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# Relaxed State of Energy in Incompressible Fluid and Incompressible MHD Fluid

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Two examples of application of the general principle to find relaxed states are presented to lead to two new results of the relaxed flow profile given by  $\nabla \times \mathbf{u} = \kappa \mathbf{u}$  in the incompressible viscous fluid and the relaxed state given by  $\{ \nabla \times \mathbf{B} = \lambda \mathbf{B} \text{ and } \nabla \times \mathbf{u} = \kappa \mathbf{u} \}$  in the incompressible viscous MHD fluid. The former result suggests that some common phenomena are expected to be observed in the reversed field pinch plasma and in the incompressible viscous fluid. The latter one is expected to have some connections with the field profile of  $\mathbf{B}$  and the flow pattern of  $\mathbf{u}$  in the magmas as a result of the earth dynamo.

**Keywords :** thought analysis, general principle to find relaxed states, relaxed state of energy, without helicity, incompressible viscous fluid, incompressible viscous MHD fluid, RFP, earth dynamo,

The general principle to find relaxed states has been reported by the present author, using a thought analysis on relaxation itself.<sup>1)</sup> Here, the word "thought analysis" means that we investigate logical structures, ideas or thoughts used in the objects being studied, and try to find some key elements for improvement and/or some other new thoughts which involve generality.<sup>1-3)</sup> An example of the application of the general principle to the energy relaxation of the MHD plasma has been shown in ref.1 to lead to the relaxed state of the force-free field,  $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ , and the mode transition condition of the relaxed state, which are derived by Taylor.<sup>4,5)</sup> Remarkable points of the applied theory in ref.1 are the followings: (a) The relaxed state of  $\nabla \times \mathbf{B} = \lambda \mathbf{B}$  and the mode transition condition can be derived from the general principle without using "helicity", whose concept is essential in the theory by Taylor.<sup>4,5)</sup> (b) The applied theory permits the quasi-steady energy flow through the boundary surface, as is indeed the case in most experiments, and leads to a more general relaxed state of  $2\eta \mathbf{j} = \alpha \mathbf{A}$  for plasmas having spatially dependent resistivity  $\eta$ . This result leads directly to the experimental fact of  $\mathbf{j} = 0$  near the wall, as is indeed the case in all experiments where  $\eta$  goes up to infinity near the boundary wall. The new theory in ref.1 was motivated by the recent experimental data in the ZP-2 device, which is a simple toroidal Z pinch without toroidal coils for the toroidal flux and therefore has no initial total helicity,<sup>6,7)</sup> and also motivated by the three-dimensional MHD simulations leading to the relaxed state given nearly by  $\nabla \times \mathbf{B} = \lambda \mathbf{B}$  without using any equations for the quantity of helicity.<sup>8)</sup> In this letter, other two examples of the application of the general principle for finding relaxed states to the incompressible viscous fluid and the incompressible viscous MHD fluid are presented to lead to internal structures of the relaxed fluid flow profile which are similar to the relaxed state of  $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ . The internal structure of the relaxed state in the incompressible MHD fluid derived here may have some possibility to be connected to some parts of

the earth dynamo.<sup>9)</sup>

The thought analysis on the concept "relaxation" itself of a dynamical system that consists of quantities  $\mathbf{q}(t, \mathbf{x})$  leads us to the following general principle to find the relaxed state of  $W$  in the system, as was shown in detail in ref.1,

$$\begin{aligned} \text{relaxed state of } W &\equiv \text{quasi-steady and minimum } |dW/dt| \text{ state} \\ &\text{for the value of } W \text{ at the relaxed state.} \end{aligned} \quad (1)$$

Here,  $t$  is time,  $\mathbf{x}$  denotes  $m$ -dimensional space variables, and  $\mathbf{q}$  represents a set of quantities having  $n$  elements, some of which are vectors such as  $\mathbf{B}$  and  $\mathbf{j}$ , and others are scalars such as the mass density, the energy density, the specific entropy and so on. Time evolutions of  $\mathbf{q}$  are given by definite equations such as the conservation laws of mass, momentum, and energy, and the Maxwell equations or the laws ruling the dynamical system in general sense. The global quantity of  $W(t)$  is defined as  $W(t) = \int w(\mathbf{q}) d\mathbf{x}$  by integrating one element,  $w$ , such as the energy density in  $\mathbf{q}$  over the space volume. This general principle of eq.(1) would be common for all dynamical systems including physical systems, biological systems, and/or economical systems. We note clearly here that the value of  $W$  in eq.(1) is not the initial value but the value at the point of the relaxed state, and  $W$  is not an invariant for time. Using the variational technique in order to find the internal structures or characteristic distributions in the quantities of  $\mathbf{q}(t^o, \mathbf{x})$  of the relaxed state with respect to  $\mathbf{x}$ , we obtain the following mathematical expression for the general principle of eq.(1),

