distributions of uniformly distributed random numbers.
- Fig.4. The distributions of normally distributed random numbers.
- Fig.5. The distributions of random numbers with Planck distribution.
- Fig.6. Chi-square goodness-of-fit tests for uniformly distributed random numbers.
- Fig.7. Relative errors of uniformly distributed random numbers.
- Fig.8. Computational time to generate random numbers.
- Fig.9. The block diagram of the system for generating random numbers with arbitrary distributions.
- Fig.10. The basic time charts of the system for generating random numbers.
- Fig.11. The photographs of the system for generating random numbers.

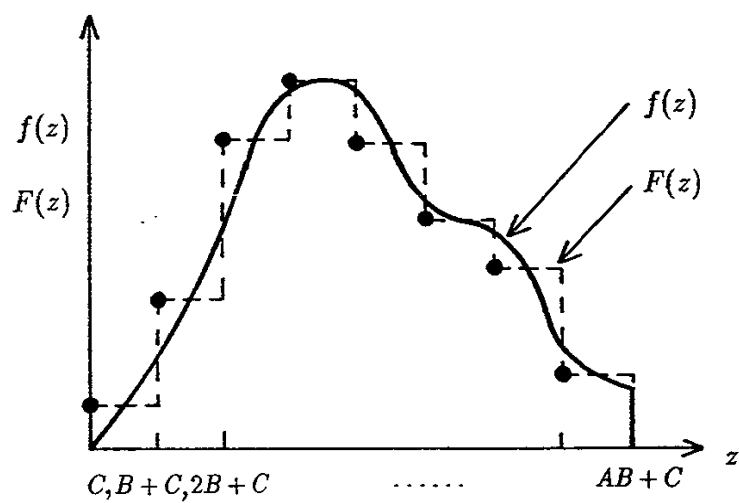


Fig.1

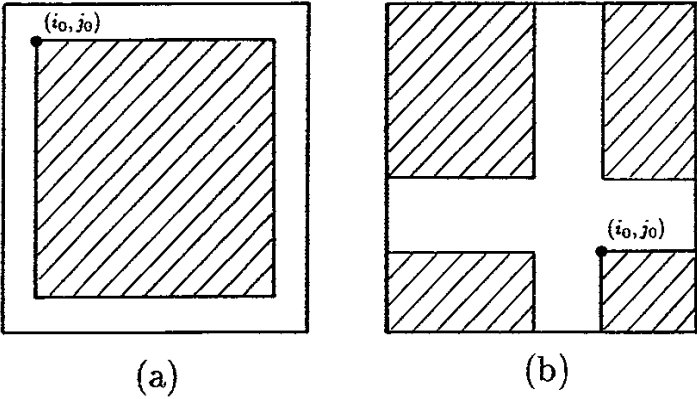
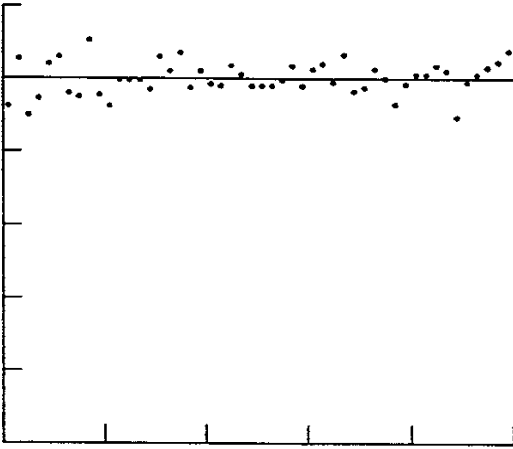
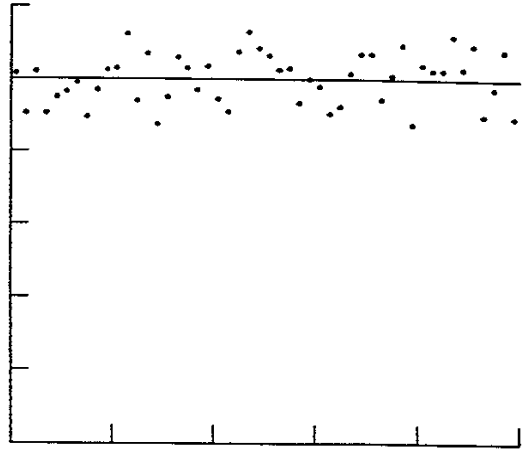


