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TWO-STREAM INSTABILITY FOR A LIGHT ION BEAM-PLASMA SYSTEM  
WITH EXTERNAL MAGNETIC FIELD

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**Abstract**

For inertial confinement fusion, a focused light ion beam (LIB) is required to propagate stably through a chamber to a target. We have pointed out that the applied external magnetic field is important for LIB propagation. To investigate the influence of the external magnetic field on the LIB propagation, we analysed the electrostatic dispersion relation of magnetized light ion beam-plasma system. The particle in-cell (PIC) simulation results are presented for a light ion beam-plasma system with external magnetic field.

**Keywords:** Inertial Confinement Fusion, Light Ion Beam Propagation, Two-stream Instability, Magnetized Light Ion Beam-plasma System, PIC Simulation

## 1. Introduction

In the investigation of inertial confinement fusion, light ion beam (LIB) is seriously considered as a possible driver for pellet fusion. Many authors (Okada and Niu 1981; Okada and Schmidt 1987; Ottinger et al. 1981) have investigated the propagation of LIB through a fusion target chamber. For the propagation of LIB, an electromagnetic filamentation instability has been investigated in a light ion beam-plasma system with external magnetic field (Okada and Niu 1988). It has been found that the external magnetic field has the tendency to stabilize the filamentation instability. In this article an electrostatic instability analysis is done by using the dispersion relations of the two-stream instability and PIC simulation code to study the influence of the external magnetic field.

## 2. Electrostatic two-stream instability

We introduce a cylindrical polar coordinate system ( $r, \theta, z$ ) with  $z$ -axis along the axis of symmetry. To obtain a dispersion relation for the electrostatic two-stream instability, we adopt the linearized Vlasov equation for the  $j$ -th component perturbed distribution function  $\delta f_j(x, p, t)$  ( $j = b, e, i$ ) of the form

$$\begin{aligned} & \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{x}} + e_j \frac{\mathbf{v} \times (B_0^z \mathbf{e}_z + B^\theta \mathbf{e}_\theta)}{c} \cdot \frac{\partial}{\partial \mathbf{p}} \right) \delta f_j(x, p, t) \\ &= -e_j \left( \delta \mathbf{E}(x, t) + \frac{\mathbf{v} \times \delta \mathbf{B}(x, t)}{c} \right) \cdot \frac{\partial}{\partial \mathbf{p}} f_j^0(H, P_\theta, P_z) \end{aligned} \quad (1)$$

where  $B_0^z \mathbf{e}_z$  is a uniform magnetic field in the direction of beam propagation,  $B^\theta \mathbf{e}_\theta$  is an azimuthal self-magnetic field,  $e_j$  is the charge of the  $j$ -th component,  $H$  is the total energy,  $P_\theta$  is the canonical angular momentum,  $P_z$  is the axial canonical momentum.

The equilibrium distribution function  $f_j^0(H, P_\theta, P_z)$  for the ion beam ( $j = b$ ), the plasma electrons ( $j = e$ ) and ions ( $j = i$ ) can be approximated in the beam-plasma system by

$$f_b^0(H, P_\theta, P_z) = \frac{n_b}{2\pi m_b T_b} \exp\left(-\frac{1}{2m_b T_b} (p_r^2 + (p_\theta - m_b r \Omega_{rb})^2)\right) \delta(p_z - m_b V_{zb}) \quad (2)$$

$$f_j^0(H, P_\theta, P_z) = \frac{n_j}{(2\pi m_j T_j)^{\frac{3}{2}}} \exp\left(-\frac{1}{2m_j T_j} (p_r^2 + (p_\theta - m_j r \Omega_{rj})^2 + (p_z - m_j V_{zj})^2)\right), j = e, i, \quad (3)$$

where  $n_b$  is the beam density,  $T_b$  is the beam thermal energy,  $m_b$  is the beam mass,  $\Omega_{rb}$  is the beam angular velocity,  $V_{zb}$  is the axial velocity of the beam,  $n_j$  is the background plasma density ( $j = e, i$ ),  $T_j$  is the plasma thermal energy,  $m_j$  is the plasma mass,  $\Omega_{rj}$  is the plasma angular velocity and  $V_{zj}$  is the axial velocity of the plasma. We obtain the linear dispersion relation for the two-stream instability (Davidson 1984) as:

$$\begin{aligned} 0 = 1 + \sum_{j=b,e,i} \frac{4\pi e_j^2}{k^2} \sum_{n=-\infty}^{\infty} 2\pi \int_0^\infty dp_\perp p_\perp \int_{-\infty}^\infty dp_z J_n^2 \left( \frac{k_\perp p_\perp / m_j}{\Omega_{rj}^+ - \Omega_{rj}^-} \right) \\ \times \frac{(n(\Omega_{rj}^+ - \Omega_{rj}^-) \frac{m_j}{p_\perp} \frac{\partial}{\partial p_\perp} + k_z \frac{\partial}{\partial p_z}) f_j^0}{\omega - l\omega_{rj} - n(\Omega_{rj}^+ - \Omega_{rj}^-) - k_z v_z}. \end{aligned} \quad (4)$$

