ISSN 0915-633X

NATIONAL INSTITUTE FOR FUSION SCIENCE

Statistical Analysis of Anomalous Transport in Resistive Interchange Turbulence

H. Sugama and M. Wakatani

(Received – Nov. 22, 1991)

NIFS-125 Dec. 1991

RESEARCH REPORT
NIFS Series

NAGOYA, JAPAN

Statistical Analysis of Anomalous Transport in Resistive Interchange Turbulence

Hideo Sugama and Masahiro Wakatani †

National Institute for Fusion Science, Chikusaku, Nagoya 464–01

† *Plasma Physics Laboratory, Kyoto University, Gokasho, Uji 611*

Abstract

A new anomalous transport model for resisistive interchange turbulence is derived from statistical analysis applying two-scale direct-interaction approximation to resistive magnetohydrodynamic equations with a gravity term. Our model is similar to the K-ϵ model for eddy viscosity of turbulent shear flows in that anomalous transport coefficients are expressed in terms of by the turbulent kinetic energy K and its dissipation rate ϵ while K and ϵ are determined by transport equations. This anomalous transport model can describe some nonlocal effects such as those from boundary conditions which cannot be treated by conventional models based on the transport coefficients represented by locally determined plasma parameters.

Keywords: anomalous transport, two-scale direct-interaction approximation, resistive interchange turbulence, resistive MHD

§1. INTRODUCTION

Conventional treatments for anomalous transport[1,2] have been based on the local transport coefficients (D or χ) which are expressed as functions of local plasma parameters such as local density n, temperature T, magnetic field B and a number of gradient scale length L_n, L_T, L_s, $\cdots$:

$$D \text{ or } \chi = F(n, T, B, L_n, L_T, L_s, \cdots).$$

These treatments assume that the mixing length l and the time scale τ of the turbulence responsible for the anomalous transport are determined by the local plasma parameters. However it is possible that the turbulence structure has nonlocal nature and the validity of the expression for the local transport coefficients as given above is limited. For example, the conventional transport models did not treat the radial propagation of turbulence energy[3] which can bring about fluctuations and the resultant anomalous transport even in the linearly stable regions.

Here we present a K-ϵ type model for the analysis of anomalous transport in the resistive intechange turbulence. A K-ϵ model was originally proposed for modeling the turbulent (or eddy) viscosity of the large Reynolds number turbulent shear flow[4] and its theoretical formulation was advanced by Yoshizawa using two-scale direct-interaction approximation (TSDIA).[5-7] TSDIA is based on Kraichnan's direct-interaction approximation (DIA)[8,9] and the two-scale expansion technique[10] utilizing the fact that the characteristic scale of the turbulent fluctuations is much smaller than that of the mean fields. In the K-ϵ model the turbulent kinetic energy $K \equiv \frac{1}{2}\langle v^2 \rangle$ and its viscous dissipation rate ϵ characterize the local turbulence spectral structure and their temporal and spatial variations are governed by transport equations. The turbulent transport coefficient is given by $D \sim K^2/\epsilon$, which has some nonlocal properties not included in the conventional expressions since the mixing length $l \sim K^{3/2}/\epsilon$, the turbulent time scale $\tau \sim K/\epsilon$ and the turbulent transport coefficient $D \sim l^2/\tau \sim K^2/\epsilon$ are determined not locally but globally by the solution of K-ϵ transport equations.

The resistive interchange turbulence is driven by the density or pressure gradient combined with the gravity force (which may be produced by the magnetic curvature) and has been extensively studied as a cause of anomalous transport in the peripheral region of stellarator plasmas.[11–13] In this article we derive the K-ϵ anomalous transport model for the resistive interchange turbulence by applying TSDIA to the resistive magnetohydrodynamic (MHD) equations with a gravity term. The governing equations of the resistive interchange turbulence have a similar structure to those of the thermally-driven turbulence[14,15] in which the temperature gradient and the buoyancy force play roles of the density gradient and the gravity force in the former. Then, in our starting equations, the velocity field couples to the magnetic field as well as to the density field through the gravity term. It is considerably difficult to treat these couplings directly so that we solve them perturbatively by using another expansion parameter.

This paper is organized as follows. In §2 the fundamental equations describing the resistive interchange turbulence are explained. In §3 TSDIA is applied to the fundamental equations given in §2 and the turbulent transport coefficients are represented by the wavenumber spectra of the response and correlation functions. In §4 the turbulent diffusivities are rewritten in terms of K and ϵ by using the inertial-range theory for the lowest-order turbulent fields. The transport equations for K and ϵ are described in §5. Finally, discussions are given in §6.

§2. Fundamental Equations

Here we employ the following magnetohydrodynamical (MHD) equations in order to describe the resistive interchange turbulence :

$$\left(\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\nabla\right)\boldsymbol{v}=\nabla p^{*}+\frac{1}{4\pi\rho_0}\boldsymbol{B}\cdot\nabla\boldsymbol{B}+\frac{\rho}{\rho_0}\boldsymbol{g}+\nu\nabla^2\boldsymbol{v} \tag{2.1}$$

$$\left(\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\nabla\right)\rho=S \tag{2.2}$$

$$\left(\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\nabla\right)\boldsymbol{B}=\boldsymbol{B}\cdot\nabla\boldsymbol{v}+\frac{c^2\eta}{4\pi}\nabla^2\boldsymbol{B} \tag{2.3}$$

where $\boldsymbol{v}$ denotes the plasma flow velocity, $\boldsymbol{B}$ the magnetic field, ρ the mass density, $\boldsymbol{g}$ the acceleration due to gravity, ν the kinematic viscosity, η the resistivity, c the light velocity in the vacuum and $p^* = (p+B^2/8\pi)/\rho_0$ the sum of the kinematic pressure p and the magnetic pressure $B^2/8\pi$ divided by the averaged mass $\rho_0 = \langle\rho\rangle$. In the above equations (2.1)–(2.3) we used the solenoidal condition for $\boldsymbol{B}$ and $\boldsymbol{v}$

$$\nabla\cdot\boldsymbol{B}=0, \qquad \nabla\cdot\boldsymbol{v}=0 \tag{2.4}$$

in which the latter represents the incompressibility. In the momentum balance equation (2.1), the spatial and temporal variation of the mass density ρ are neglected except in the acceleration term $\rho\boldsymbol{g}$ according to the Boussinesq approximation. The source term S is included in the density equation (2.2).

The turbulent quantities are divided into the average and fluctuating parts as

$$\boldsymbol{v}=\boldsymbol{V}+\tilde{\boldsymbol{v}}, \qquad \boldsymbol{V}=\langle\boldsymbol{v}\rangle \tag{2.5}$$

$$\boldsymbol{B}=\boldsymbol{B}_0+\tilde{\boldsymbol{b}}, \qquad \boldsymbol{B}_0=\langle\boldsymbol{B}\rangle \tag{2.6}$$

$$\rho=\rho_0+\tilde{\rho}, \qquad \rho_0=\langle\rho\rangle \tag{2.7}$$

$$p^*=P+\tilde{p}, \qquad P=\langle p^*\rangle \tag{2.8}$$

where $\langle\cdot\rangle$ denotes the ensemble average.

From eqs.(2.1)-(2.8), we obtain the following equations for the average and turbulent parts of energy

$$
\begin{aligned}
&\left(\frac{\partial}{\partial t}+\boldsymbol{V}\cdot\nabla\right)\left(\frac{1}{2}\rho_0 V^2+\frac{1}{8\pi}B^2+\rho_0\Phi_g\right)\\
&= \nabla\cdot\left[-\rho_0 P\boldsymbol{V}+\frac{1}{4\pi}(\boldsymbol{V}\cdot\boldsymbol{B}_0)\boldsymbol{B}_0+\left(-\rho_0\langle\tilde{\boldsymbol{v}}\tilde{\boldsymbol{v}}\rangle+\frac{1}{4\pi}\langle\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}\rangle\right)\cdot\boldsymbol{V}\right.\\
&\qquad\left.+\frac{1}{4\pi}(\langle\tilde{\boldsymbol{b}}\tilde{\boldsymbol{v}}\rangle-\langle\tilde{\boldsymbol{v}}\tilde{\boldsymbol{b}}\rangle)\cdot\boldsymbol{B}_0-\Phi_g\langle\tilde{\rho}\tilde{\boldsymbol{v}}\rangle+\frac{\mu}{2}\nabla V^2+\left(\frac{c}{4\pi}\right)^2\frac{\eta}{2}\nabla B_0^2\right]\\
&\quad+\left(\rho_0\langle\tilde{\boldsymbol{v}}\tilde{\boldsymbol{v}}\rangle-\frac{1}{4\pi}\langle\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}\rangle\right):\nabla\boldsymbol{V}+\frac{1}{4\pi}(\langle\tilde{\boldsymbol{b}}\tilde{\boldsymbol{v}}\rangle-\langle\tilde{\boldsymbol{v}}\tilde{\boldsymbol{b}}\rangle):\nabla\boldsymbol{B}_0\\
&\quad+S\Phi_g-\mu\left\langle\frac{\partial V^a}{\partial x^b}\frac{\partial V^a}{\partial x^b}\right\rangle-\left(\frac{c}{4\pi}\right)^2\eta\left\langle\frac{\partial B_0^a}{\partial x^b}\frac{\partial B_0^a}{\partial x^b}\right\rangle-\langle\tilde{\rho}\tilde{\boldsymbol{v}}\rangle\cdot\boldsymbol{g}
\end{aligned}
\tag{2.9}
$$