$$\delta F = 0, \quad (2)$$

$$\delta^2 F > 0, \quad (3)$$

$$\delta q_j = 0 \text{ at the boundary.} \quad (4)$$

Here,  $t^\circ$  denotes the quasi-steady phase,  $F$  is the functional defined by  $F = |dW/dt| - \alpha W$ ;  $\delta F$  and  $\delta^2 F$  are the first and second variations of  $F$ ;  $\alpha$  is the Lagrange multiplier, and eq.(4) is the boundary conditions of the variations  $\delta \mathbf{q}(\mathbf{x})$  for the case that the boundary values of some elements  $q_j$  in  $\mathbf{q}$  are given at the relaxed state in the quasi-steady phase.

We now apply the general principle of eqs.(1) - (4) to the incompressible fluid which is described by the Navier-Stokes equation

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \nu \nabla^2 \mathbf{u}, \quad (5)$$

where  $\rho$ ,  $\mathbf{u}$ , and  $p$  are the fluid mass density, the fluid velocity, and the pressure, respectively,  $\nu$  is the coefficient of viscosity, and  $\nabla \cdot \mathbf{u} = 0$ . We pick up here the flow energy  $W_f = \int (\rho u^2/2) dv$  of the system and look for the relaxed state of  $W_f$ , which is therefore the global quantity  $W$  in the general principle. ( If the internal energy is assumed to be negligible compared to  $W_f$ , then the relaxed state of  $W_f$  is equivalent to the relaxed state of energy of the system. ) Using eq.(5), the vector formula of  $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot \nabla \times \mathbf{a} - \mathbf{a} \cdot \nabla \times \mathbf{b}$ ,  $\nabla \cdot \mathbf{u} = 0$ , and the Gauss theorem, we obtain  $dW_f/dt = -\int \nu \boldsymbol{\omega} \cdot \boldsymbol{\omega} dv + \oint \{ \nu(\mathbf{u} \times \boldsymbol{\omega}) - p\mathbf{u} \} \cdot d\mathbf{s}$ , where  $\oint$  denotes the surface integral over the boundary, and  $\boldsymbol{\omega} = \nabla \times \mathbf{u}$  is the vorticity. We assume here  $\nu$  to be constant, for simplicity. Substituting  $W_f$  and  $|dW_f/dt|$  respectively to  $W$  and  $|dW/dt|$  in the general principle of eqs.(1)-(4) to find the relaxed state, we obtain the followings,

$$\delta F = \int (2\nu \delta \boldsymbol{\omega} \cdot \boldsymbol{\omega} - \alpha \rho \delta \mathbf{u} \cdot \mathbf{u}) dv = 0, \quad (6)$$

$$\delta^2 F = \int (2\nu \delta \boldsymbol{\omega} \cdot \delta \boldsymbol{\omega} - \alpha \rho \delta \mathbf{u} \cdot \delta \mathbf{u}) dv > 0, \quad (7)$$

where the values of quantities on the boundary surface in  $dW_f/dt$  are assumed to be given so that the surface integral terms vanish in both  $\delta F$  and  $\delta^2 F$  by the boundary conditions of eq.(4), for simplicity. The boundary conditions are given here as

$\{ \delta \mathbf{u} = 0, \delta \omega = 0, \delta p = 0; \text{ at the boundary } \}$ . Using  $\delta \omega = \nabla \times \delta \mathbf{u}$ ,  $\omega = \nabla \times \mathbf{u}$ ,  $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot \nabla \times \mathbf{a} - \mathbf{a} \cdot \nabla \times \mathbf{b}$ , and the Gauss theorem again, we obtain the followings from eqs.(6) and (7),

$$\delta F = 2\nu \int \delta \mathbf{u} \cdot (\nabla \times \nabla \times \mathbf{u} - \frac{\alpha \rho}{2\nu} \mathbf{u}) dv = 0, \quad (8)$$

$$\delta^2 F = 2\nu \int \delta \mathbf{u} \cdot (\nabla \times \nabla \times \delta \mathbf{u} - \frac{\alpha \rho}{2\nu} \delta \mathbf{u}) dv > 0, \quad (9)$$

where the surface integral terms vanish in both  $\delta F$  and  $\delta^2 F$  by the same boundary conditions used at eqs.(6) and (7). We then obtain the Euler-Lagrange equation for arbitrary variations of  $\delta \mathbf{u}$  from eq.(8) as follows,

$$\nabla \times \nabla \times \mathbf{u} = \frac{\alpha \rho}{2\nu} \mathbf{u}. \quad (10)$$

