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Cross Field Ion Motion at Sawtooth Crash

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Abstract

Turbulent ion motion which is caused by the sawtooth crash is analyzed in the framework of the magnetic stochasticity model of the sawtooth crash. The average current profile changes much more slowly than the pressure profile after the onset of the crash, so that inhomogeneity of the parallel current along the field line is generated. This causes the perpendicular current with a short scale length, leading to the ion cross field motion.

Keywords: Disruption, Sawtooth Crash, Magnetic Stochasticity, Current Profile, Ion Motion, Inertial Effect

Recently the interest in the sawtooth[1,2] has been revived by the detailed experimental observations. One of the most striking observations is that the q value at the axis, q(0), remains well below unity during the whole period of the sawtooth oscillations, while the electron temperature profile shows a complete flattening[3-5]. (q is the safety factor.) This has urged to renew the physics modelling of the sawtooth crash. A possible explanation for it is to take into account of the global stochasticity, which is caused by the toroidicity and $m=1$ magnetic perturbation[6-9], rather than the rearrangement of poloidal flux by reconnection[10]. This process allows a rapid rearrangement of the plasma pressure: at the same time, the average current profile, and hence the q-profile, change at a slow time scale of the resistive (or current-diffusive) time, leading to a small change of q(0) at the crash.

This model needs experimental verification. At the same time, further theoretical study is still necessary. One of the problems is a 'paradox' associated with the pressure balance. This paradox is as follows[11]. Within the framework of magneto-hydrodynamic (MHD) model, the post-state of the fast crash would be approximated as an equilibrium. For the simplicity, flattening of the pressure, $\nabla p=0$, is assumed. Then the force balance equation $J\times B=0$ leads to $J=\alpha B$. Noting relations $\nabla\cdot B=0$ and $\nabla\cdot J=0$ for the steady state, one has $\nabla\alpha\cdot B=0$, showing that the parallel current is constant along the field line. It is well known that, in a state of the global stochasticity, the field line at any minor radius r is connected to the line going through near the axis (see Fig.1). This means that $J_{/\!/}(r) = J_{/\!/}(0)$, indicating q profile should also be flattened. These considerations seem to give a contradiction within the stochasticity model of the sawtooth.

In this note we analyze the ion motion at the crash. We examine the ion inertia effect, and keep the $\mathbf{v}\nabla\cdot\mathbf{v}$ and viscous term in the equation of motion. These terms are balanced by the force associated with the cross field current, the divergence of which is balanced by $\nabla_{/\!/}\cdot J_{/\!/}$ term. By this mechanism, the parallel current need not be constant along the field line, allowing the average q-profile to change at a slow resistive (or current-diffusive) time scale within the stochastic field lines.

We write the current density J as

$$\mathbf{J} = \alpha\mathbf{B} + \mathbf{j}. \tag{1}$$

As is discussed above, we assume that

$$\nabla p = 0 \tag{2}$$

for the post-crash state in order to keep simplicity. The equation of motion is written as

$$mn(\dot{\mathbf{v}} + \mathbf{v}\nabla\cdot\mathbf{v} - \nabla\cdot\mu\nabla\mathbf{v}) = \mathbf{j}\times\mathbf{B}, \tag{3}$$

where m is the ion mass, n is the ion density, $\mathbf{v}$ is the velocity, and μ is the ion viscosity. The charge neutrality $\nabla\cdot\mathbf{J}=0$ holds within a short time scale, and this condition is satisfied in the post-crash state. We have

$$\nabla\alpha \cdot \mathbf{B} = -\nabla \cdot \mathbf{j}. \tag{4}$$

This relation indicates that, when the average current profile is sustained unchanged in the braided magnetic field configuration, the perpendicular current with a short scale length appears. Figure 2 illustrates schematically the exchange of the current between magnetic field lines. The perpendicular current $\mathbf{j}$ generates the cross field ion motion.