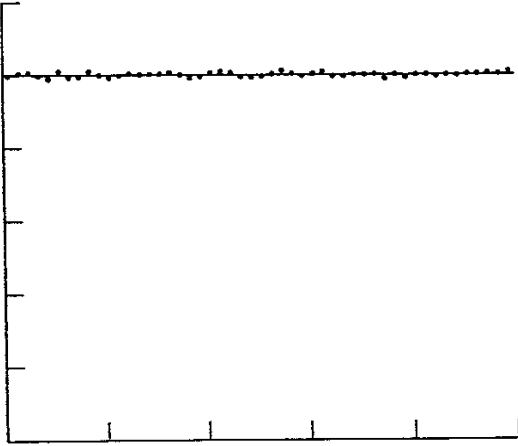
Fig.2



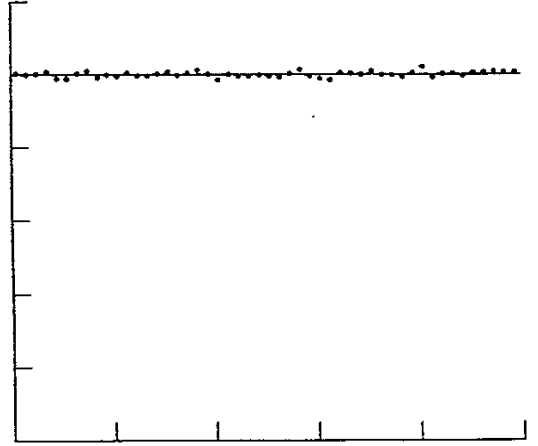
(a) the proposed method  
(the number of random numbers is  $1 \times 10^2$ )



(c) the library of Pro Fortran-77  
(  $1 \times 10^2$ )



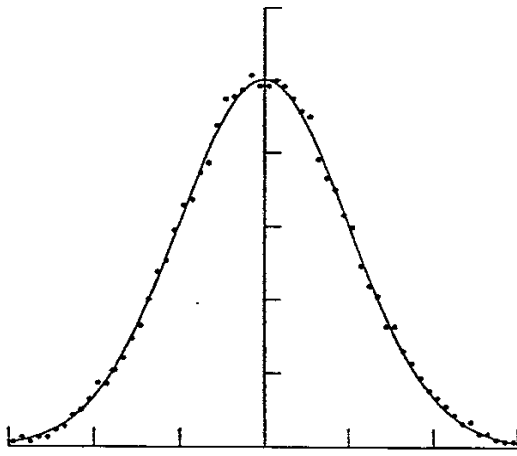
(b) the proposed method  
( $100 \times 10^2$ )



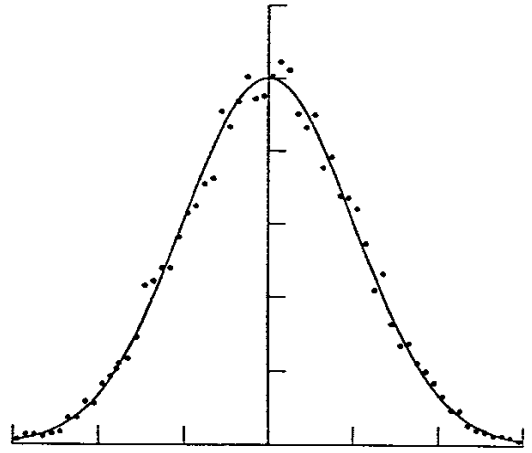
(d) the library of Pro Fortran-77  
( $100 \times 10^2$ )

Fig.3

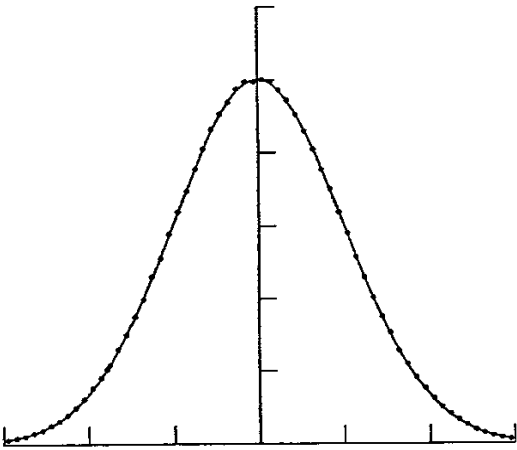




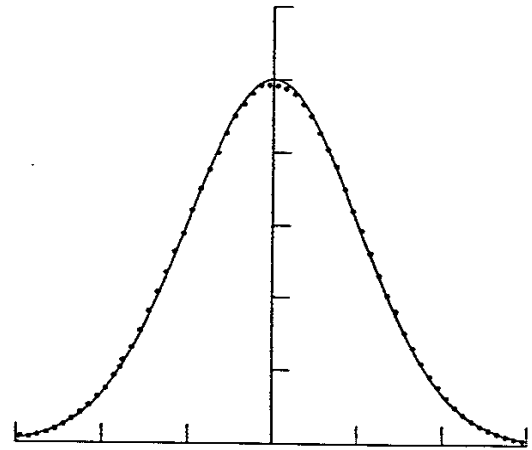
(a) the proposed method  
(  $1 \times 10^2$  )



(c) the central limit thorem  
(  $1 \times 10^2$  )

