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REFINED THEORY OF DIAMAGNETIC EFFECT IN STELLARATORS

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ABSTRACT

The difference $\delta\Phi$ between toroidal magnetic flux Φ_p through the plasma cross-section and the same flux Φ_v of a vacuum magnetic field is calculated analytically for "conventional" stellarators with planar circular axis. It has been done without limitations of aspect ratio, shape and position of a plasma. The results obtained show weak dependence of $\delta\Phi/\Phi_p$ on the geometry of equilibrium configuration. It proves that diamagnetic measurements can be considered as a reliable basis for the direct evaluation of stored plasma energy at various experimental conditions.

Keywords : stellarator, diamagnetic measurements, plasma equilibrium

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1. INTRODUCTION

Diamagnetic measurements are important part of plasma diagnostics in fusion devices. The measurements themselves and apparatus used are rather simple. The main physical problem is how to interpret data obtained.

Existing theory predicts linear dependence of measured diamagnetic signal $\delta\Phi$ in stellarators on the average $\bar{\beta}$ (ratio of plasma and magnetic pressures) with coefficient $-\Phi_p/2$ [1-6]. Here, Φ_p is the toroidal magnetic flux through the plasma column, $\delta\Phi$ is the difference between Φ_p and the same flux of a vacuum magnetic field. This proportionality coefficient has been calculated in [1-6] for a straight stellarator with nonshifted circular averaged magnetic surfaces. Such a model seems to be inadequate for shifted or elongated configurations which can be produced in helical systems (stellarators) CHS [7] and ATF [8] with relatively low aspect ratios $A = 5 \div 7$. It is necessary, therefore, to extend the theory, evaluating $\delta\Phi$ in more realistic geometry than in Refs [1-6].

This problem seems to be natural from theoretical point of view, but it is also of a practical importance. Configurational effects on plasma confinement in stellarators are now under extensive studies. Experiments on helical systems Heliotron E [9] and CHS [10] have shown a strong influence of the geometry of magnetic configuration on the main plasma parameters. One of them is the stored plasma energy which is evaluated usually from diamagnetic measurements.

In this paper we calculate analytically $\delta\Phi$ for "conventional" stellarators with planar circular geometrical axis. Our aim is to find a relationship for $\delta\Phi$ through $\bar{\beta}$ for a plasma with arbitrary aspect ratio, shape and position of magnetic surfaces.

In Sec.2 general expression for $\delta\Phi$ is derived. It is used to obtain more clear dependence of $\delta\Phi$ on $\bar{\beta}$ and plasma geometry in Sec.3. In Sec.4 we use simplified analytical model to estimate contribution to $\delta\Phi$ due to magnetic axis shift. Summary and discussion are presented in Sec.5. In the Appendix some formulae useful for calculating equilibrium quantities in stellarator approximation are given.

2. GENERAL EXPRESSION FOR $\delta\Phi$

We are going to calculate a difference between toroidal fluxes of equilibrium ($\mathbf{B}$) and vacuum ($\mathbf{B}_V$) magnetic fields through the transverse cross-section $S_\perp$ of a plasma column:

$$\delta\Phi \equiv \Phi_p - \Phi_V = \int_{S_\perp} (\mathbf{B} - \mathbf{B}_V) d\mathbf{S}_\perp . \tag{1}$$

With the help of identity

$$\int_{V_p} \mathbf{q}\nabla\zeta d\tau = \int_{V_p} \mathrm{div}\zeta\mathbf{q} d\tau = 2\pi \int_{S_\perp} \mathbf{q} d\mathbf{S}_\perp + \int_{S_p} \zeta\mathbf{q} d\mathbf{S}_p \tag{2}$$

which is valid for any divergence-free vector $\mathbf{q}$, we can rewrite Eq. (1) in the form

$$\delta\Phi = \frac{1}{2\pi} \int_{V_p} (\mathbf{B} - \mathbf{B}_V)\nabla\zeta d\tau - \frac{1}{2\pi} \int_{S_p} \zeta(\mathbf{B} - \mathbf{B}_V) d\mathbf{S}_p . \tag{3}$$

Here V_p is the plasma volume, S_p is the plasma boundary surface, ζ is the toroidal angle varying from 0 to 2π.