We estimate the magnitude of the ion velocity. The magnitude of the magnetic field is assumed to be constant without losing generality of the argument. The inhomogeneity of α ($\equiv J_{/\!/}/B$) along the filed line is estimated by

$$\nabla\alpha \simeq [\alpha(0)-\alpha(r)]/\ell \tag{5}$$

where ℓ is the connection length of the field line between two radial points r=0 and r=r. By using the diffusion constant of the field line D_M, we have

$$\ell = r^2/D_M. \tag{6}$$

The typical perpendicular scale length, 1/k, is estimated to be approximately equal to those of the magnetic islands which generate the magnetic stochasticity. In the case of magnetic braiding at the sawtooth crash, the field line braiding is caused by the overlapping of the secondary islands. Their typical mode numbers are of the order of 10 [7-9]. We write

$$\nabla\cdot\mathbf{j} \simeq k|j|. \qquad (7)$$

Combining Eqs.(5), (6) and (7), we have

$$j \simeq \frac{D_M}{r^2 k}\{J_{/\!/}(0) - J_{/\!/}(r)\} \qquad (8)$$

in the perpendicular direction.

The ion motion driven by this current is given from Eq.(3). The $\mathbf{j}\times\mathbf{B}$ force is given as

$$F \simeq \frac{D_M}{r^2 k}\frac{\Delta J_{/\!/}}{J_{/\!/}}\frac{B^2}{q(0)\mu_0 R} \qquad (9)$$

where $\Delta J_{/\!/} = \{J_{/\!/}(0) - J_{/\!/}(r)\}$, and we used the relation $J_{/\!/}(0) = B/q(0)\mu_0 R$. We consider the case where the viscous term is small. In this case, we estimate $\mathbf{v}\nabla\cdot\mathbf{v}$ by kv^2, and have

$$mnkv^2 \simeq \frac{D_M}{r^2 k}\frac{\Delta J_{/\!/}}{J_{/\!/}}\frac{B^2}{q(0)\mu_0 R} \qquad (10)$$

The ion velocity is given in a normalized form as

$$\left(\frac{v}{v_{Ti}}\right)^2 = \frac{3D_M}{R\beta r^2 k^2}\frac{\Delta J_{//}}{J_{//}} \tag{11}$$

where v_{Ti} is the ion thermal velocity $\sqrt{T_i/m}$, and β is defined as $B^2/\mu_0 = 3nT/\beta$ (i.e., we assume that $T_i=T_e$).

The flow velocity given by Eq.(11) is evaluated for typical plasma parameters. In the quasilinear limit, D_M is given as[9]

$$D_M \simeq R[\tilde{B}_r R/Br_1]^2$$

where r_1 is the minor radius of the q=1 radius, and D_M is of the order of 10^{-4}m for the parameters of JET. Taking typical values of β=3%, $\Delta J_{//}/J_{//}$=1/4, $\tilde{B}_r R/Br_1 \sim 10^{-2}$, and rk=10, we have

$$v/v_{Ti} \simeq 1/200. \tag{12}$$

This small ion velocity is enough to sustain the inhomogeneity of the parallel current along the magnetic field line.

In summary, we studied the short-scale-length ion motion generated by the sawtooth crash in a magnetic stochasticity model of the sawtooth. The q(r) profile can changes at a slow time scale (such as the resistive time scale), so that the average current is almost unchanged after the crash takes place and the pressure profile is flattened. The inhomogeneity of the parallel current along the filed lines, which extends from the center to the inversion radius, is compensated by the cross field current with a small scale length. This cross field current generates the ion motion, the magnitude of which is much smaller than the thermal velocity. This analysis predicts the ion motion, resolving the 'paradox' quoted in the introduction.

The analysis here indicates the importance of the ion inertia in the force balance. The important role of the electron inertia in the Ohm's law has also been pointed out[12]. It is also noted that a small amplitude variation of ∇p can also balance with $\mathbf{j}$. Equation (10) (or (11)) is derived with the help of the analytic simplification. The quantitative result would be obtained by examining the results of numerical simulations.

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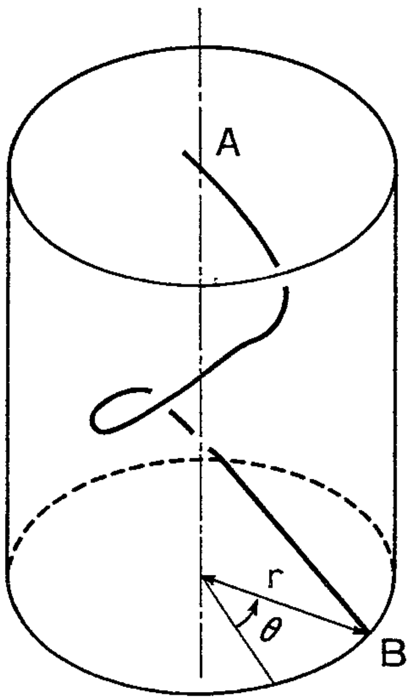
Figure Captions