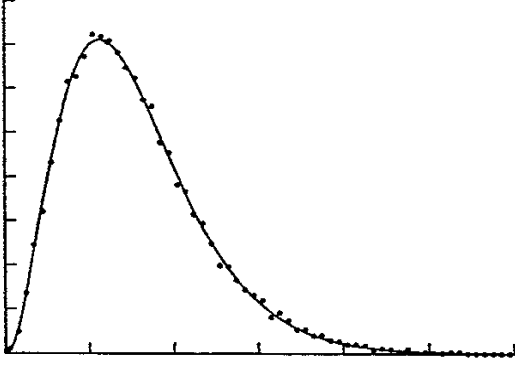


(b) the proposed method  
(  $100 \times 10^2$  )

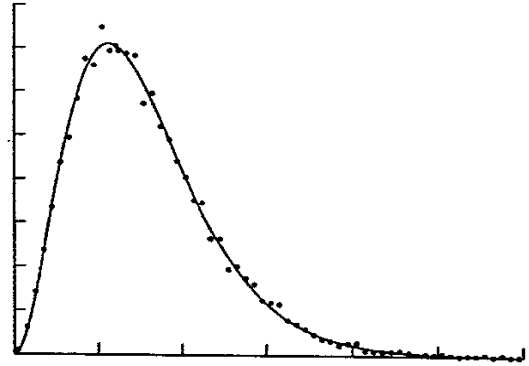


(d) the central limit thorem  
(  $100 \times 10^2$  )

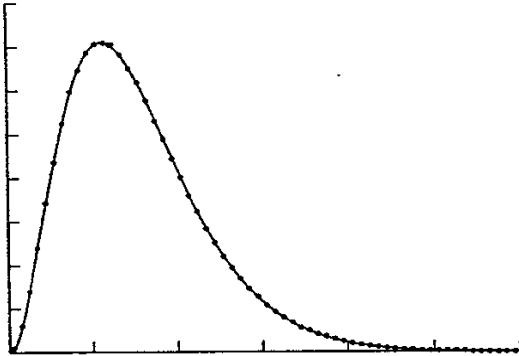
Fig.4



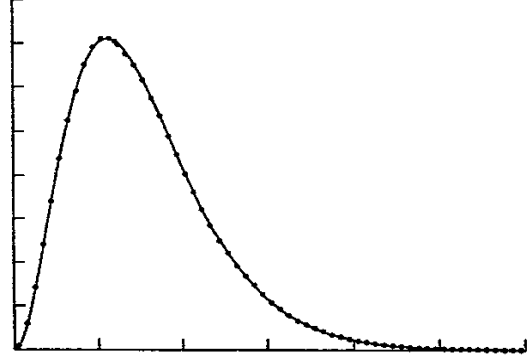
(a) the proposed method  
(  $1 \times 100^2$  )



(c) the acceptance-rejection method  
(  $1 \times 100^2$  )



(b) the proposed method  
(  $100 \times 100^2$  )



(d) the acceptance-rejection method  
(  $100 \times 100^2$  )