$$
\begin{aligned}
&\left(\frac{\partial}{\partial t}+\boldsymbol{V}\cdot\nabla\right)\left(\frac{1}{2}\rho_0\langle\tilde{v}^2\rangle+\frac{1}{8\pi}\langle\tilde{b}^2\rangle\right)\\
&= \nabla\cdot\left[-\frac{1}{2}\rho_0\langle\tilde{v}^2\tilde{\boldsymbol{v}}\rangle-\frac{1}{8\pi}\langle\tilde{b}^2\tilde{\boldsymbol{v}}\rangle-\langle\tilde{p}\tilde{\boldsymbol{v}}\rangle+\frac{1}{4\pi}\langle\tilde{\boldsymbol{v}}\cdot\tilde{\boldsymbol{b}}\rangle\boldsymbol{B}_0+\frac{1}{4\pi}\langle(\tilde{\boldsymbol{v}}\cdot\tilde{\boldsymbol{b}})\tilde{\boldsymbol{b}}\rangle\right.\\
&\qquad\left.+\frac{\mu}{2}\nabla\langle\tilde{v}^2\rangle+\left(\frac{c}{4\pi}\right)^2\frac{\eta}{2}\nabla\langle\tilde{b}^2\rangle\right]\\
&\quad+\left(-\rho_0\langle\tilde{\boldsymbol{v}}\tilde{\boldsymbol{v}}\rangle+\frac{1}{4\pi}\langle\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}\rangle\right):\nabla\boldsymbol{V}+\frac{1}{4\pi}(\langle\tilde{\boldsymbol{v}}\tilde{\boldsymbol{b}}\rangle-\langle\tilde{\boldsymbol{b}}\tilde{\boldsymbol{v}}\rangle):\nabla\boldsymbol{B}_0\\
&\quad+\langle\tilde{\rho}\tilde{\boldsymbol{v}}\rangle\cdot\boldsymbol{g}-\mu\left\langle\frac{\partial\tilde{v}^a}{\partial x^b}\frac{\partial\tilde{v}^a}{\partial x^b}\right\rangle-\left(\frac{c}{4\pi}\right)^2\eta\left\langle\frac{\partial\tilde{b}^a}{\partial x^b}\frac{\partial\tilde{b}^a}{\partial x^b}\right\rangle
\end{aligned}
\tag{2.10}
$$

where we assumed $\boldsymbol{g}$ to be expressed in terms of a time-independent potential Φ_g as $\boldsymbol{g} = -\nabla\Phi_g$. Here we treated $\mu \equiv \rho_0\nu$ and η as constants and the density source term S as a non-random quantity. In eqs.(2.9) and (2.10), terms in the form of $\nabla\cdot[\cdots]$ represent transport of energy. We can see that the production of the turbulent energy due to the gradients of the mean velocity and magnetic field are transferred from the mean part of energy since the corresponding turbulent energy production terms appear in eqs.(2.9) and (2.10) with opposite signs. The turbulent energy production due to the gravity, which is essential to the resistive interchange turbulence, also comes from the loss of the mean part of energy as can be seen from the signs of $\langle\tilde{\rho}\tilde{\boldsymbol{v}}\rangle\cdot\boldsymbol{g}$ in eqns.(2.9) and (2.10). In addition to the energy supply due to the transport through the plasma boundary surface, the mean energy are produced by the density source term $S\Phi_g$ which represents the supply of particles with gravity potential. Both the mean and turbulent energy equations (2.9) and (2.10) contain the viscous and Ohmic dissipation terms, which are proportional to μ and η, respectively.

In the following sections, we assume that $\boldsymbol{V} \equiv \langle \boldsymbol{v} \rangle = 0$ since it makes the resistive interchange turbulence problem simpler by eliminating the turbulent energy production due to the gradient of the mean velocity.

§3. Formulation

When $\boldsymbol{V}=0$, the equation for $\tilde{\boldsymbol{v}}$, $\tilde{\boldsymbol{b}}$, $\tilde{\rho}$ and $\tilde{p}$ are obtained from eqs.(2.1)–(2.4) as

$$\frac{\partial}{\partial t}\tilde{\boldsymbol{v}}-\frac{1}{4\pi\rho_0}(\boldsymbol{B}_0\cdot\nabla\tilde{\boldsymbol{b}}+\tilde{\boldsymbol{b}}\cdot\nabla\boldsymbol{B}_0)$$
$$+\nabla\cdot\left(\tilde{\boldsymbol{v}}\tilde{\boldsymbol{v}}-\frac{1}{4\pi\rho_0}\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}-\langle\tilde{\boldsymbol{v}}\tilde{\boldsymbol{v}}\rangle+\frac{1}{4\pi\rho_0}\langle\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}\rangle\right)=-\nabla\tilde{p}+\frac{\tilde{\rho}}{\rho_0}\boldsymbol{g}+\nu\nabla^2\tilde{\boldsymbol{v}} \tag{3.1}$$

$$\frac{\partial}{\partial t}\tilde{\boldsymbol{b}}-\boldsymbol{B}_0\cdot\nabla\tilde{\boldsymbol{v}}+\tilde{\boldsymbol{v}}\cdot\nabla\boldsymbol{B}_0+\nabla\cdot\left(\tilde{\boldsymbol{v}}\tilde{\boldsymbol{b}}-\tilde{\boldsymbol{b}}\tilde{\boldsymbol{v}}-\langle\tilde{\boldsymbol{v}}\tilde{\boldsymbol{b}}\rangle+\langle\tilde{\boldsymbol{b}}\tilde{\boldsymbol{v}}\rangle\right)=\frac{c^2\eta}{4\pi}\nabla^2\tilde{\boldsymbol{b}} \tag{3.2}$$

$$\frac{\partial}{\partial t}\tilde{\rho}+\nabla\cdot(\tilde{\rho}\tilde{\boldsymbol{v}}-\langle\tilde{\rho}\tilde{\boldsymbol{v}}\rangle)=-\tilde{\boldsymbol{v}}\cdot\nabla\rho_0. \tag{3.3}$$

$$\nabla^2\tilde{p}=-\nabla\nabla:\left(\tilde{\boldsymbol{v}}\tilde{\boldsymbol{v}}-\frac{1}{4\pi\rho_0}\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}-\langle\tilde{\boldsymbol{v}}\tilde{\boldsymbol{v}}\rangle+\frac{1}{4\pi\rho_0}\langle\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}\rangle\right)+\frac{1}{2\pi\rho_0}\nabla\boldsymbol{B}_0:\nabla\tilde{\boldsymbol{b}}+\frac{\boldsymbol{g}}{\rho_0}\cdot\nabla\tilde{\rho} \tag{3.4}$$

In eqs.(3.1) and (3.4), ρ_0 and $\boldsymbol{g}$ are treated as constants according to the Boussinesq approximation.

Here we apply TSDIA[5] to eqs.(3.1)–(3.4). A random field $f(\boldsymbol{x},t)$ such as $\boldsymbol{v}$, $\boldsymbol{B}$, ρ and p is written as

$$f(\boldsymbol{x},t)=F(\boldsymbol{X},T)+\tilde{f}(\boldsymbol{x},\boldsymbol{X};t,T),\qquad F=\langle f\rangle \tag{3.5}$$

where $\boldsymbol{X}\equiv\delta\boldsymbol{x}$ and $T\equiv\delta t$ represent weak spatial and temporal dependence of the mean field and δ the small expansion parameter. The fluctuation part $\tilde{f}$ is Fourier transformed with respective to $\boldsymbol{x}$ as

$$\tilde{f}(\boldsymbol{x},\boldsymbol{X};t,T)=\int d^3\boldsymbol{k}f(\boldsymbol{k},\boldsymbol{X};t,T)\exp(i\boldsymbol{k}\cdot\boldsymbol{x}) \tag{3.6}$$

where $\boldsymbol{X}$ is treated as a parameter. Then we expand $f(\boldsymbol{k};t)\equiv f(\boldsymbol{k},\boldsymbol{X};t,T)$ in powers of δ as

$$f(\boldsymbol{k};t)=f_0(\boldsymbol{k};t)+\delta f_1(\boldsymbol{k};t)+\delta^2 f_2(\boldsymbol{k};t)+\cdots. \tag{3.7}$$