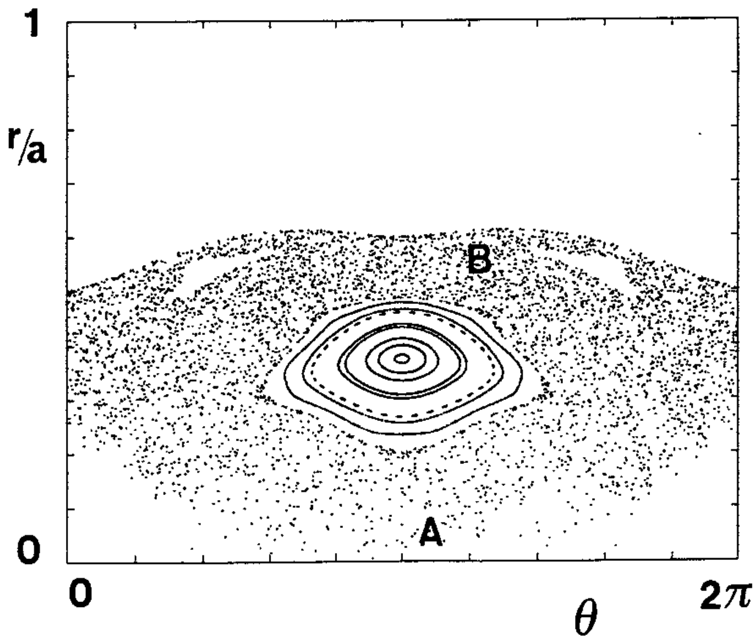
Fig.1 Schematic drawing of the field line (a) and the Poincare plot of the filed line on the r-θ plane (b). [r:minor radius and θ: poloidal angle.]

Fig.2 Schematic drawing of the field lines and current (a) and the radial profile of the averaged current (b). Bold arrows indicate the current.

Fig. 1

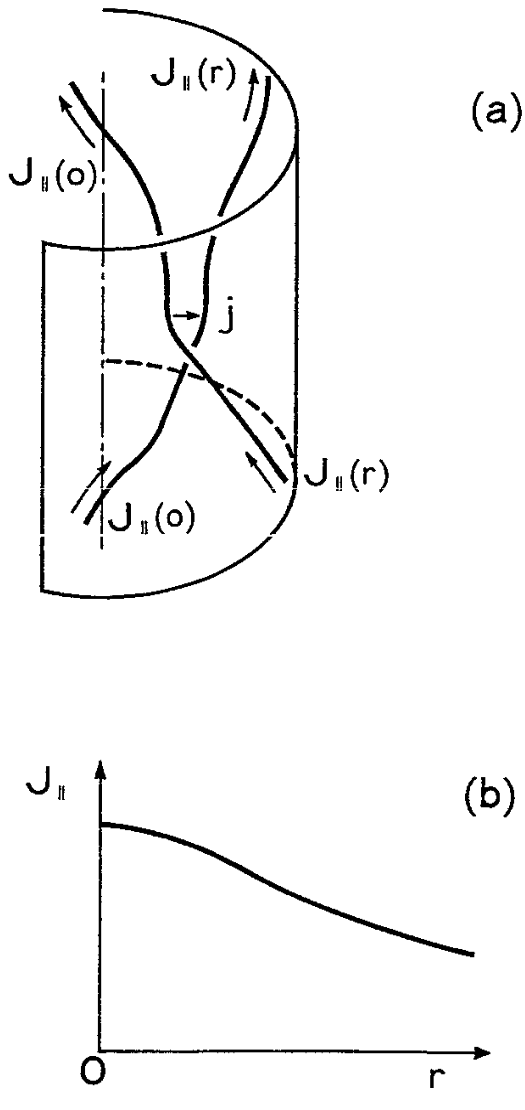


(a)



(b)

Fig. 2



(a)
$J_{\parallel}(r)$
$J_{\parallel}(o)$
j
$J_{\parallel}(r)$
$J_{\parallel}(o)$
(b)
$J_{\parallel}$
O
r

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