Contribution from the helical fields to $\delta\Phi$ is much smaller than that from the main toroidal field and can be omitted in Eqs (1), (3). Formally, it corresponds to the neglecting of pressure-induced changes of the helical field.

Next, we can drop out the last term in Eq.(3). It can be done by several reasons. First, it contains the difference between averaged toroidal field and the same vacuum field at the plasma boundary which should be small enough. Second, toroidal component n_t of the unit vector $\mathbf{n}$ normal to the surface S_p is $n_t = -\mathbf{n}\mathbf{B}_{pol} / B_t$, where $\mathbf{B}_{pol}$ is the poloidal field. So, we have an additional small parameter B_{pol} / B_t in the last integral in Eq.(3):

$$\delta B_t n_t = -\delta B_t \frac{\mathbf{B}_{pol}}{B_t} \mathbf{n}. \tag{4}$$

And, third, n_t oscillates over ζ, which should result in a further reduction of the integral. Note, that for axisymmetric systems the last term of Eq.(3) is identically zero.

In conventional stellarators with planar circular geometric axis the axisymmetric part ($\overline{\mathbf{B}}$) of a magnetic field can be represented as [11]

$$\overline{\mathbf{B}} = \frac{1}{2\pi}\left[\nabla(\psi - \psi_V)\nabla\zeta\right] + \frac{F}{2\pi}\nabla\zeta. \tag{5}$$

It allows to reduce, finally, general expression for $\delta\Phi$ to the

$$\delta\Phi = \frac{1}{4\pi^2}\int_{V_p}\frac{F - F_b}{r^2}d\tau \tag{6}$$

where F_b is the boundary (vacuum) value of F, which corresponds to the vacuum toroidal field, r is the radius: $|\nabla\zeta|^2 = 1/r^2$.

Similar value has been calculated in the above mentioned articles [1-6] for a straight stellarator. It means that r was considered as a constant in Eq.(6). This approximation, as can be seen from Eq.(6), should not lead to significant error. Nevertheless, it can be noticeable for compact devices such as CHS. So, we shall take into account the toroidal nature of longitudinal field, which is also important to eliminate limitations in the previous theories.

Shafranov shift or shift of magnetic surfaces produced by an external vertical field enters into the Eq.(6) through the F function. It is clear, that $\delta\Phi$ depends not only on the value of $|F - F_b|$, but also on the position of its maximum. In outward shifted configurations it is divided by larger value of r (another r is cancelled with that in $d\tau$). As a result, in this case $\delta\Phi$ dependence on $\overline{\beta}$ should be weaker than that for nonshifted or inward shifted configurations. We can expect it at high $\overline{\beta}$'s in low-aspect-ratio systems.

For calculations we shall use another form of Eq.(6):

$$\delta\Phi = \frac{1}{4\pi^2}\int_{V_p}\left(F - F_b\right)\left\langle\frac{1}{r^2}\right\rangle dV = -\frac{1}{4\pi^2}\int_{V_p}\frac{dF}{dV}\int_{V}\frac{d\tau}{r^2}dV. \tag{7}$$

Here brackets <...> denote averaging over the layer between adjacent magnetic surfaces,

$$\langle f \rangle = \frac{d}{dV}\int_V f d\tau, \tag{8}$$

V is the volume bounded by a magnetic surface. To reduce Eq.(6) to Eq.(7), one should remember that $F - F_b$ is a constant on any magnetic surface. Kruskal-Kulsrud equation

$$p'V' = -(\mu J' + F')\Phi' \tag{9}$$

(the direct consequence of equilibrium equation $\nabla p = [\mathbf{jB}]$) allows us to express $F'(V)$ in Eq.(7) via plasma pressure p, longitudinal current J and rotational transform μ. With its help we get

$$\delta\Phi = \frac{1}{4\pi^2}\int_{V_p}\left(\frac{dp}{d\Phi} + \mu\frac{dJ}{dV}\right)\int_V \frac{d\tau}{r^2}dV. \tag{10}$$

Primes in Eq.(9) denote the derivative with respect to arbitrary "label" of magnetic surfaces, Φ is the toroidal flux enclosed by a magnetic surface.