Applying these procedures to $\boldsymbol{v}$, $\boldsymbol{B}$, ρ and p and substituting eq.(3.6) into eqs.(3.1)–(3.3) yield $O(\delta^0)$ equations

$$(\partial_t+\nu k^2)v_0^\alpha(\boldsymbol{k})+iM_S^{\alpha ab}(\boldsymbol{k})\overset{\Delta}{\sum}[v_0^a(\boldsymbol{p})v_0^b(\boldsymbol{q})-(4\pi\rho_0)^{-1}b_0^a(\boldsymbol{p})b_0^b(\boldsymbol{q})]$$
$$=i(4\pi\rho_0)^{-1}k^a B_0^a b_0^\alpha(\boldsymbol{k})+D^{\alpha a}(\boldsymbol{k})(g^a/\rho_0)\rho_0(\boldsymbol{k}) \tag{3.8}$$

$$(\partial_t + (c^2\eta/4\pi)k^2)b_0^\alpha(\boldsymbol{k}) + iM_A^{\alpha ab}(\boldsymbol{k})\overset{\triangle}{\sum}[v_0^a(\boldsymbol{p})b_0^b(\boldsymbol{q}) - v_0^b(\boldsymbol{p})b_0^a(\boldsymbol{q})] = ik^a B_0^a v_0^\alpha(\boldsymbol{k}) \tag{3.9}$$

$$\partial_t\rho_0(\boldsymbol{k}) + ik^a\overset{\triangle}{\sum} v_0^a(\boldsymbol{p})\rho_0(\boldsymbol{q}) = 0 \tag{3.10}$$

$$p_0(\boldsymbol{k}) = -(k^ak^b/k^2)\overset{\triangle}{\sum}[v_0^a(\boldsymbol{p})v_0^b(\boldsymbol{q}) - (4\pi\rho_0)^{-1}b_0^a(\boldsymbol{p})b_0^b(\boldsymbol{q})] - (g^a/\rho_0)(k^a/k^2)\rho_0(\boldsymbol{k}) \tag{3.11}$$

and $O(\delta^1)$ equations

$$\begin{aligned}(\partial_t + \nu k^2)v_1^\alpha(\boldsymbol{k}) &+ 2iM_S^{\alpha ab}(\boldsymbol{k})\overset{\triangle}{\sum}[v_0^a(\boldsymbol{p})v_1^b(\boldsymbol{q}) - (4\pi\rho_0)^{-1}b_0^a(\boldsymbol{p})b_1^b(\boldsymbol{q})]\\
&= i(4\pi\rho_0)^{-1}k^aB_0^ab_1^\alpha(\boldsymbol{k}) + D^{\alpha a}(\boldsymbol{k})(g^a/\rho_0)\rho_1(\boldsymbol{k}) + 2ik^{-2}M_S^{ab\alpha}(\boldsymbol{k})(g^b/\rho_0)\partial_a\rho_0(\boldsymbol{k})\\
&\quad +(4\pi\rho_0)^{-1}[D_1^{\alpha a}(\boldsymbol{k})(\partial_bB_0^a)b_0^b(\boldsymbol{k}) + B_0^a\partial_ab_0^\alpha(\boldsymbol{k})] - (\partial_T - 2i\nu k^a\partial_a)v_0^\alpha(\boldsymbol{k})\\
&\quad +M_1^{abc\alpha}(\boldsymbol{k})\overset{\triangle}{\sum}\partial_c[v_0^a(\boldsymbol{p})v_0^b(\boldsymbol{q}) - (4\pi\rho_0)^{-1}b_0^a(\boldsymbol{p})b_0^b(\boldsymbol{q})]\end{aligned} \tag{3.12}$$

$$\begin{aligned}(\partial_t + (c^2\eta/4\pi)k^2)b_1^\alpha(\boldsymbol{k}) &+ iM_A^{\alpha ab}(\boldsymbol{k})\overset{\triangle}{\sum}[v_0^a(\boldsymbol{p})b_1^b(\boldsymbol{q}) - v_0^b(\boldsymbol{p})b_1^a(\boldsymbol{q}) + v_1^a(\boldsymbol{p})b_0^b(\boldsymbol{q}) - v_1^b(\boldsymbol{p})b_0^a(\boldsymbol{q})]\\
&= ik^aB_0^av_0^\alpha(\boldsymbol{k}) - (\partial_aB_0^\alpha)v_0^a(\boldsymbol{k}) + B_0^a\partial_av_0^\alpha(\boldsymbol{k})\\
&\quad -(\partial_T - 2i(c^2\eta/4\pi)k^a\partial_a)b_0^\alpha(\boldsymbol{k}) - \overset{\triangle}{\sum}\partial_a[v_0^a(\boldsymbol{p})b_0^\alpha(\boldsymbol{q}) - v_0^\alpha(\boldsymbol{p})b_0^a(\boldsymbol{q})]\end{aligned} \tag{3.13}$$

$$\partial_t\rho_1(\boldsymbol{k}) + ik^a\overset{\triangle}{\sum}[v_0^a(\boldsymbol{p})\rho_1(\boldsymbol{q}) + v_1^a(\boldsymbol{p})\rho_0(\boldsymbol{q})] = -(\partial_a\rho_0)v_0^a(\boldsymbol{k}) - \partial_T\rho_0(\boldsymbol{k}) - \overset{\triangle}{\sum}\partial_a[v_0^a(\boldsymbol{p})\rho_0(\boldsymbol{q})] \tag{3.14}$$

$$\begin{aligned}p_1(\boldsymbol{k}) &= -2(k^ak^b/k^2)\overset{\triangle}{\sum}[v_0^a(\boldsymbol{p})v_1^b(\boldsymbol{q}) - (4\pi\rho_0)^{-1}b_0^a(\boldsymbol{p})b_1^b(\boldsymbol{q})] - (g^a/\rho_0)(k^a/k^2)\rho_1(\boldsymbol{k})\\
&\quad -2i(c^2\eta/4\pi)(k^a/k^2)(\partial_bB_0^a)b_0^a(\boldsymbol{k}) - k^{-2}D_1^{ab}(\boldsymbol{k})(g^a/\rho_0)\partial_b\rho_0(\boldsymbol{k})\\
&\quad +2iM_2^{abc}(\boldsymbol{k})\overset{\triangle}{\sum}\partial_c[v_0^a(\boldsymbol{p})v_0^b(\boldsymbol{q}) - (4\pi\rho_0)^{-1}b_0^a(\boldsymbol{p})b_0^b(\boldsymbol{q})]\end{aligned} \tag{3.15}$$

where $\partial_t \equiv \partial/\partial t$, $\partial_T \equiv \partial/\partial T$, $\partial_\alpha \equiv \partial/\partial X^\alpha$ and

$$\overset{\triangle}{\sum} \equiv \int d^3\boldsymbol{p}\int d^3\boldsymbol{q}\,\delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}) \tag{3.16}$$

$$M_S^{\alpha\beta\gamma}(\boldsymbol{k}) \equiv \frac{1}{2}[k^\beta D^{\alpha\gamma}(\boldsymbol{k}) + k^\gamma D^{\alpha\beta}(\boldsymbol{k})] \tag{3.17}$$

$$M_A^{\alpha\beta\gamma}(\boldsymbol{k}) \equiv \frac{1}{2}[k^\beta\delta^{\alpha\gamma} - k^\gamma\delta^{\alpha\beta}] \tag{3.18}$$

$$D^{\alpha\beta}(\boldsymbol{k}) \equiv \delta^{\alpha\beta} - k^\alpha k^\beta/k^2 \tag{3.19}$$

$$D_1^{\alpha\beta}(\boldsymbol{k}) \equiv \delta^{\alpha\beta} - 2k^\alpha k^\beta/k^2 \tag{3.20}$$

$$M_1^{\alpha\beta\gamma\delta}(\boldsymbol{k}) \equiv -\delta^{\alpha\beta}\delta^{\gamma\delta} + (k^\alpha k^\beta/k^2)\delta^{\gamma\delta} + 2(k^\alpha k^\delta/k^2)\delta^{\beta\gamma} - 2(k^\alpha k^\beta k^\gamma k^\delta/k^4) \tag{3.21}$$

$$M_2^{\alpha\beta\gamma}(\boldsymbol{k}) \equiv (k^\alpha/k^2)\delta^{\beta\gamma} - (k^\alpha k^\beta k^\gamma/k^4). \tag{3.22}$$

It is very difficult to solve eqs.(3.8)–(3.15) directly since the velocity, magnetic field and density fluctuations couple each other. Then in order to treat the resistive interchange turbulence analytically, we introduce another expansion parameter λ into eqs.(3.8)–(3.15) as

$$\eta \to \lambda^{-1}\eta, \qquad g^\alpha \to \lambda g^\alpha. \tag{3.23}$$

These orderings are equivalent to those given in refs.15 and 16. The first one in eq.(3.23) is introduced because we consider the peripheral plasma region where the resistivity is large and the anomalous transport are dominant. We expand each term of eq.(3.7) as

$$\begin{aligned} f_0(\boldsymbol{k};t) &= f_{00}(\boldsymbol{k};t) + \lambda f_{01}(\boldsymbol{k};t) + \lambda^2 f_{02}(\boldsymbol{k};t) + \cdots \\ f_1(\boldsymbol{k};t) &= f_{10}(\boldsymbol{k};t) + \lambda f_{11}(\boldsymbol{k};t) + \lambda^2 f_{12}(\boldsymbol{k};t) + \cdots \end{aligned} \tag{3.24}$$

and substitute them into eqs.(3.8)–(3.15). From (3.9) and (3.13) we have

$$b_{00}^\alpha(\boldsymbol{k};t) = b_{10}^\alpha(\boldsymbol{k};t) = 0 \tag{3.25}$$

$$b_{01}^\alpha(\boldsymbol{k};t) = i(4\pi/c^2\eta)(k^a B_0^a/k^2)v_{00}^\alpha(\boldsymbol{k};t). \tag{3.26}$$