Fig.5

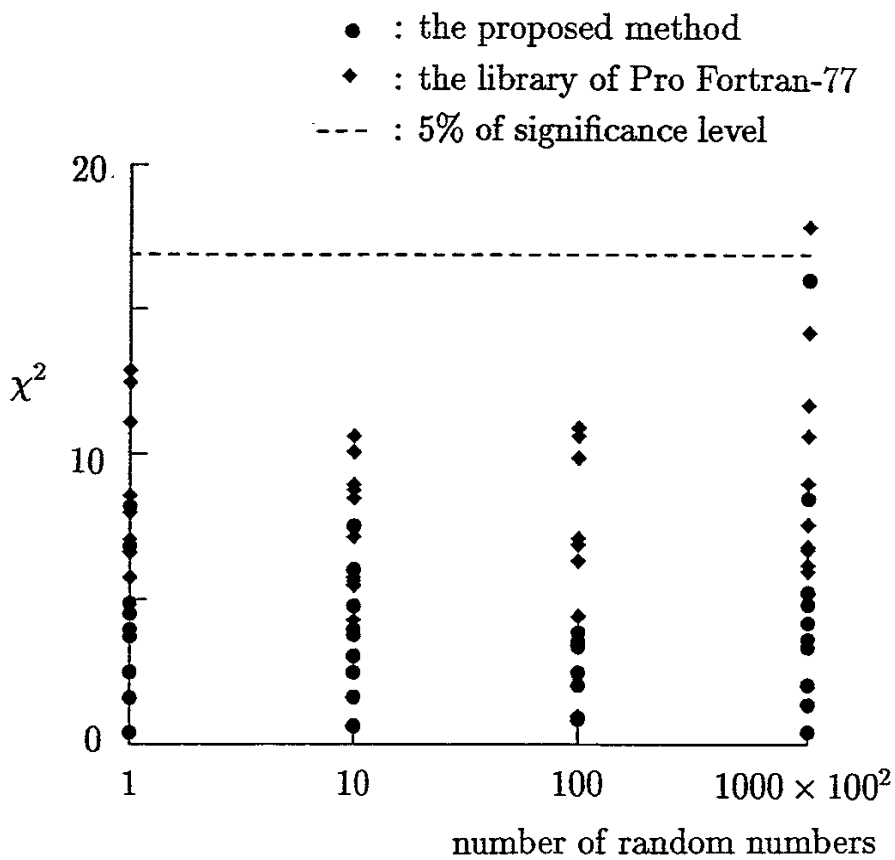


Fig.6

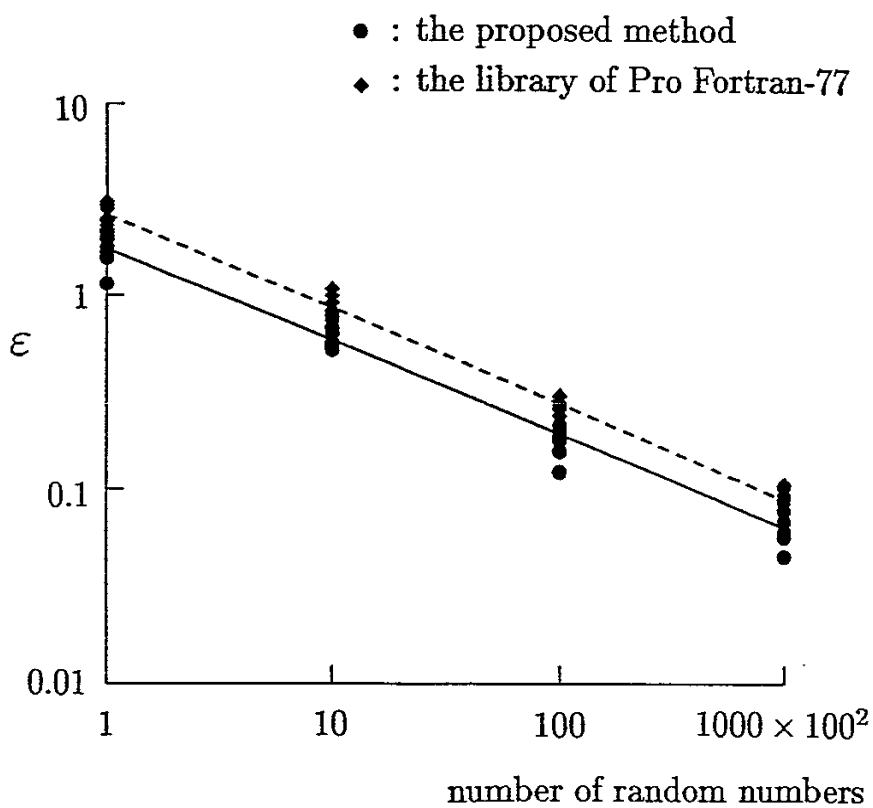


Fig.7

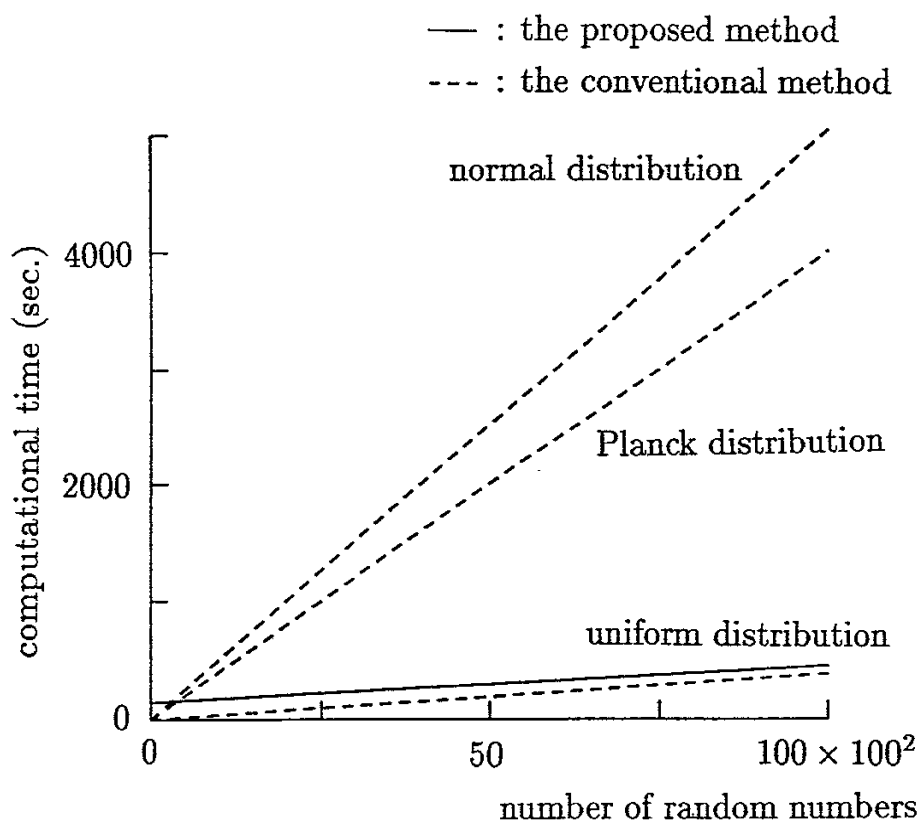


Fig.8

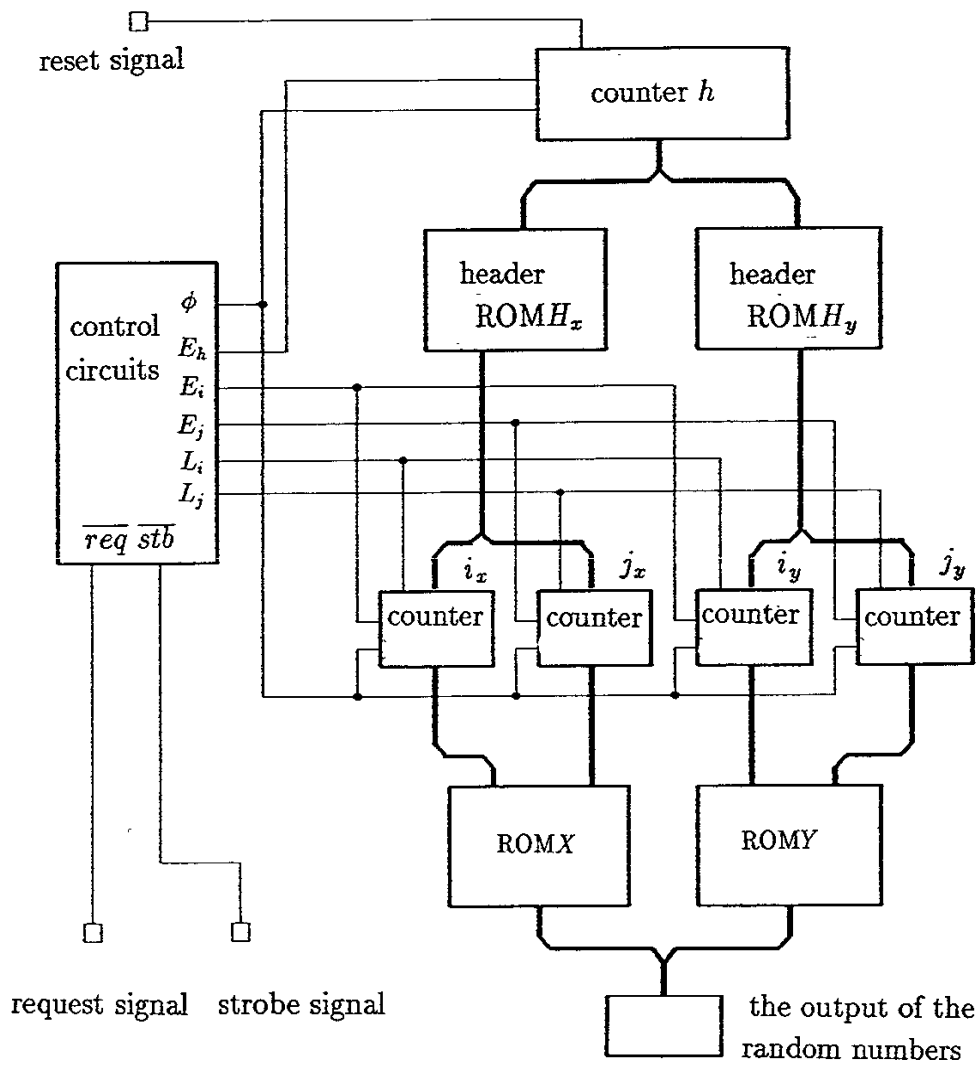


Fig.9

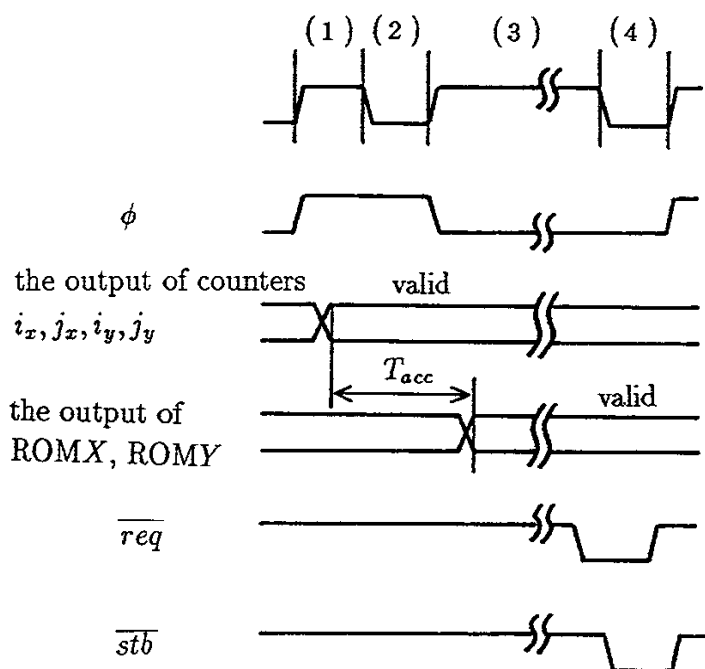


Fig.10

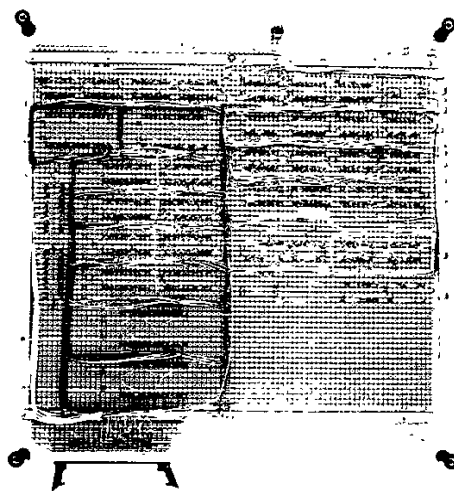
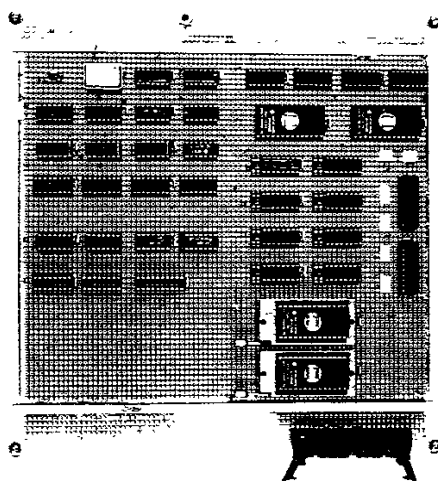


Fig.11



## Recent Issues of NIFS Series

- |          |   |
|----------|---|
| NIFS-61  | T.Yabe and H.Hoshino, <i>Two- and Three-Dimensional Behavior of Rayleigh-Taylor and Kelvin-Helmholz Instabilities</i> ; Oct. 1990                                       |
| NIFS-62  | H.B. Stewart, <i>Application of Fixed Point Theory to Chaotic Attractors of Forced Oscillators</i> ; Nov. 1990  |
| NIFS-63  | K.Konn., M.Mituhashi, Yoshi H.Ichikawa, <i>Soliton on Thin Vortex Filament</i> ; Dec. 1990  |
| NIFS-64  | K.Itoh, S.-I.Itoh and A.Fukuyama, <i>Impact of Improved Confinement on Fusion Research</i> ; Dec. 1990  |