In the following we consider a currentless plasma with $J = 0$.

3. $\delta\Phi$ IN CURRENTLESS STELLARATORS

For currentless plasma Eq.(10) can be written as

$$\delta\Phi = -\frac{\bar{p}V_p}{F_b} + \int_{V_p}\frac{dp}{d\Phi}\left[\frac{1}{4\pi^2}\int_V\frac{d\tau}{r^2} - \frac{V}{F_b}\frac{d\Phi}{dV}\right]dV, \tag{11}$$

where $\bar{p}$ is the volume-averaged pressure:

$$\bar{p} = \frac{1}{V_p}\int_{V_p} p dV. \tag{12}$$

The second term in Eq.(11) is much smaller than the first one, $\bar{p}V_p / F_b$, because terms in square brackets almost cancel each other. To evaluate it analytically, we need an expression for $\Phi'(V)$.

For any toroidal system with nested flux surfaces this function is given by

$$\frac{d\Phi}{dV} = \frac{1}{2\pi}\langle \mathbf{B}\nabla\zeta \rangle. \tag{13}$$

In a stellarator

$$\mathbf{B} = \overline{\mathbf{B}} + \tilde{\mathbf{B}} \tag{14}$$

with axisymmetric component of a magnetic field given by Eq.(5). So we have

$$\frac{d\Phi}{dV} = \frac{F}{4\pi^2}\left\langle \frac{1}{r^2} \right\rangle + \frac{1}{2\pi}\langle \tilde{\mathbf{B}}\nabla\zeta \rangle. \tag{15}$$

The contribution of helical field, $\tilde{\mathbf{B}}$, to $\Phi'(V)$ can be easily calculated with the help of widely used stellarator expansion. It was originally proposed by Greene and Johnson [12] for large-aspect-ratio stellarators (see also Ref. [13]), but its extension for low-aspect-ratio systems is possible [11, 14]. Simple calculations (see Appendix) result in

$$\left\langle \tilde{\mathbf{B}}\nabla\zeta \right\rangle = -\frac{B_0}{R}\left\langle \Omega^0 \right\rangle, \tag{16}$$

where B_0 is the toroidal field on geometrical axis of a stellarator, R is the radius of this axis,

$$\Omega^0 = \frac{\left\langle \tilde{\mathbf{B}}^2 \right\rangle_\zeta}{B_0^2}, \tag{17}$$

$\langle ... \rangle_\zeta$ stands for averaging over the toroidal angle ζ. Finally, Eq.(15) leads to

$$\frac{d\Phi}{dV} = \frac{F}{4\pi^2}\left\langle \frac{1}{r^2} - \frac{\Omega^0}{R^2} \right\rangle, \tag{18}$$

where the last term is much smaller than the first one, but its contribution to the varying part of $\Phi'(V)$ is important.

We can neglect the difference between F and F_b when we substitute expression (18) into Eq.(11), to obtain

$$\delta\Phi = -\frac{\bar{p}V_p}{F_b} - \frac{1}{4\pi^2}\int_{V_p}\frac{dp}{d\Phi}V\left[V\frac{d}{dV}\overline{\frac{1}{r^2}} - \frac{\langle\Omega^0\rangle}{R^2}\right]dV. \tag{19}$$

Here

$$\overline{\frac{1}{r^2}} = \frac{1}{V}\int_V \frac{d\tau}{r^2}. \tag{20}$$

In a more compact form

$$\delta\Phi = -\frac{\bar{p}V_p}{F_b}(1 - \delta_S + \delta_H), \tag{21}$$

where

$$\delta_S \equiv -\frac{F_b}{4\pi^2}\frac{1}{\bar{p}V_p}\int_{V_p}\frac{dp}{d\Phi}V^2\frac{d}{dV}\overline{\frac{1}{r^2}}dV, \tag{22}$$

$$\delta_H \equiv -\frac{F_b}{4\pi^2}\frac{1}{\bar{p}V_p}\int_{V_p}\frac{dp}{d\Phi}V\frac{\langle\Omega^0\rangle}{R^2}dV. \tag{23}$$