The relation given in eq.(3.26) corresponds to the electrostatic approximation used for the resistive interchange modes.[11–13] The equations for v_{00}^α and ρ_{00} are given from eqs.(3.8) and (3.10) as

$$(\partial_t + \nu k^2)v_{00}^\alpha(\boldsymbol{k}) + iM_S^{\alpha ab}(\boldsymbol{k})\sum^{\Delta} v_{00}^a(\boldsymbol{p})v_{00}^b(\boldsymbol{q}) = 0 \tag{3.27}$$

$$\partial_t\rho_{00}(\boldsymbol{k}) + ik^a\sum^{\Delta} v_{00}^a(\boldsymbol{p})\rho_{00}(\boldsymbol{q}) = 0. \tag{3.28}$$

These equations have the same form as those for the velocity and the passive scalar in homogeneous turbulence, respectively, except the implicit dependence on $\boldsymbol{X}$ and T. From eqs.(3.8), (3.10), (3.12) and (3.14), we obtain

$$v_{01}^\alpha(\boldsymbol{k};t) = -(1/\rho_0 c^2\eta)(k^a B_0^a/k)^2 \int_{-\infty}^{t} dt_1 \hat{F}^{\alpha b}(\boldsymbol{k};t,t_1)v_{00}^b(\boldsymbol{k};t_1)$$

$$+(g^a/\rho_0)\int_{-\infty}^{t} dt_1 \hat{G}^{\alpha a}(\boldsymbol{k};t,t_1)\rho_{00}(\boldsymbol{k};t_1) \tag{3.29}$$

$$\begin{aligned}
\rho_{01}(\boldsymbol{k};t) &= -ik^a \sum^{\Delta}\int_{-\infty}^{t} dt_1 \hat{G}_\rho(\boldsymbol{k};t,t_1)v_{01}^a(\boldsymbol{p};t_1)\rho_{00}(\boldsymbol{q};t_1) \\
&= (ik^a/\rho_0 c^2\eta)(k^b B_0^b/k)^2 \sum^{\Delta}\int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 \hat{G}_\rho(\boldsymbol{k};t,t_1)\hat{F}^{ac}(\boldsymbol{p};t_1,t_2)v_{00}^c(\boldsymbol{p};t_2)\rho_{00}(\boldsymbol{q};t_1) \\
&\quad -i(g^b/\rho_0)k^a \sum^{\Delta}\int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 \hat{G}_\rho(\boldsymbol{k};t,t_1)\hat{G}^{ab}(\boldsymbol{k};t_1,t_2)\rho_{00}(\boldsymbol{p};t_2)\rho_{00}(\boldsymbol{q};t_1) \qquad (3.30) \\
v_{10}^\alpha(\boldsymbol{k};t) &= -\int_{-\infty}^{t} dt_1 \hat{F}^{\alpha a}(\boldsymbol{k};t,t_1)(\partial_T - 2i\nu k^b\partial_b)v_{00}^a(\boldsymbol{k};t_1) \\
&\quad + M_1^{abcd}(\boldsymbol{k})\sum^{\Delta}\int_{-\infty}^{t} dt_1 \hat{F}^{\alpha d}(\boldsymbol{k};t,t_1)\partial_c[v_{00}^a(\boldsymbol{p};t_1)v_{00}^b(\boldsymbol{q};t_1)] \qquad (3.31) \\
\rho_{10}(\boldsymbol{k};t) &= -ik^a \sum^{\Delta}\int_{-\infty}^{t} dt_1 \hat{G}_\rho(\boldsymbol{k};t,t_1)v_{10}^a(\boldsymbol{p};t_1)\rho_{00}(\boldsymbol{q};t_1) \\
&\quad -(\partial_a\rho_0)\int_{-\infty}^{t} dt_1 \hat{G}_\rho(\boldsymbol{k};t,t_1)v_{00}^a(\boldsymbol{k};t_1) - \int_{-\infty}^{t} dt_1 \hat{G}_\rho(\boldsymbol{k};t,t_1)\partial_T\rho_{00}(\boldsymbol{k};t_1) \\
&\quad -\sum^{\Delta}\int_{-\infty}^{t} dt_1 \hat{G}_\rho(\boldsymbol{k};t,t_1)\partial_a[v_{00}^a(\boldsymbol{p};t_1)\rho_{00}(\boldsymbol{q};t_1)] \qquad (3.32)
\end{aligned}$$

where we have used the response functions $\hat{F}^{\alpha\beta}(\boldsymbol{k};t,t')$, $\hat{G}^{\alpha\beta}(\boldsymbol{k};t,t')$ and $\hat{G}_\rho(\boldsymbol{k};t,t')$ defined by

$$(\partial_t + \nu k^2)\hat{F}^{\alpha\beta}(\boldsymbol{k};t,t') + 2iM_S^{\alpha ab}(\boldsymbol{k})\sum^{\Delta} v_{00}^a(\boldsymbol{p};t)\hat{F}^{b\beta}(\boldsymbol{q};t,t') = \delta^{\alpha\beta}\delta(t-t') \tag{3.33}$$

$$(\partial_t + \nu k^2)\hat{G}^{\alpha\beta}(\boldsymbol{k};t,t') + 2iM_S^{\alpha ab}(\boldsymbol{k})\sum^{\Delta} v_{00}^a(\boldsymbol{p};t)\hat{G}^{b\beta}(\boldsymbol{q};t,t') = D^{\alpha\beta}(\boldsymbol{k})\delta(t-t') \tag{3.34}$$

$$\partial_t\hat{G}_\rho(\boldsymbol{k};t,t') + ik^a\sum^{\Delta} v_{00}^a(\boldsymbol{p};t)\hat{G}_\rho(\boldsymbol{q};t,t') = \delta(t-t'). \tag{3.35}$$

Using eqs.(3.29)–(3.32), we can express turbulent transport quantities in terms of the response functions and the fluctuation spectra. For example, the turbulent convective transport of mass $\langle\tilde{\rho}\tilde{v}^\alpha\rangle$ is given by

$$\begin{aligned}
\langle\tilde{\rho}\tilde{v}^\alpha\rangle &= \langle\tilde{\rho}_{00}\tilde{v}_{00}^\alpha\rangle + \delta[\langle\tilde{\rho}_{10}\tilde{v}_{00}^\alpha\rangle + \langle\tilde{\rho}_{00}\tilde{v}_{10}^\alpha\rangle] \\
&\quad + \lambda[\langle\tilde{\rho}_{01}\tilde{v}_{00}^\alpha\rangle + \langle\tilde{\rho}_{00}\tilde{v}_{01}^\alpha\rangle] + O(\delta^2,\delta\lambda,\lambda^2) \qquad (3.36)
\end{aligned}$$

with

$$\langle\tilde{\rho}_{00}\tilde{v}_{00}^\alpha\rangle = \langle\tilde{\rho}_{00}\tilde{v}_{10}^\alpha\rangle = \langle\tilde{\rho}_{01}\tilde{v}_{00}^\alpha\rangle = 0 \tag{3.37}$$

$$\begin{aligned}
\langle \tilde{\rho}_{10} \tilde{v}^{\alpha}_{00} \rangle &= \int d^3\boldsymbol{k} \langle \rho_{10}(\boldsymbol{k}) v^{\alpha}_{00}(-\boldsymbol{k}) \rangle / \delta(0) \\
&= -\partial_a \rho_0 \int d^3\boldsymbol{k} \int_{-\infty}^{t} dt_1 \langle \hat{G}_\rho(\boldsymbol{k}; t, t_1) v^{a}_{00}(\boldsymbol{k}; t_1) v^{\alpha}_{00}(-\boldsymbol{k}; t_1) \rangle / \delta(0) \\
&= -\partial_a \rho_0 \int d^3\boldsymbol{k} \int_{-\infty}^{t} dt_1 G_\rho(\boldsymbol{k}; t, t_1) Q^{a\alpha}(\boldsymbol{k}; t_1, t) \qquad (3.38)
\end{aligned}$$

and

$$\begin{aligned}
\langle \tilde{\rho}_{00} \tilde{v}^{\alpha}_{01} \rangle &= \int d^3\boldsymbol{k} \langle \rho_{00}(\boldsymbol{k}) v^{\alpha}_{01}(-\boldsymbol{k}) \rangle / \delta(0) \\
&= (g^a/\rho_0) \int d^3\boldsymbol{k} \int_{-\infty}^{t} dt_1 \langle \hat{G}^{\alpha a}(\boldsymbol{k}; t, t_1) \rho_{00}(\boldsymbol{k}; t_1) \rho_{00}(-\boldsymbol{k}; t_1) \rangle / \delta(0) \\
&= (g^a/\rho_0) \int d^3\boldsymbol{k} \int_{-\infty}^{t} dt_1 G^{\alpha a}(\boldsymbol{k}; t, t_1) Q_\rho(\boldsymbol{k}; t_1, t). \qquad (3.39)
\end{aligned}$$

Here we defined the average response and correlation functions as

$$G^{\alpha\beta}(\boldsymbol{k}; t, t') \equiv \langle \hat{G}^{\alpha\beta}(\boldsymbol{k}; t, t') \rangle \qquad (3.40)$$

$$F^{\alpha\beta}(\boldsymbol{k}; t, t') \equiv \langle \hat{F}^{\alpha\beta}(\boldsymbol{k}; t, t') \rangle \qquad (3.41)$$

$$G_\rho(\boldsymbol{k}; t, t') \equiv \langle \hat{G}_\rho(\boldsymbol{k}; t, t') \rangle \qquad (3.42)$$

$$Q^{\alpha\beta}(\boldsymbol{k}; t, t') \equiv \langle v^{\alpha}_{00}(\boldsymbol{k}; t) v^{\beta}_{00}(-\boldsymbol{k}; t') \rangle / \delta(0) \qquad (3.43)$$

$$Q_\rho(\boldsymbol{k}; t, t') \equiv \langle \rho_{00}(\boldsymbol{k}; t) \rho_{00}(-\boldsymbol{k}; t') \rangle / \delta(0). \qquad (3.44)$$

In eq.(3.38) and (3.39), we used DIA[8,9] to replace $\langle \hat{G}_\rho v^{\alpha}_{00} v^{\beta}_{00} \rangle / \delta(0)$ and $\langle \hat{G}^{\alpha\beta} \rho_{00} \rho_{00} \rangle / \delta(0)$ by $G_\rho Q^{\alpha\beta}$ and $G^{\alpha\beta} Q_\rho$, respectively.