| NIFS -65 | A.Fukuyama, S.-I.Itoh and K. Itoh, <i>A Consistency Analysis on the Tokamak Reactor Plasmas</i> ; Dec. 1990   |
| NIFS-66  | K.Itoh, H. Sanuki, S.-I. Itoh and K. Tani, <i>Effect of Radial Electric Field on <math>\alpha</math>-Particle Loss in Tokamaks</i> ; Dec. 1990                          |
| NIFS-67  | K.Sato, and F.Miyawaki, <i>Effects of a Nonuniform Open Magnetic Field on the Plasma Presheath</i> ; Jan.1991   |
| NIFS-68  | K.Itoh and S.-I.Itoh, <i>On Relation between Local Transport Coefficient and Global Confinement Scaling Law</i> ; Jan. 1991   |
| NIFS-69  | T.Kato, K.Masai, T.Fujimoto, F.Koike, E.Källne, E.S.Marmor and J.E.Rice, <i>He-like Spectra Through Charge Exchange Processes in Tokamak Plasmas</i> ; Jan.1991         |
| NIFS-70  | K. Ida, H. Yamada, H. Iguchi, K. Itoh and CHS Group, <i>Observation of Parallel Viscosity in the CHS Heliotron/Torsatron</i> ; Jan.1991                                 |
| NIFS-71  | H. Kaneko, <i>Spectral Analysis of the Heliotron Field with the Toroidal Harmonic Function in a Study of the Structure of Built-in Divertor</i> ; Jan. 1991             |
| NIFS-72  | S. -I. Itoh, H. Sanuki and K. Itoh, <i>Effect of Electric Field Inhomogeneities on Drift Wave Instabilities and Anomalous Transport</i> ; Jan. 1991                     |
| NIFS-73  | Y.Nomura, Yoshi.H.Ichikawa and W.Horton, <i>Stabilities of Regular Motion in the Relativistic Standard Map</i> ; Feb. 1991  |
| NIFS-74  | T.Yamagishi, <i>Electrostatic Drift Mode in Toroidal Plasma with Minority Energetic Particles</i> , Feb. 1991   |
| NIFS-75  | T.Yamagishi, <i>Effect of Energetic Particle Distribution on Bounce Resonance Excitation of the Ideal Ballooning Mode</i> , Feb. 1991                                   |
| NIFS-76  | T.Hayashi, A.Tadei, N.Ohyabu and T.Sato, <i>Suppression of Magnetic Surface Breeding by Simple Extra Coils in Finite Beta Equilibrium of Helical System</i> ; Feb. 1991 |