The first integral for eq.(10) is given as follows,

$$\nabla \times \mathbf{u} = \pm \kappa \mathbf{u}, \quad (11)$$

$$|\kappa| = \sqrt{\frac{\alpha \rho}{2\nu}}, \quad (12)$$

where the Lagrange multiplier  $\alpha$  is assumed to be positive. It is easy to show that eq.(11) is the solution of eq.(10), by substituting eq.(11) to the result of  $\nabla \times \{\text{eq.(11)}\}$  to lead to eq.(10). Since the mathematical structure of eq.(11) is the same as that of the force-free field  $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ , it is easy to show that the Bessel Function model ( BFM ) for the reversed field pinch ( RFP ) plasma is also applicable to the flow profiles of  $\mathbf{u}$  in the relaxed state of the cylindrical incompressible fluid.<sup>1,4,5,10)</sup>

We next discuss the mode transition point of the relaxed state, by using eq.(9), in the same way as the theory for the RFP plasma.<sup>1,4,5,10)</sup> We consider here the following associated eigenvalue problem for the critical perturbation  $\delta \mathbf{u}$  that makes the volume integral term of  $\delta^2 F$  in eq.(9) become zero:

$$\nabla \times \nabla \times \delta \mathbf{u}_i - \kappa_i^2 \delta \mathbf{u}_i = 0, \quad (13)$$

with the boundary condition of  $\delta \mathbf{u} \cdot d\mathbf{s} = 0$  at the boundary, where  $\kappa_i$  and  $\delta \mathbf{u}_i$  denote the eigenvalue and the eigensolution, respectively. The eigensolution  $\delta \mathbf{u}_i$  of eq.(13) is also the solution of  $\nabla \times \delta \mathbf{u}_i = \pm \kappa_i \delta \mathbf{u}_i$ . Substituting the eigensolution  $\delta \mathbf{u}_i$  into eq.(9) and using eq.(13), we obtain the following:

$$\delta^2 F = 2\nu(\kappa_i^2 - \kappa^2) \int \delta \mathbf{u}_i \cdot \delta \mathbf{u}_i dv > 0, \quad (14)$$

where eq.(12) is used. Since eq.(14) is required for all eigenvalues, we obtain the following condition for the relaxed state with the minimum  $|dW_f/dt|$ ,

$$\kappa_{-1} < \kappa < \kappa_1, \quad (15)$$

where  $\kappa_{-1}$  and  $\kappa_1$  are the largest of the negative and the smallest of the positive eigenvalues, respectively. When the value of  $\kappa$  goes out of the condition eq.(15), then the mixed mode, which consists with the basic mode by eq.(11) with  $\kappa = \kappa_1$  and the lowest eigenmode by eq.(13), becomes the relaxed state with the minimum value of  $|dW_f/dt|$ , just the same as the case of the force-free field  $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ .<sup>1,4,5,10)</sup> Since the mathematical structures of eqs.(6) - (15) for the fluid flow velocity  $\mathbf{u}$  are the same as those of equations for the magnetic field  $\mathbf{B}$  used in the RFP plasma,<sup>1)</sup> some common phenomena are expected to be observed in the RFP plasma and in the incompressible fluid, like as the saw tooth oscillation in the former and its corresponding phenomenon in the latter.

We next show another example of the application of the general principle of eqs.(1) - (4) to the incompressible MHD fluid which is described by the following extended Navier-Stokes equation,

$$\rho \frac{d\mathbf{u}}{dt} = \mathbf{j} \times \mathbf{B} - \nabla p + \nu \nabla^2 \mathbf{u}. \quad (16)$$

We pick up here the magnetic energy,  $W_m$ , and the flow energy,  $W_f$ , of the system and look for the relaxed state of  $W = W_m + W_f = \int (B^2/2\mu_o + \rho u^2/2) dv$ . ( If the internal energy is assumed to be negligible compared to this  $W$ , then the relaxed state of  $W$  is equivalent to the relaxed state of energy of the system. ) Using eq.(16), Maxwell's equations, Ohm's law of  $\eta \mathbf{j} = \mathbf{E} + \mathbf{u} \times \mathbf{B}$ ,  $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot \nabla \times \mathbf{a} - \mathbf{a} \cdot \nabla \times \mathbf{b}$ , and the Gauss theorem again, we obtain  $dW/dt = - \int ( \eta \mathbf{j} \cdot \mathbf{j} + \nu \boldsymbol{\omega} \cdot \boldsymbol{\omega} ) dv + \oint \{ \nu (\mathbf{u} \times \boldsymbol{\omega}) - p \mathbf{u} - \mathbf{E} \times \mathbf{H} \} \cdot d\mathbf{s}$ . We assume here the resistivity  $\eta$  to be constant, for simplicity. Substituting  $W$  and  $|dW/dt|$  shown above respectively to  $W$  and  $|dW/dt|$  in the general principle of eqs.(1)-(4) to find the relaxed state and using the same procedure used from eq.(6) to eq.(9), we obtain the followings which correspond to eqs.(8) and (9),