Putting $\delta \to 1$, $\lambda \to 1$ and $\partial_\alpha \equiv \partial/\partial X^\alpha \to \partial/\partial x^\alpha$, we obtain from eqs.(3.36)–(3.39)

$$\begin{aligned}
\langle \tilde{\rho} \tilde{v}^{\alpha} \rangle &= -\frac{\partial \rho_0}{\partial x^a} \int d^3\boldsymbol{k} \int_{-\infty}^{t} dt_1 G_\rho(\boldsymbol{k}; t, t_1) Q^{a\alpha}(\boldsymbol{k}; t_1, t) \\
&\quad + \frac{g^a}{\rho_0} \int d^3\boldsymbol{k} \int_{-\infty}^{t} dt_1 G^{\alpha a}(\boldsymbol{k}; t, t_1) Q_\rho(\boldsymbol{k}; t_1, t). \qquad (3.45)
\end{aligned}$$

Thus, in this case, the turbulent transport of mass consist of two terms: the first term is a familar one which contains the density gradient while the second one is due to the gravity. We can see that the turbulent diffusion tensor is a functional of the wavenumber spectra of the density response and the velocity fluctuations while the coefficient of the gravity term is a functional of the wavenumber spectra of the velocity response and the density fluctuations. The gravity term in the turbulent transport of mass resulted from the coupling of the density fluctuation to the velocity (or momentum) equation through the gravity. The similar convection term due to the buoyancy force is obtained by TSDIA for the thermally-driven turbulence.[15]

For a passive scalar $\theta = \Theta + \tilde{\theta}$ $(\Theta = \langle\theta\rangle)$ which satisfies

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla\right)\theta = \kappa\nabla^2\theta, \tag{3.46}$$

the turbulent convection of θ is calculated in the same way from TSDIA

$$\langle\tilde{\theta}\tilde{v}^\alpha\rangle = -\frac{\partial\Theta}{\partial x^a}\int d^3\boldsymbol{k}\int_{-\infty}^{t} dt_1 G_\theta(\boldsymbol{k};t,t_1)Q^{a\alpha}(\boldsymbol{k};t_1,t) \tag{3.47}$$

where the average response function $G_\theta(\boldsymbol{k};t,t')$ is defined by

$$G_\theta(\boldsymbol{k};t,t') \equiv \langle\hat{G}_\theta(\boldsymbol{k};t,t')\rangle \tag{3.48}$$

$$(\partial_t + \kappa k^2)\hat{G}_\theta(\boldsymbol{k};t,t') + ik^a\sum^{\Delta} v_{00}^a(\boldsymbol{p};t)\hat{G}_\theta(\boldsymbol{q};t,t') = \delta(t-t'). \tag{3.49}$$

Similarly we obtain the Reynolds stress up to $O(\delta,\lambda)$

$$\begin{aligned} -\langle\tilde{v}^\alpha\tilde{v}^\beta\rangle &= -\int d^3\boldsymbol{k}Q^{\alpha\beta}(\boldsymbol{k};t,t) + \int d^3\boldsymbol{k}\int_{-\infty}^{t} dt_1 F^{\alpha a}(\boldsymbol{k};t,t_1)\partial_T Q^{a\beta}(\boldsymbol{k};t_1,t) \\ &+(\rho_0 c^2\eta)^{-1}B_0^a B_0^b\int d^3\boldsymbol{k}(k^a k^b/k^2)\int_{-\infty}^{t} dt_1[F^{\alpha c}(\boldsymbol{k};t,t_1)Q^{c\beta}(\boldsymbol{k};t_1,t) + (\alpha\leftrightarrow\beta)]. \end{aligned} \tag{3.50}$$

Here we did not show the turbulent transport terms due to the mean velocity shear by assuming that $\boldsymbol{V} \equiv \langle\boldsymbol{v}\rangle = 0$ although we can express those terms in the same way as above when the mean velocity shear exists.

§4. Evaluation of Turbulent Convection

Here our concerns are in the evaluation of turbulent convection given by eqs.(3.45) and (3.47). For that purpose, we need to obtain the expressions for the wavenumber spectra of the fluctuations and the response functions of $O(\delta^0\lambda^0)$ fields. Here it is the simplest way to express them by using the inertial-range theory since $O(\delta^0\lambda^0)$ fields satisfy the same equations as for homogeneous turbulence as seen in eqs.(3.27) and (3.28). Then we assume $O(\delta^0\lambda^0)$ fields to be homogeneous and isotropic so that

$$Q^{\alpha\beta}(\boldsymbol{k};t,t') = D^{\alpha\beta}(\boldsymbol{k})Q(k;t,t') \tag{4.1}$$

$$Q_\rho(\boldsymbol{k};t,t') = Q_\rho(k;t,t') \tag{4.2}$$

$$G^{\alpha\beta}(\boldsymbol{k};t,t') = D^{\alpha\beta}(\boldsymbol{k})G(k;t,t') \tag{4.3}$$

$$F^{\alpha\beta}(\boldsymbol{k};t,t') = \delta^{\alpha\beta}F(k;t,t') \tag{4.4}$$

$$G_\rho(\boldsymbol{k};t,t') = G_\rho(k;t,t'). \tag{4.5}$$

The inertial-range theory[17−19] shows that

$$Q(k;t,t') = \sigma(k)\exp[-\omega(k)|t-t'|] \tag{4.6}$$

$$Q_\rho(k;t,t') = \sigma_\rho(k)\exp[-\omega_\rho(k)|t-t'|] \tag{4.7}$$

$$G(k;t,t') = \exp[-\omega(k)(t-t')]H(t-t') \tag{4.8}$$

$$F(k;t,t') = \exp[-\omega_f(k)(t-t')]H(t-t') \tag{4.9}$$

$$G_\rho(k;t,t') = \exp[-\omega_\rho(k)(t-t')]H(t-t') \tag{4.10}$$

with

$$\sigma(k) = 0.12\epsilon^{2/3}k^{-11/3} \tag{4.11}$$

$$\sigma_\rho(k) = 0.066\epsilon_\rho\epsilon^{-1/3}k^{-11/3} \tag{4.12}$$

$$\omega(k) = 0.42\epsilon^{1/3}k^{2/3} \tag{4.13}$$

$$\omega_f(k) = 0.767\omega(k) \tag{4.14}$$

$$\omega_\rho(k) = 1.60\omega(k) \tag{4.15}$$

where $H(t-t')$ is the unit step function, ϵ and ϵ_ρ the transfer rates of the energy and the density fluctuation variance in the wavenumber space, respectively. As seen from eqs.(3.35) and (3.49), we can put $G_\theta = G_\rho$ in the inertial-convection range where both the equations are identical.

Substituting eqs.(4.1)–(4.15) into eqs.(3.45) and (3.47), we obtain

$$\begin{aligned} \langle \tilde{\rho}\tilde{v}^\alpha \rangle &= -D_\rho \frac{\partial \rho_0}{\partial x^\alpha} + D_g \frac{g^a}{\rho_0} && (4.16) \\ \langle \tilde{\theta}\tilde{v}^\alpha \rangle &= -D_\theta \frac{\partial \Theta}{\partial x^\alpha} && (4.17) \end{aligned}$$

where

$$\begin{aligned} D_\rho &= D_\theta \\ &= \frac{2}{3}\int_{k_m}^{\infty} 4\pi k^2 dk \int_{-\infty}^{t} dt_1 G_\rho(k;t,t_1) Q(k;t_1,t) \\ &= 0.0596\epsilon^{1/3} l^{4/3} && (4.18) \\ D_g &= \frac{2}{3}\int_{k_m}^{\infty} 4\pi k^2 dk \int_{-\infty}^{t} dt_1 G(k;t,t_1) Q_\rho(k;t_1,t). \\ &= 0.0426\epsilon_\rho \epsilon^{-2/3} l^{4/3}. && (4.19) \end{aligned}$$

In eqs.(4.18) and (4.19), the lower limit of the wavenumber integral is given by

$$k_m = 2\pi/l \tag{4.20}$$

where l is the characteristic length of the largest energy-containing eddies.