- NIFS-77 N. Ohyabu, *High Temperature Divertor Plasma Operation*; Feb. 1991
- NIFS-78 K. Kusano, T. Tamano and T. Sato, *Simulation Study of Toroidal Phase-Locking Mechanism in Reversed-Field Pinch Plasma*; Feb. 1991
- NIFS-79 K. Nagasaki, K. Itoh and S. -I. Itoh, *Model of Divertor Biasing and Control of Scrape-off Layer and Divertor Plasmas*; Feb. 1991
- NIFS-80 K. Nagasaki and K. Itoh, *Decay Process of a Magnetic Island by Forced Reconnection*; Mar. 1991
- NIFS-81 K. Takahata, N. Yanagi, T. Mito, J. Yamamoto, O. Motojima and LHDDesign Group, K. Nakamoto, S. Mizukami, K. Kitamura, Y. Wachi, H. Shinohara, K. Yamamoto, M. Shibui, T. Uchida and K. Nakayama, *Design and Fabrication of Forced-Flow Coils as R&D Program for Large Helical Device*; Mar. 1991
- NIFS-82 T. Aoki and T. Yabe, *Multi-dimensional Cubic Interpolation for ICF Hydrodynamics Simulation*; Apr. 1991
- NIFS-83 K. Ida, S.-I. Itoh, K. Itoh, S. Hidekuma, Y. Miura, H. Kawashima, M. Mori, T. Matsuda, N. Suzuki, H. Tamai, T. Yamauchi and JFT-2M Group, *Density Peaking in the JFT-2M Tokamak Plasma with Counter Neutral Beam Injection*; May 1991
- NIFS-84 A. Iiyoshi, *Development of the Stellarator/Heliotron Research*; May 1991
- NIFS-85 Y. Okabe, M. Sasao, H. Yamaoka, M. Wada and J. Fujita, *Dependence of Au<sup>-</sup> Production upon the Target Work Function in a Plasma-Sputter-Type Negative Ion Source*; May 1991
- NIFS-86 N. Nakajima and M. Okamoto, *Geometrical Effects of the Magnetic Field on the Neoclassical Flow, Current and Rotation in General Toroidal Systems*; May 1991
- NIFS-87 S. -I. Itoh, K. Itoh, A. Fukuyama, Y. Miura and JFT-2M Group, *ELMy-H mode as Limit Cycle and Chaotic Oscillations in Tokamak Plasmas*; May 1991
- NIFS-88 N. Matsunami and K. Itoh, *High Resolution Spectroscopy of H<sup>+</sup> Energy Loss in Thin Carbon Film*; May 1991
- NIFS-89 H. Sugama, N. Nakajima and M. Wakatani, *Nonlinear Behavior of Multiple-Helicity Resistive Interchange Modes near Marginally Stable States*; May 1991
- NIFS-90 H. Hojo and T. Hatori, *Radial Transport Induced by Rotating RF Fields and Breakdown of Intrinsic Ambipolarity in a Magnetic Mirror*; May 1991
- NIFS-91 M. Tanaka, S. Murakami, H. Takamaru and T. Sato, *Macroscale Implicit, Electromagnetic Particle Simulation of Inhomogeneous and Magnetized Plasmas in Multi-Dimensions*; May 1991

- NIFS-92 S. - I. Itoh, *H-mode Physics, -Experimental Observations and Model Theories-, Lecture Notes, Spring College on Plasma Physics, May 27 - June 21 1991 at International Centre for Theoretical Physics ( IAEA UNESCO ) Trieste, Italy ; Jun. 1991*
- NIFS-93 Y. Miura, K. Itoh, S. - I. Itoh, T. Takizuka, H. Tamai, T. Matsuda, N. Suzuki, M. Mori, H. Maeda and O. Kardaun, *Geometric Dependence of the Scaling Law on the Energy Confinement Time in H-mode Discharges; Jun. 1991*
- NIFS-94 H. Sanuki, K. Itoh, K. Ida and S. - I. Itoh, *On Radial Electric Field Structure in CHS Torsatron / Heliotron; Jun. 1991*
- NIFS-95 K. Itoh, H. Sanuki and S. - I. Itoh, *Influence of Fast Ion Loss on Radial Electric Field in Wendelstein VII-A Stellarator; Jun. 1991*
- NIFS-96 S. - I. Itoh, K. Itoh, A. Fukuyama, *ELMy-H mode as Limit Cycle and Chaotic Oscillations in Tokamak Plasmas; Jun. 1991*
- NIFS-97 K. Itoh, S. - I. Itoh, H. Sanuki, A. Fukuyama, *An H-mode-Like Bifurcation in Core Plasma of Stellarators; Jun. 1991*
- NIFS-98 H. Hojo, T. Watanabe, M. Inutake, M. Ichimura and S. Miyoshi, *Axial Pressure Profile Effects on Flute Interchange Stability in the Tandem Mirror GAMMA 10; Jun. 1991*
- NIFS-99 A. Usadi, A. Kageyama, K. Watanabe and T. Sato, *A Global Simulation of the Magnetosphere with a Long Tail : Southward and Northward IMF; Jun. 1991*
- NIFS-100 H. Hojo, T. Ogawa and M. Kono, *Fluid Description of Ponderomotive Force Compatible with the Kinetic One in a Warm Plasma ; July 1991*
- NIFS-101 H. Momota, A. Ishida, Y. Kohzaki, G. H. Miley, S. Ohi, M. Ohnishi K. Yoshikawa, K. Sato, L. C. Steinhauer, Y. Tomita and M. Tuszewski *Conceptual Design of D-<sup>3</sup>He FRC Reactor "ARTEMIS" ; July 1991*
- NIFS-102 N. Nakajima and M. Okamoto, *Rotations of Bulk Ions and Impurities in Non-Axisymmetric Toroidal Systems ; July 1991*
- NIFS-103 A. J. Lichtenberg, K. Itoh, S. - I. Itoh and A. Fukuyama, *The Role of Stochasticity in Sawtooth Oscillation ; Aug. 1991*
- NIFS-104 K. Yamazaki and T. Amano, *Plasma Transport Simulation Modeling for Helical Confinement Systems; Aug. 1991*
- NIFS-105 T. Sato, T. Hayashi, K. Watanabe, R. Horiuchi, M. Tanaka, N. Sawairi and K. Kusano, *Role of Compressibility on Driven Magnetic Reconnection ; Aug. 1991*
- NIFS-106 Qian Wen - Jia, Duan Yun - Bo, Wang Rong - Long and H. Narumi, *Electron Impact Excitation of Positive Ions - Partial Wave Approach in Coulomb - Eikonal Approximation ; Sep. 1991*