Both terms, δ_S and δ_H, are much smaller than unity. The first one, δ_S, has a clear geometrical nature. It is directly related with relative shift of magnetic surfaces. It vanishes for nonshifted configurations with $\overline{r^{-2}}$=const. This term is positive, when magnetic axis is shifted outward. It can be due to Shafranov shift at finite β's, or due to external vertical field which produces nonuniform shift of magnetic surfaces in stellarators with a shear. For inward shifted configurations δ_S should be negative.

The second value, δ_H, appeared in $\delta\Phi$ because we retained $\langle\Omega^0\rangle$ in Eq.(18). At the same time we omitted the term with $\tilde{\mathbf{B}}-\tilde{\mathbf{B}}_V$ in Eq.(3), which, probably, could give a contribution to $\delta\Phi$ of the same order as δ_H. In other words, accuracy of our calculations is not sufficient enough to claim that δ_H in Eq.(21) should be given exactly by Eq.(23). Or, to say so, we should add to the right side of Eq.(23) another small term due to β-induced helical fields. It will not change the structure of Eq.(21), only small value δ_H can be, probably, changed.

In any case, our calculations show that for a currentless plasma of any shape in conventional stellarators with arbitrary aspect ratio the $\delta\Phi$ value is related with $\overline{\beta}$ through Eq.(21), where δ_S and δ_H are much smaller than unity. In the large-aspect-ratio limit, for a plasma with nonshifted circular averaged magnetic surfaces it is reduced to the previously obtained formula [1-6]

$$\frac{\delta\Phi}{\Phi_p} = -\frac{\overline{\beta}}{2}, \tag{24}$$

where $\Phi_p = \pi b^2 B_0$ is the flux of toroidal field through the plasma cross-section, b is its average radius, and $\overline{\beta} = 2\overline{p}/B_0^2$. Eq.(21) shows that the same equality (with $\Phi_p = B_0 S_\perp$) holds for "noncircular" plasma also, because $V_p / \Phi_p \approx 2\pi R / B_0$, $F \approx 2\pi R B_0$.

We shall concentrate, next, on the dependence of $\delta\Phi$ on the shift of magnetic surfaces. We should expect, as was said above, smaller increase of $\delta\Phi$ with β-raise in configurations with more outward shifted magnetic axis. It follows, that Shafranov shift should lead to weak nonlinearity in $\delta\Phi$ dependence on $\overline{\beta}$ at high enough β's. To investigate such effects, we shall use simplified analytical model.

4. ANALYTICAL TREATMENT OF $\delta\Phi$ DEPENDENCE ON MAGNETIC AXIS SHIFT

For stellarators with circular shifted averaged magnetic surfaces (model widely used in analytical studies) we get

$$V = 2\pi^2 a^2 R_c, \qquad \int_V \frac{d\tau}{r^2} = \frac{V}{R_c^2}, \tag{25}$$

where a is the minor radius of the cross-section of averaged magnetic surface, $R_c = R + \Delta$ is its major radius (position of its center), Δ is the shift of this magnetic surface a=const relative to the geometrical center of the initial nonshifted configuration. In this case

$$\frac{1}{4\pi^2}\frac{V^2}{V_p}\frac{d}{dV}\overline{\frac{1}{r^2}} = -\frac{1}{V'}\frac{a^4\Delta'}{b^2 R R_c}, \tag{26}$$

where b is the minor radius of a plasma. We calculate δ_S in the lowest approximation, omitting higher-order toroidal corrections. They are not important because the term δ_S in Eq.(21) is small itself. So, we can disregard the difference between R and R_c in Eq.(26) and substitute $\Phi'(a)=2\pi a B_0$ in Eq.(22) to obtain

$$\delta_S = \frac{1}{Rb^2}\int_0^b \frac{p'}{\bar{p}} a^3 \Delta' da. \tag{27}$$