Using eq.(3.50) and the inertial-range energy spectrum given above, we obtain the turbulent kinetic energy K to the lowest order

$$K \equiv \frac{1}{2}\langle \tilde{v}^a \tilde{v}^a \rangle = \int_{k_m}^{\infty} 4\pi k^2 dk Q(k;t,t) = 0.665\epsilon^{2/3} l^{2/3}. \tag{4.21}$$

From eqs.(4.18), (4.19) and (4.21), we have

$$\begin{aligned} D_\rho &= D_\theta = 0.135 K^2 \epsilon^{-1} && (4.22) \\ D_g &= 0.096 K^2 \epsilon^{-2} \epsilon_\rho. && (4.23) \end{aligned}$$

Thus the turbulent transport coefficients D_ρ, D_θ and D_g are expressed in terms of the turbulent kinetic energy K, the energy transfer rate ϵ and the transfer rate of the density fluctuation variance ϵ_ρ. In the next section, we will consider how K, ϵ and ϵ_ρ are determined.

§5. Equations for K and ϵ

When $\boldsymbol{V}=0$, eq.(2.10) reduces to

$$\begin{aligned}\frac{\partial}{\partial t}E_T &= \langle\tilde{\rho}\tilde{\boldsymbol{v}}\rangle\cdot\left(\frac{\boldsymbol{g}}{\rho_0}\right)-\epsilon-\epsilon_m\\ &+\nabla\cdot\left[-\frac{1}{2}\langle\tilde{v}^2\tilde{\boldsymbol{v}}\rangle-\frac{1}{\rho_0}\langle\tilde{p}\tilde{\boldsymbol{v}}\rangle+\frac{\nu}{2}\nabla\langle\tilde{v}^2\rangle+\left(\frac{c}{4\pi}\right)^2\frac{\eta}{2\rho_0}\nabla\langle\tilde{b}^2\rangle\right]\end{aligned} \tag{5.1}$$

where

$$E_T \equiv \frac{1}{2}\langle\tilde{v}^2\rangle+\frac{1}{8\pi\rho_0}\langle\tilde{b}^2\rangle\simeq\frac{1}{2}\langle\tilde{v}^2\rangle\equiv K \tag{5.2}$$

$$\epsilon \equiv \nu\left\langle\frac{\partial\tilde{v}^a}{\partial x^b}\frac{\partial\tilde{v}^a}{\partial x^b}\right\rangle \tag{5.3}$$

$$\epsilon_m \equiv \left(\frac{c}{4\pi}\right)^2\frac{\eta}{\rho_0}\left\langle\frac{\partial\tilde{b}^a}{\partial x^b}\frac{\partial\tilde{b}^a}{\partial x^b}\right\rangle \tag{5.4}$$

Here we used the ordering in terms of δ and λ in §3 in order to neglect higher-order terms in eq.(2.10) and derive the above equations. Substituting eq.(3.26) into eq.(5.4) and using eq.(3.43), we can express the ohmic dissipation dissipation ϵ_m as

$$\epsilon_m=\alpha\frac{B_0^2}{\rho_0 c^2\eta}K. \tag{5.5}$$

where the parameter α is defined by

$$\alpha=2\frac{\int d^3\boldsymbol{k}(k_{\parallel}/k)^2Q^{aa}(\boldsymbol{k})}{\int d^3\boldsymbol{k}Q^{aa}(\boldsymbol{k})}=\frac{\int d^3\boldsymbol{k}(k_{\parallel}/k)^2Q^{aa}(\boldsymbol{k})}{K}. \tag{5.6}$$

Here $k_{\parallel}\equiv\boldsymbol{k}\cdot\boldsymbol{B}_0/B_0$ denotes the wavenumber along the mean magnetic field $\boldsymbol{B}_0$ and the value of the parameter α becomes $\alpha=2/3$ when the isotropic spectrum eq.(4.1) is used.

In the transport terms of the form $\nabla\cdot[\cdots]$, the viscosity and resistivity terms are ignorably smaller than the others in strong turbulence and the latter terms are approximately expressed for the non-MHD case by Yoshizawa using TSDIA.[6] Here we adopt the following simplest expression used successfully for the non-MHD flow[4]

$$\nabla\cdot\left[-\frac{1}{2}\langle\tilde{v}^2\tilde{\boldsymbol{v}}\rangle-\frac{1}{\rho_0}\langle\tilde{p}\tilde{\boldsymbol{v}}\rangle\right]=\nabla\cdot\left(C_K\frac{K^2}{\epsilon}\nabla K\right) \tag{5.7}$$

where C_K is a non-dimensional numeriscal constant. Equation (5.7) implies that transport of K is also given by the turbulent diffusivity $\sim K^2/\epsilon$.

The derivation of the equation for the energy dissipation rate ϵ is more difficult than for the energy K although Yoshizawa derived it by using TSDIA and some additional assumption.[7] Here we assume that, if the transport terms of K and ϵ are ignored, the rate of change of ϵ is proportional to that of K :

$$\frac{1}{\epsilon}\frac{\partial \epsilon}{\partial t} = \frac{\gamma}{K}\frac{\partial K}{\partial t} \tag{5.8}$$

where γ is a numerical constant. This is similar to the equation for ϵ given by Yoshizawa[7] except that the latter is assumed to hold including the transport terms. Equation (5.8) ensures that K and ϵ can take the stationary and finite values simultaneously in the homogeneous case where the density gradient, the gravity force, K and ϵ are all uniform so that the transport terms of the form $\nabla \cdot [\cdots]$ vanish. In the inhomogeneous case, we add the transport term for ϵ using the turbulent diffusivity $\sim K^2/\epsilon$ in the same way as in eq.(5.7).

The equation for ϵ_ρ is obtained in the similar manner to that for ϵ.[20] However it is complicated since it containes the density fluctuation variance $K_\rho = \langle \tilde{\rho}^2 \rangle$ as an unknown field variable. Thus we simply estimate ϵ_ρ in terms of $\epsilon_T \equiv \epsilon + \epsilon_m$, $\nabla \rho_0$ and $\boldsymbol{g}/\rho_0$ as

$$\epsilon_\rho = \epsilon_T \frac{|\nabla \rho_0|}{|\boldsymbol{g}/\rho_0|} \tag{5.9}$$

which holds exactly for $-\nabla \rho_0 \parallel \boldsymbol{g}$ in the stationary and homogeneous case since we obtain $\epsilon_T = \langle \tilde{\rho} \boldsymbol{v} \rangle \cdot \boldsymbol{g}/\rho_0$ and $\epsilon_\rho = -\langle \tilde{\rho} \boldsymbol{v} \rangle \cdot \nabla \rho_0$ in that case.

Finally, our model equations for K and ϵ are summarized as follows

$$\frac{\partial K}{\partial t} = \langle \tilde{\rho} \tilde{\boldsymbol{v}} \rangle \cdot \left(\frac{\boldsymbol{g}}{\rho_0} \right) - \epsilon - \alpha \frac{B_0^2}{\rho_0 c^2 \eta} K + \nabla \cdot \left(C_K \frac{K^2}{\epsilon} \nabla K \right) \tag{5.10}$$

$$\frac{\partial \epsilon}{\partial t} = \gamma \frac{\epsilon}{K} \langle \tilde{\rho} \tilde{\boldsymbol{v}} \rangle \cdot \left(\frac{\boldsymbol{g}}{\rho_0} \right) - \gamma \frac{\epsilon^2}{K} - \gamma \alpha \frac{B_0^2}{\rho_0 c^2 \eta} \epsilon + \nabla \cdot \left(C_\epsilon \frac{K^2}{\epsilon} \nabla \epsilon \right) \tag{5.11}$$

where $\langle \tilde{\rho} \tilde{\boldsymbol{v}} \rangle$ is given by eqs.(4.16), (4.22), (4.23) and (5.9) for the case of $-\nabla \rho_0 \parallel \boldsymbol{g}$ as

$$\langle \tilde{\rho} \tilde{\boldsymbol{v}} \rangle = -\left(C_\rho \frac{K^2}{\epsilon} + C_g \alpha \frac{B_0^2}{\rho_0 c^2 \eta} \frac{K^3}{\epsilon^2} \right) \nabla \rho_0. \tag{5.12}$$

Solving eqs.(5.10)–(5.12) for K and ϵ, we can obtain the turbulent diffusivity D_θ for the passive scalar θ by

$$D_\theta = C_\theta \frac{K^2}{\epsilon}. \tag{5.13}$$

Numerical constants in eqs.(5.10)–(5.13) are given empirically or theoretically by TSDIA in the case of the isotropic $O(\delta^0\lambda^0)$ turbulent fields as

$$C_\rho = 0.231, \qquad C_g = 0.096, \qquad C_\theta = 0.135, \qquad C_K = 0.09,$$
$$C_\epsilon = 0.07, \qquad \alpha = 0.667, \qquad \gamma = 1.70. \tag{5.14}$$

The equations for K and ϵ have the similar structures to each other in that both of them contain the transport terms, the viscous and Ohmic dissipation terms, and the production terms due to the turbulent density flux conbined with the gravity. We can see that, if $-\nabla\rho_0 \cdot \boldsymbol{g} > 0$, interchange modes are unstable and turbulence production is positive. It is the transport terms that give the nonlocal effects on the turbulent diffusivities, which have been not included in the conventional treatment of the plasma anomalous transport based on the turbulent diffusivities represented by the local plasma parameters.