- NIFS-107 S. Murakami and T. Sato, *Macroscale Particle Simulation of Externally Driven Magnetic Reconnection*; Sep. 1991
- NIFS-108 Y. Ogawa, T. Amano, N. Nakajima, Y. Ohyabu, K. Yamazaki, S. P. Hirshman, W. I. van Rij and K. C. Shaing, *Neoclassical Transport Analysis in the Banana Regime on Large Helical Device (LHD) with the DKES Code*; Sep. 1991
- NIFS-109 Y. Kondoh, *Thought Analysis on Relaxation and General Principle to Find Relaxed State*; Sep. 1991
- NIFS-110 H. Yamada, K. Ida, H. Iguchi, K. Hanatani, S. Morita, O. Kaneko, H. C. Howe, S. P. Hirshman, D. K. Lee, H. Arimoto, M. Hosokawa, H. Idei, S. Kubo, K. Matsuoka, K. Nishimura, S. Okamura, Y. Takeiri, Y. Takita and C. Takahashi, *Shafranov Shift in Low-Aspect-Ratio Heliotron / Torsatron CHS* ; Sep 1991
- NIFS-111 R. Horiuchi, M. Uchida and T. Sato, *Simulation Study of Stepwise Relaxation in a Spheromak Plasma* ; Oct. 1991
- NIFS-112 M. Sasao, Y. Okabe, A. Fujisawa, H. Iguchi, J. Fujita, H. Yamaoka and M. Wada, *Development of Negative Heavy Ion Sources for Plasma Potential Measurement* ; Oct. 1991
- NIFS-113 S. Kawata and H. Nakashima, *Tritium Content of a DT Pellet in Inertial Confinement Fusion* ; Oct. 1991
- NIFS-114 M. Okamoto, N. Nakajima and H. Sugama, *Plasma Parameter Estimations for the Large Helical Device Based on the Gyro-Reduced Bohm Scaling* ; Oct. 1991
- NIFS-115 Y. Okabe, *Study of Au<sup>-</sup> Production in a Plasma-Sputter Type Negative Ion Source* ; Oct. 1991
- NIFS-116 M. Sakamoto, K. N. Sato, Y. Ogawa, K. Kawahata, S. Hirokura, S. Okajima, K. Adati, Y. Hamada, S. Hidekuma, K. Ida, Y. Kawasumi, M. Kojima, K. Masai, S. Morita, H. Takahashi, Y. Taniguchi, K. Toi and T. Tsuzuki, *Fast Cooling Phenomena with Ice Pellet Injection in the JIPP T-IIU Tokamak*; Oct. 1991
- NIFS-117 K. Itoh, H. Sanuki and S. -I. Itoh, *Fast Ion Loss and Radial Electric Field in Wendelstein VII-A Stellarator*; Oct. 1991
- NIFS-118 Y. Kondoh and Y. Hosaka, *Kernel Optimum Nearly-analytical Discretization (KOND) Method Applied to Parabolic Equations <<KOND-P Scheme>>*; Nov. 1991
- NIFS-119 T. Yabe and T. Ishikawa, *Two- and Three-Dimensional Simulation Code for Radiation-Hydrodynamics in ICF*; Nov. 1991
- NIFS-120 S. Kawata, M. Shiromoto and T. Teramoto, *Density-Carrying Particle Method for Fluid* ; Nov. 1991