It follows from here that

$$\delta_S = C\frac{b}{R}\frac{\Delta_{ax}}{b}, \tag{28}$$

where Δ_{ax} is the shift of magnetic axis and C is the factor of order of unity depending on pressure distribution and $\Delta(a)$ profile:

$$C = -\int_0^1 \frac{p}{\bar{p}}\frac{d}{dx}\left(x^3\frac{\Delta'(x)}{\Delta_{ax}}\right)dx. \tag{29}$$

Here x = a/b is the normalized dimensionless minor radius. For peacked pressure profiles the value of C should be smaller than that for the flat ones. For

$$\Delta = \Delta_{ax}\left(1-x^2\right), \qquad p = p_0\left(1-x^2\right) \tag{30}$$

we have $\bar{p} = p_0/2$ and C = 4/3. For

$$\Delta = \Delta_{ax}\left(1-\frac{3}{2}x^2+\frac{x^4}{2}\right), \qquad p = p_0\left(1-x^2\right)^2 \tag{31}$$

we receive C = 0.9.

Calculating C value analytically, one should remember that $\Delta(a)$ depends on p(a) and boundary (or initial) conditions. Both cases mentioned here represent self-consistent solutions of equilibrium problem for shearless stellarators only with nonshifted magnetic surfaces at $\bar{\beta}=0$. In general, they should be considered as not more than simple model used for estimates. Such estimates show, for example, that for the CHS torsatron contribution to $\delta\Phi$ due to δ_S term can be in some regimes of the order of 10%.

We can evaluate now $\delta\Phi$ dependence on $\overline{\beta}$ in shearless stellarators with account of Shafranov shift. It can be done if we put

$$\Delta_{ax} = \frac{\overline{\beta}}{2\beta_{eq}^{0}} b \qquad (32)$$

in Eq.(30), where

$$\beta_{eq}^{0} \equiv \mu^{2} \frac{b}{R}, \qquad (33)$$

μ=const is the rotational transform. In this case we obtain

$$\frac{\delta\Phi}{\pi b^{2} B_{0}} = -\frac{\overline{\beta}}{2}\left(1 - \frac{2}{3}\frac{b}{R}\frac{\overline{\beta}}{\beta_{eq}^{0}}\right) \qquad (34)$$

with $\overline{\beta}=2\overline{p}/B_0^2$. Even at $\overline{\beta}$ close to equilibrium limit the last term in Eq.(34) could not be larger than 0.7b/R. It means that simple formula (21) with $\delta_S=0$ and $\delta_H=0$ can be considered as a reliable basis for interpretation of diamagnetic measurements at any $\overline{\beta}$. Nonlinear dependence of $\delta\Phi$ on $\overline{\beta}$ can be seen in numerical calculations. Our analysis explains this phenomena.

In elongated configurations Shafranov shift is suppressed to some extent. In this case toroidal corrections in Eq.(21) can be also of no much practical importance. We are not able to discuss here in more details the contribution to $\delta\Phi$ due to helical field. It is clear, nevertheless, that it should be small. For δ_H given by Eq.(23) we have

$$\delta_H \propto \frac{1}{2}\frac{b}{R}\mu_b \frac{mb}{R} \quad , \qquad (35)$$

where μ_b is the rotational transform at the plasma edge, m is the number of periods of helical field. Function $\langle\Omega^{0}\rangle$ in Eq.(23) depends on the radius measured from the geometrical axis of "nonshifted" vacuum configuration. As a result, δ_H should also depend on Δ_{ax}, which can be seen in strongly shifted configurations. It means, that in such cases small deviation of $\delta\Phi/\Phi_p$ from that given by Eq.(24) can be somewhat different than obtained from δ_S only.