In order to examine the nonlocal nature of the anomalous transport, let us consider the stationary states. Then we have from eqs.(5.10) and (5.11)

$$P_K - \epsilon_T + \nabla \cdot \left(C_K \frac{K^2}{\epsilon} \nabla K \right) = 0 \tag{5.15}$$

$$\nabla \cdot \left(C_K \frac{K^2}{\epsilon} \nabla K \right) - \gamma^{-1} \frac{K}{\epsilon} \nabla \cdot \left(C_\epsilon \frac{K^2}{\epsilon} \nabla \epsilon \right) = 0 \tag{5.16}$$

where we put $P_K = \langle \tilde{\rho} \tilde{\boldsymbol{v}} \rangle \cdot (\boldsymbol{g}/\rho_0)$. If the balance of local production and dissipation $P_K = \epsilon_T$ is assumed to hold, the characteristic time of turbulence $\tau \equiv K/\epsilon$ is specified as seen from eqs.(5.10) and (5.12) although both K and ϵ are not determined simultaneously by the local balance equation alone. When the characteristic time τ is regarded as a constant, either eq.(5.15) or (5.16) yields

$$- C_K \frac{K^2}{\epsilon} \nabla K \equiv h = \text{const.} \tag{5.17}$$

which represents a constant turbulent energy flux. Then we have the solution

$$K(x) = [K^2(x_0) - (2h/C_K\tau)(x - x_0)]^{1/2} \tag{5.18}$$

where the x-axis is taken in the direction of ∇K. It is seen that the turbulent eneregy K and the anomalous transport coefficient $D \sim K^2/\epsilon = K\tau$ at a point $(x = x)$ are affected by the turbulent energy K and the turbulent energy flux given at a distant point $(x = x_0)$ even if the characteristic time of turbulence τ is locally determined. This simple example suggests the importance of the boundary conditions in our anomalous transport model.

§6. Discussions

We have presented the new model of the anomalous transport in the resistive interchange turbulence, which can treat nonlocal properties of the anomalous transport coefficients not included in its conventional expressions in terms of local plasma parameters only. The turbulent diffusivity is represented by the turbulent kinetic energy K and its dissipation rate ϵ as in eq.(5.13) while K and ϵ are determined nonlocally by solving their transport equations (5.10)–(5.12). Hence, in our model, the boundary conditions at the plasma surface possibly affect the inside transport as seen in §5. Furthermore, the anomalous transport may occur in the linearly stable region. The density profile is required to solve the K-ϵ equations so that it is desirable to combine them with a transport code solving the density and temperature profiles using the anomalous transport coefficients given by K and ϵ. These new transport analyses are under investigation and the results will be reported elsewhere.

In the derivation of the present model, the lowest-order $O(\delta^0\lambda^0)$ turbulent fields are assumed to be isotropic. Therefore the resulting equations appear to contain no magnetic shear effects on the wavenumber spectra. The place where the anisotropy of the wavenumber spectra due to the magnetic shear occurs manifestly is the factor α in the ohmic dissipation term as seen in eq.(5.6). If the magnetic shear is significantly large, the fluctuations with large wavenumbers along the magnetic field are strongly damped so that the value of α becomes much smaller than the value $\frac{2}{3}$ for the case of the isotropic spectrum. This effect should be involved in analyzing resistive interchange turbulence. However, if we include the magnetic shear into the lowest order equations, they become difficult to solve and have no simple spectral solutions such as those for the inertial range of isotropic turbulence. Then the simplest way to treat the magnetic shear effect is to express the factor α in terms of the magnetic shear length L_s and the mixing length $l \sim K^{3/2}/\epsilon$ as

$$\alpha \sim (l/L_s)^2 \sim K^3/(\epsilon L_s)^2$$

which was derived by replacing $k_{\|}/k$ with l/L_s in eq.(5.6). Futher improvement of the model of the local turbulence structure may give the more accurate estimation of α and other numerical constants in the K-ϵ equations or suggest other quantities instead of K and ϵ to characterize the local turbulence spectrum.

As a future work, we are planning the extensions of our transport model to treat the mean velocity shear effects relating to the H-mode plasma and to include the two-fluid effects by formulation based on the Braginskii equations.

Acknowledgements

The authors thank Professor Okamoto for his encouragements of this work.

Refferences

1) P. C. Liewer: Nucl. Fusion **25** (1985) 543.

2) W. A. Houlberg, D. W. Ross, G. Bateman, S. C. Cowley, P. C. Efthimion, W. W. Pfeiffer, G. D. Porter, E. D. Shumaker, L. E. Sugiyama and J. C. Wiley: Phys. Fluids B **2** (1990) 2913.

3) N. Prakash, C. K. Chu and A. Hasegawa: Phys. Fluids **29** (1986) 2426.

4) P. Bradshaw, T. Cebeci and J. H. Whitelaw: *Engineering Calculation Method for Turbulent Flow* (Academic, London, 1981) p.37.

5) A. Yoshizawa: Phys. Fluids **27** (1984) 1377.

6) A. Yoshizawa: J. Phys. Soc. Jpn. **51** (1982) 2326.

7) A. Yoshizawa: Phys. Fluids **30** (1987) 628.

8) R. H. Kraichnan: J. Fluid Mech. **5** (1959) 497.

9) D. C. Leslie: *Developments in the Theory of Turbulence* (Clarendon Press, Oxford, 1973).

10) A. H. Nayfeh: *Perturbation Methods* (Wiley, New York, 1973), p.243.

11) H. Sugama and M. Wakatani: J. Phys. Soc. Jpn. **57** (1988) 2010.

12) B. A. Carreras, L. Garcia and P. H. Diamond: Phys. Fluids **30** (1987) 1388.

13) G. S. Lee and B. A. Carreras: Phys. Fluids B **1** (1989) 119.

14) F. G. Busse: Rep. Prog. Phys. **41** (1978) 1929.

15) A. Yoshizawa: J. Phys. Soc. Jpn. **52** (1983) 1194.

16) Y. Shimomura: J. Phys. Soc. Jpn. **57** (1988) 2365.

17) A. Yoshizawa: J. Phys. Soc. Jpn. **45** (1978) 1734.

18) A. Yoshizawa: J. Phys. Soc. Jpn. **47** (1979) 659.

19) A. Yoshizawa and M. Sakiyama: J. Phys. Soc. Jpn. **48** (1980) 301.

20) A. Yoshizawa: J. Fluid Mech. **195** (1988) 541.

Recent Issues of NIFS Series

NIFS-66 K.Itoh, H. Sanuki, S.-I. Itoh and K. Tani,*Effect of Radial Electric Field on α-Particle Loss in Tokamaks;* Dec. 1990

NIFS-67 K.Sato,and F.Miyawaki,*Effects of a Nonuniform Open Magnetic Field on the Plasma Presheath*; Jan.1991

NIFS-68 K.Itoh and S.-I.Itoh,*On Relation between Local Transport Coefficient and Global Confinement Scaling Law;* Jan. 1991

NIFS-69 T.Kato, K.Masai, T.Fujimoto,F.Koike, E.Källne, E.S.Marmar and J.E.Rice,*He-like Spectra Through Charge Exchange Processes in Tokamak Plasmas;* Jan.1991

NIFS-70 K. Ida, H. Yamada, H. Iguchi, K. Itoh and CHS Group, *Observation of Parallel Viscosity in the CHS Heliotron/Torsatron* ; Jan.1991

NIFS-71 H. Kaneko, *Spectral Analysis of the Heliotron Field with the Toroidal Harmonic Function in a Study of the Structure of Built-in Divertor* ; Jan. 1991

NIFS-72 S. -I. Itoh, H. Sanuki and K. Itoh, *Effect of Electric Field Inhomogeneities on Drift Wave Instabilities and Anomalous Transport* ; Jan. 1991

NIFS-73 Y.Nomura, Yoshi.H.Ichikawa and W.Horton, *Stabilities of Regular Motion in the Relativistic Standard Map*; Feb. 1991

NIFS-74 T.Yamagishi, *Electrostatic Drift Mode in Toroidal Plasma with Minority Energetic Particles,* Feb. 1991

NIFS-75 T.Yamagishi, *Effect of Energetic Particle Distribution on Bounce Resonance Excitation of the Ideal Ballooning Mode*, Feb. 1991

NIFS-76 T.Hayashi, A.Tadei, N.Ohyabu and T.Sato, *Suppression of Magnetic Surface Breading by Simple Extra Coils in Finite Beta Equilibrium of Helical System;* Feb. 1991

NIFS-77 N. Ohyabu, *High Temperature Divertor Plasma Operation*; Feb. 1991

NIFS-78 K.Kusano, T. Tamano and T. Sato, *Simulation Study of Toroidal Phase-Locking Mechanism in Reversed-Field Pinch Plasma*; Feb. 1991

NIFS-79 K. Nagasaki, K. Itoh and S. -I. Itoh, *Model of Divertor Biasing and Control of Scrape-off Layer and Divertor Plasmas*; Feb. 1991

NIFS-80 K. Nagasaki and K. Itoh, *Decay Process of a Magnetic Island by Forced Reconnection*; Mar. 1991

NIFS-81 K. Takahata, N. Yanagi, T. Mito, J. Yamamoto, O.Motojima and LHDDesign Group, K. Nakamoto, S. Mizukami, K. Kitamura, Y. Wachi, H. Shinohara, K. Yamamoto, M. Shibui, T. Uchida and K. Nakayama, *Design and Fabrication of Forced-Flow Coils as R&D Program for Large Helical Device*; Mar. 1991

NIFS-82 T. Aoki and T. Yabe, *Multi-dimensional Cubic Interpolation for ICF Hydrodynamics Simulation*; Apr. 1991