$$\delta F = \int \left\{ \frac{2\eta}{\mu_o^2} \delta \mathbf{B} \cdot (\nabla \times \nabla \times \mathbf{B} - \frac{\alpha \mu_o}{2\eta} \mathbf{B}) + 2\nu \delta \mathbf{u} \cdot (\nabla \times \nabla \times \mathbf{u} - \frac{\alpha \rho}{2\nu} \mathbf{u}) \right\} dv = 0, \quad (17)$$

$$\delta^2 F = \int \left\{ \frac{2\eta}{\mu_o^2} \delta \mathbf{B} \cdot (\nabla \times \nabla \times \delta \mathbf{B} - \frac{\alpha \mu_o}{2\eta} \delta \mathbf{B}) + 2\nu \delta \mathbf{u} \cdot (\nabla \times \nabla \times \delta \mathbf{u} - \frac{\alpha \rho}{2\nu} \delta \mathbf{u}) \right\} dv > 0, \quad (18)$$

where the surface integral terms vanish in both  $\delta F$  and  $\delta^2 F$  by the similar boundary conditions used at eqs.(6) and (7). We then obtain the Euler-Lagrange equations for arbitrary variations of  $\delta \mathbf{u}$  and  $\delta \mathbf{B}$  from eq.(17), and also their first integrals, corresponding to eqs.(10) - (12), as follows,

$$\nabla \times \nabla \times \mathbf{B} = \lambda^2 \mathbf{B}, \quad (19)$$

$$\nabla \times \nabla \times \mathbf{u} = \kappa^2 \mathbf{u}, \quad (20)$$

$$\nabla \times \mathbf{B} = \pm \lambda \mathbf{B}, \quad (21)$$



$$\nabla \times \mathbf{u} = \pm \kappa \mathbf{u}, \quad (22)$$

$$|\lambda| = \sqrt{\frac{\alpha \mu_o}{2\eta}}, \quad (23)$$

$$|\kappa| = \sqrt{\frac{\alpha \rho}{2\nu}}, \quad (24)$$

where the Lagrange multiplier  $\alpha$  is assumed to be positive. We can also discuss the mode transition points of the relaxed state for  $\mathbf{B}$  and  $\mathbf{u}$ , by using eq.(18) in the same way from eq.(13) to eq.(15).<sup>1,5,10)</sup>

Various combinations of solutions for eqs.(21) and (22) are expected to be observed as the field profile of  $\mathbf{B}$  and the flow pattern of  $\mathbf{u}$  in the relaxed states of energy of the incompressible MHD fluid, corresponding to the amount of the magnetic energy  $W_m$  and the flow energy  $W_f$ . Some of them would possibly be connected to the field profile of  $\mathbf{B}$  and the flow pattern of  $\mathbf{u}$  in the magmas as a result of the earth dynamo.<sup>9)</sup> When we consider the flow pattern of the electron fluid in the experimental RFP plasma, gross features of  $\mathbf{B}$  and  $\mathbf{u}$  in the relaxed state are expected to be given by eqs.(21) and (22) with some necessary corrections by the compressibility. One of the simplest solutions for eqs.(21) and (22) is the solution for the case with  $\mathbf{B} = \gamma \mathbf{u}$  with a constant value of  $\gamma$ . In this case, eqs.(21) and (23) become to be equivalent to eqs.(22) and (24), and there exists a relation of  $\mu_o/\eta = \rho/\nu$  from eqs.(23) and (24).

In conclusion, we have presented two examples of the application of the general principle to find relaxed states and have derived the relaxed state of eq.(11) in the incompressible viscous fluid and that of eqs.(21) and (22) in the incompressible viscous MHD fluid. The common mathematical structures of eqs.(6) - (15) for the fluid flow velocity  $\mathbf{u}$  and the equations for the magnetic field  $\mathbf{B}$  used in the RFP plasma<sup>1)</sup> suggest that some common phenomena are expected to be observed in the RFP plasma and in the incompressible fluid. The relaxed state described by eqs.(21) and (22) in the

incompressible HMD fluid are expected to be connected to the field profile of  $\mathbf{B}$  and the flow pattern of  $\mathbf{u}$  in the magmas as a result of the earth dynamo.<sup>9)</sup>

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