5. SUMMARY AND DISCUSSION

The difference $\delta\Phi$ between toroidal magnetic flux Φ_p through the plasma column and similar flux Φ_V of a vacuum magnetic field has been calculated analytically for a conventional stellarator. It has been done without limitations on aspect ratio, shape and position of a plasma. We have shown that the ratio $\delta\Phi/\Phi_p$ is not very sensitive to the geometry of configuration. We confirmed by this that previous result, Eq.(24), obtained in Refs [1-6] for straight stellarators with nonshifted circular averaged magnetic surfaces, should not lead to serious mistakes when applied to real devices at different experimental conditions. At the same time we found that the ratio $\delta\Phi/\Phi_p$ depends on the relative shift of magnetic surfaces. This effect should be seen in numerical calculations and, probably, can be of some importance for shifted configurations in stellarators with a shear. In such systems nonuniform shift of magnetic surfaces appears when vertical field is applied. As a result, for outward shift we should have weaker dependence of $\delta\Phi/\Phi_p$ on $\bar{\beta}$ than that given by Eq.(24), and more stronger one for inward shift. This dependence can be slightly nonlinear at high β's due to Shafranov shift.

Analytic solutions give physical insight of the problem and reveal main dependencies. Several questions should be addressed to numerical simulations for complete clarifying the problem. First, on the plasma boundary $\mathbf{Bn}=0$, but vacuum field $\mathbf{B}_V$ can have nonvanishing normal component. It means that $\delta\Phi$ can be slightly different in different ζ=const cross-sections. In general, this effect should be small and can be disregarded. Nevertheless, the question remains to be opened: in what cases (maybe, exotic ones) this effect can be noticeable? Second, $\tilde{\mathbf{B}} - \tilde{\mathbf{B}}_V$ is small, but, probably, in some cases its contribution to $\delta\Phi$ can be of the same order than that given by δ_H in Eq.(21). It can be easily checked numerically. Third, in experiments not exactly $\delta\Phi$ value is measured by a diamagnetic loop, but the change of magnetic flux through the surface enclosed by the loop. At finite β the helical field in the gap between plasma and a loop is also not the same as vacuum field at β=0. So, one can wonder whether it is important or not. The final answer should be given by the precise numerical analysis. All questions listed here are kind of theoretical ones, with no much practical importance. Because the main, largest part (if not the whole) of measured diamagnetic signal is given by $\delta\Phi$.

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APPENDIX

In stellarators flux function Ψ subject to the equation

$$\mathbf{B}\nabla\Psi = 0 \tag{A.1}$$

can be represented as

$$\Psi = \psi + \tilde{\psi}, \tag{A.2}$$

where ψ is the axisymmetric part of Ψ and $\tilde{\psi}$ is the helical one. Eq.(A.1) is then naturally divided on two parts:

$$\overline{\mathbf{B}}\nabla\psi = -\left\langle\tilde{\mathbf{B}}\nabla\tilde{\psi}\right\rangle_\zeta, \tag{A.3}$$

$$\overline{\mathbf{B}}\nabla\tilde{\psi} = -\tilde{\mathbf{B}}\nabla\psi - \left(\tilde{\mathbf{B}}\nabla\tilde{\psi} - \left\langle\tilde{\mathbf{B}}\nabla\tilde{\psi}\right\rangle_\zeta\right). \tag{A.4}$$

Here and in what follows

$$\bar{f} \equiv \langle f\rangle_\zeta = \frac{1}{2\pi}\int_0^{2\pi} f d\zeta, \quad \tilde{f} = f - \langle f\rangle_\zeta, \quad \hat{\tilde{f}} = \int \tilde{f} d\zeta. \tag{A.5}$$