NIFS-83 K. Ida, S.-I. Itoh, K. Itoh, S. Hidekuma, Y. Miura, H. Kawashima, M. Mori, T. Matsuda, N. Suzuki, H. Tamai, T.Yamauchi and JFT-2M Group, *Density Peaking in the JFT-2M Tokamak Plasma with Counter Neutral Beam Injection* ; May 1991

NIFS-84 A. Iiyoshi, *Development of the Stellarator/Heliotron Research*; May 1991

NIFS-85 Y. Okabe, M. Sasao, H. Yamaoka, M. Wada and J. Fujita, *Dependence of Au^- Production upon the Target Work Function in a Plasma-Sputter-Type Negative Ion Source;* May 1991

NIFS-86 N. Nakajima and M. Okamoto, *Geometrical Effects of the Magnetic Field on the Neoclassical Flow, Current and Rotation in General Toroidal Systems*; May 1991

NIFS-87 S. -I. Itoh, K. Itoh, A. Fukuyama, Y. Miura and JFT-2M Group, *ELMy-H mode as Limit Cycle and Chaotic Oscillations in Tokamak Plasmas;* May 1991

NIFS-88 N.Matsunami and K.Kitoh, *High Resolution Spectroscopy of H^+ Energy Loss in Thin Carbon Film;* May 1991

NIFS-89 H. Sugama, N. Nakajima and M.Wakatani, *Nonlinear Behavior of Multiple-Helicity Resistive Interchange Modes near Marginally Stable States;* May 1991

NIFS-90 H. Hojo and T.Hatori, *Radial Transport Induced by Rotating RF Fields and Breakdown of Intrinsic Ambipolarity in a Magnetic Mirror*; May 1991

NIFS-91 M. Tanaka, S. Murakami, H. Takamaru and T.Sato, *Macroscale Implicit, Electromagnetic Particle Simulation of Inhomogeneous and Magnetized Plasmas in Multi-Dimensions;* May 1991

NIFS-92 S. - I. Itoh, *H-mode Physics, -Experimental Observations and Model Theories-, Lecture Notes, Spring College on Plasma Physics, May 27 - June 21 1991 at International Centre for Theoretical Physics (IAEA UNESCO) Trieste, Italy ;* Jun. 1991

NIFS-93 Y. Miura, K. Itoh, S. - I. Itoh, T. Takizuka, H. Tamai, T. Matsuda, N. Suzuki, M. Mori, H. Maeda and O. Kardaun, *Geometric Dependence of the Scaling Law on the Energy Confinement Time in H-mode Discharges;* Jun. 1991

NIFS-94 H. Sanuki, K. Itoh, K. Ida and S. - I. Itoh, *On Radial Electric Field Structure in CHS Torsatron / Heliotron;* Jun. 1991

NIFS-95 K. Itoh, H. Sanuki and S. - I. Itoh, *Influence of Fast Ion Loss on Radial Electric Field in Wendelstein VII-A Stellarator;* Jun. 1991

NIFS-96 S. - I. Itoh, K. Itoh, A. Fukuyama, *ELMy-H mode as Limit Cycle and Chaotic Oscillations in Tokamak Plasmas;* Jun. 1991

NIFS-97 K. Itoh, S. - I. Itoh, H. Sanuki, A. Fukuyama, *An H-mode-Like Bifurcation in Core Plasma of Stellarators;* Jun. 1991

NIFS-98 H. Hojo, T. Watanabe, M. Inutake, M. Ichimura and S. Miyoshi, *Axial Pressure Profile Effects on Flute Interchange Stability in the Tandem Mirror GAMMA 10;* Jun. 1991

NIFS-99 A. Usadi, A. Kageyama, K. Watanabe and T. Sato, *A Global Simulation of the Magnetosphere with a Long Tail : Southward and Northward IMF;* Jun. 1991

NIFS-100 H. Hojo, T. Ogawa and M. Kono, *Fluid Description of Ponderomotive Force Compatible with the Kinetic One in a Warm Plasma* ; July 1991

NIFS-101 H. Momota, A. Ishida, Y. Kohzaki, G. H. Miley, S. Ohi, M. Ohnishi K. Yoshikawa, K. Sato, L. C. Steinhauer, Y. Tomita and M. Tuszewski *Conceptual Design of D-^{3}He FRC Reactor "ARTEMIS"* ; July 1991

NIFS-102 N. Nakajima and M. Okamoto, *Rotations of Bulk Ions and Impurities in Non-Axisymmetric Toroidal Systems* ; July 1991

NIFS-103 A. J. Lichtenberg, K. Itoh, S. - I. Itoh and A. Fukuyama, *The Role of Stochasticity in Sawtooth Oscillation* ; Aug. 1991

NIFS-104 K. Yamazaki and T. Amano, *Plasma Transport Simulation Modeling for Helical Confinement Systems;* Aug. 1991

NIFS-105 T. Sato, T. Hayashi, K. Watanabe, R. Horiuchi, M. Tanaka, N. Sawairi and K. Kusano, *Role of Compressibility on Driven Magnetic Reconnection ;* Aug. 1991

NIFS-106 Qian Wen - Jia, Duan Yun - Bo, Wang Rong - Long and H. Narumi, *Electron Impact Excitation of Positive Ions - Partial Wave Approach in Coulomb - Eikonal Approximation* ; Sep. 1991

NIFS-107 S. Murakami and T. Sato, *Macroscale Particle Simulation of Externally Driven Magnetic Reconnection;* Sep. 1991

NIFS-108 Y. Ogawa, T. Amano, N. Nakajima, Y. Ohyabu, K. Yamazaki, S. P. Hirshman, W. I. van Rij and K. C. Shaing, *Neoclassical Transport Analysis in the Banana Regime on Large Helical Device (LHD) with the DKES Code;* Sep. 1991

NIFS-109 Y. Kondoh, *Thought Analysis on Relaxation and General Principle to Find Relaxed State;* Sep. 1991

NIFS-110 H. Yamada, K. Ida, H. Iguchi, K. Hanatani, S. Morita, O. Kaneko, H. C. Howe, S. P. Hirshman, D. K. Lee, H. Arimoto, M. Hosokawa,

H. Idei, S. Kubo, K. Matsuoka, K. Nishimura, S. Okamura, Y. Takeiri, Y. Takita and C. Takahashi, *Shafranov Shift in Low-Aspect-Ratio Heliotron / Torsatron CHS* ; Sep 1991

NIFS-111 R. Horiuchi, M. Uchida and T. Sato, *Simulation Study of Stepwise Relaxation in a Spheromak Plasma ;* Oct. 1991

NIFS-112 M. Sasao, Y. Okabe, A. Fujisawa, H. Iguchi, J. Fujita, H. Yamaoka and M. Wada, *Development of Negative Heavy Ion Sources for Plasma Potential Measurement* ; Oct. 1991

NIFS-113 S. Kawata and H. Nakashima, *Tritium Content of a DT Pellet in Inertial Confinement Fusion* ; Oct. 1991

NIFS-114 M. Okamoto, N. Nakajima and H. Sugama, *Plasma Parameter Estimations for the Large Helical Device Based on the Gyro-Reduced Bohm Scaling* ; Oct. 1991

NIFS-115 Y. Okabe, *Study of* Au^- *Production in a Plasma-Sputter Type Negative Ion Source* ; Oct. 1991

NIFS-116 M. Sakamoto, K. N. Sato, Y. Ogawa, K. Kawahata, S. Hirokura, S. Okajima, K. Adati, Y. Hamada, S. Hidekuma, K. Ida, Y. Kawasumi, M. Kojima, K. Masai, S. Morita, H. Takahashi, Y. Taniguchi, K. Toi and T. Tsuzuki, *Fast Cooling Phenomena with Ice Pellet Injection in the JIPP T-IIU Tokamak;* Oct. 1991

NIFS-117 K. Itoh, H. Sanuki and S. -I. Itoh, *Fast Ion Loss and Radial Electric Field in Wendelstein VII-A Stellarator*; Oct. 1991

NIFS-118 Y. Kondoh and Y. Hosaka, *Kernel Optimum Nearly-analytical Discretization (KOND) Method Applied to Parabolic Equations <<KOND-P Scheme>>;* Nov. 1991

NIFS-119 T. Yabe and T. Ishikawa, *Two- and Three-Dimensional Simulation Code for Radiation-Hydrodynamics in ICF*; Nov. 1991

NIFS-120 S. Kawata, M. Shiromoto and T. Teramoto, *Density-Carrying Particle Method for Fluid* ; Nov. 1991

NIFS-121 T. Ishikawa, P. Y. Wang, K. Wakui and T. Yabe, *A Method for the High-speed Generation of Random Numbers with Arbitrary Distributions;* Nov. 1991

NIFS-122 K. Yamazaki, H. Kaneko, Y. Taniguchi, O. Motojima and LHD Design Group, *Status of LHD Control System Design* ; Dec. 1991

NIFS-123 Y. Kondoh, *Relaxed State of Energy in Incompressible Fluid and Incompressible MHD Fluid* ; Dec. 1991

NIFS-124 K. Ida, S. Hidekuma, M. Kojima, Y. Miura, S. Tsuji, K. Hoshino, M. Mori, N. Suzuki, T. Yamauchi and JFT-2M Group, *Edge Poloidal Rotation Profiles of H-Mode Plasmas in the JFT-2M Tokamak* ; Dec. 1991