Usually $\tilde{B}/B_t$ and rotational transform over one field period are small. It allows to drop out last term in Eq.(A.4) and to replace the operator $\overline{\mathbf{B}}\nabla$ in this equation on $B_t \mathbf{e}_\zeta \nabla$, $\mathbf{e}_\zeta$ being a unit vector along $\nabla\zeta$. After that one gets

$$\tilde{\psi} = -\frac{r}{B_t}\hat{\tilde{\mathbf{B}}}\nabla\psi \tag{A.6}$$

and, finally,

$$\Psi(\mathbf{r}) = \psi - \frac{r}{B_t}\hat{\tilde{\mathbf{B}}}\nabla\psi = \psi(\mathbf{r} - \delta\mathbf{r}), \tag{A.7}$$

where

$$\delta\mathbf{r} \equiv \frac{r}{B_t}\hat{\tilde{\mathbf{B}}}_{pol} \tag{A.8}$$

with $\tilde{\mathbf{B}}_{\mathrm{pol}}$ as a poloidal component of $\tilde{\mathbf{B}}$. It is clear from Eq.(A.7) that substitution

$$\mathbf{r} = \bar{\mathbf{r}} + \delta\mathbf{r} \tag{A.9}$$

makes $\Psi(\mathbf{r})$ two-dimensional function of new variables $\bar{r},\bar{z}$. It means that in these coordinates magnetic surfaces Ψ=const are axisymmetric. Volume element $d\tau$ is related in linear approximation with $d\bar{\tau} = \bar{r}d\bar{r}d\bar{z}d\zeta$ as

$$d\tau = (1 + \mathrm{div}\delta\mathbf{r})d\bar{\tau}. \tag{A.10}$$

So, in the same approximation we have

$$\int\limits_V f(\mathbf{r})d\tau = \int\limits_{\bar{V}} [f(\bar{\mathbf{r}}) + \mathrm{div} f\delta\mathbf{r}]d\bar{\tau} \tag{A.11}$$

which leads to

$$\int\limits_V \tilde{\mathbf{B}}\nabla\zeta d\tau = \int\limits_{\bar{V}} \mathrm{div}\frac{\tilde{B}_\zeta}{\bar{r}}\delta\mathbf{r}d\bar{\tau} \tag{A.12}$$

with $\delta\mathbf{r}$ given by Eq.(A.8). $\hat{\tilde{\mathbf{B}}}$ is a divergence-free vector. So

$$\mathrm{div}\frac{\tilde{B}_\zeta}{\bar{r}}\delta\mathbf{r} = \mathrm{div}\frac{\tilde{B}_\zeta}{B_t}\left(\hat{\tilde{\mathbf{B}}} - \hat{\tilde{B}}_\zeta\mathbf{e}_\zeta\right) = \hat{\tilde{\mathbf{B}}}\nabla\frac{\tilde{B}_\zeta}{B_t} - \mathrm{div}\frac{\hat{\tilde{B}}_\zeta\tilde{B}_\zeta}{B_t}\mathbf{e}_\zeta. \tag{A.13}$$

In stellarators β<<1 and, as a consequence, $\tilde{\mathbf{B}} \approx \tilde{\mathbf{B}}_V = \nabla\varphi$, $rB_t \approx$const. In this approximation

$$\hat{\tilde{\mathbf{B}}}\nabla\frac{\tilde{B}_\zeta}{B_t} \approx \frac{1}{rB_t}\hat{\tilde{\mathbf{B}}}\nabla\frac{\partial\varphi}{\partial\zeta} = \frac{1}{rB_t}\left(\frac{\partial}{\partial\zeta}\hat{\tilde{\mathbf{B}}}\nabla\varphi - \tilde{\mathbf{B}}\nabla\varphi\right). \tag{A.14}$$

As a result we have

$$\mathrm{div}\frac{\tilde{B}_\zeta}{\bar{r}}\delta\mathbf{r} = -\frac{\tilde{\mathbf{B}}^2}{\bar{r}B_t} + \frac{1}{\bar{r}B_t}\frac{\partial}{\partial\zeta}\left(\hat{\tilde{\mathbf{B}}}\tilde{\mathbf{B}}\right) - \mathrm{div}\frac{\hat{\tilde{B}}_\zeta\tilde{B}_\zeta}{B_t}\mathbf{e}_\zeta, \tag{A.15}$$

and after integration in Eq.(A.12) we obtain Eq.(16).

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