In (4),  $l$  is the azimuthal harmonic number,  $J_n(x)$  is the Bessel function of the first kind order  $n$ ,  $v_z = p_z/m_j$  ( $j = b, e, i$ ),  $k_z$  is the axial wavenumber,  $k_\perp = (k_x^2 + k_y^2)^{\frac{1}{2}}$  is the perpendicular wavenumber,  $p_\perp = (p_r^2 + (p_\theta - m_j r \Omega_{rj})^2)^{1/2}$  is the perpendicular momentum variable,  $\omega$  is the complex oscillation frequency, and

$$\Omega_{rj}^+ - \Omega_{rj}^- = -\varepsilon_j \omega_{cj} \left( 1 - \frac{8\pi e_j}{m_j \omega_{cj}^2} \sum_k n_k e_k \left( 1 - \frac{V_{zj} V_{zk}}{c^2} \right) \right)^{1/2} \quad (5)$$

where  $\varepsilon_j = \text{sgn}(e_j)$  and  $\omega_{cj} = |e_j| B_0^z / m_j c$ .

In the cold beam-plasma limit of (4), the dispersion relation of the two-stream instability can be approximated by

$$\begin{aligned}
1 = & \frac{k_z^2}{k^2} \left( \frac{\omega_b^2}{(\omega - l\Omega_{rb} - k_z V_{zb})^2} + \sum_{j=e,i} \frac{\omega_{pj}^2}{(\omega - l\Omega_{rj} - k_z V_{zj})^2} \right) \\
& + \frac{k_\perp^2}{k^2} \frac{\omega_b^2}{(\omega - l\Omega_{rb} - k_z V_{zb})^2 - (\Omega_{rb}^+ - \Omega_{rb}^-)^2} \\
& + \frac{k_\perp^2}{k^2} \sum_{j=e,i} \frac{\omega_{pj}^2}{(\omega - l\Omega_{rj} - k_z V_{zj})^2 - (\Omega_{rj}^+ - \Omega_{rj}^-)^2}.
\end{aligned} \tag{6}$$

For  $B_0^z = 0$  and low beam density ( $n_b \ll n_j, j = e, i$ ), the maximum growth rate of the two-stream instability can be approximately by

$$(Im \omega)_{max} = \frac{\sqrt{3}}{2} \omega_{pe} \left( \frac{n_b m_e}{2 n_e m_b} \right)^{1/3}. \tag{7}$$

In order to simulate a two-stream instability, we carry out a numerical calculation using an electrostatic particle in-cell code (Morse 1970) for beam-plasma system. The code is two-dimensional in space and velocity, respectively. External constant magnetic field  $B_0^z$  is in the  $z$ -direction and induced electric and magnetic fields are respectively indicated by  $E = (E_x, 0, E_z)$  and  $B = (B_x, 0, 0)$ . To determine the initial velocity distribution function  $f_j^0$  ( $j = b, e, i$ ) in (2) and (3), we choose the following parameters;  $v_{Tb} = \sqrt{T_b/m_b} = 0.05c$ ,  $v_{Tj} = \sqrt{T_j/m_j} = 0.01c$  ( $j = i, j$ ),  $V_{zb} = 0.1c$  and  $V_{zj} = 0.02c$  ( $j = e, i$ ). The simulation is carried out under the conditions that the grid spacing is  $0.2c/\omega_{pe}$ , the time step is  $0.1\omega_{pe}^{-1}$ , the length of simulation range is  $3.2c/\omega_{pe}$ , the number of ion beam simulation particles is 1000, the number of plasma electron simulation particles is 6000, and the number of plasma ion simulation particles is 5000. Induced electric fields for  $B_0^z = 0$  are plotted in figure 1, from which we can estimate that the maximum growth rate  $\gamma_{max}$  is about  $0.034\omega_{pe}$ . This growth rate is consistent with  $(Im \omega)_{max} \approx 0.031\omega_{pe}$  which is given by (7). Induced electric fields for  $B_0^z/(4\pi n_e m_e c^2)^{1/2} = 2.0$  are plotted in figure 2. In figure 3, the growth rates which is obtained by using PIC code are plotted versus the external magnetic field  $B_0^z$ .

### 3. Results and discussion

We have investigated a two-stream instability in a light ion beam propagation system with external magnetic field. If we assume that the external magnetic field is in the propagation direction, the field has the tendency to enhance the two-stream instability. The stability mechanism of the two-stream instability including the effects of collisions will be studied in a subsequent publication.

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## FIGURE CAPTIONS

Figure 1. Energies of induced electric fields at the linear stage with  $B_0^z=0$ .

Figure 2. Energies of induced electric fields at the linear stage with  $B_0^z/(4\pi n_e m_e c^2)^{1/2} = 2.0$ .

Figure 3. Maximum growth rates of the two-stream instability versus the external magnetic fields.

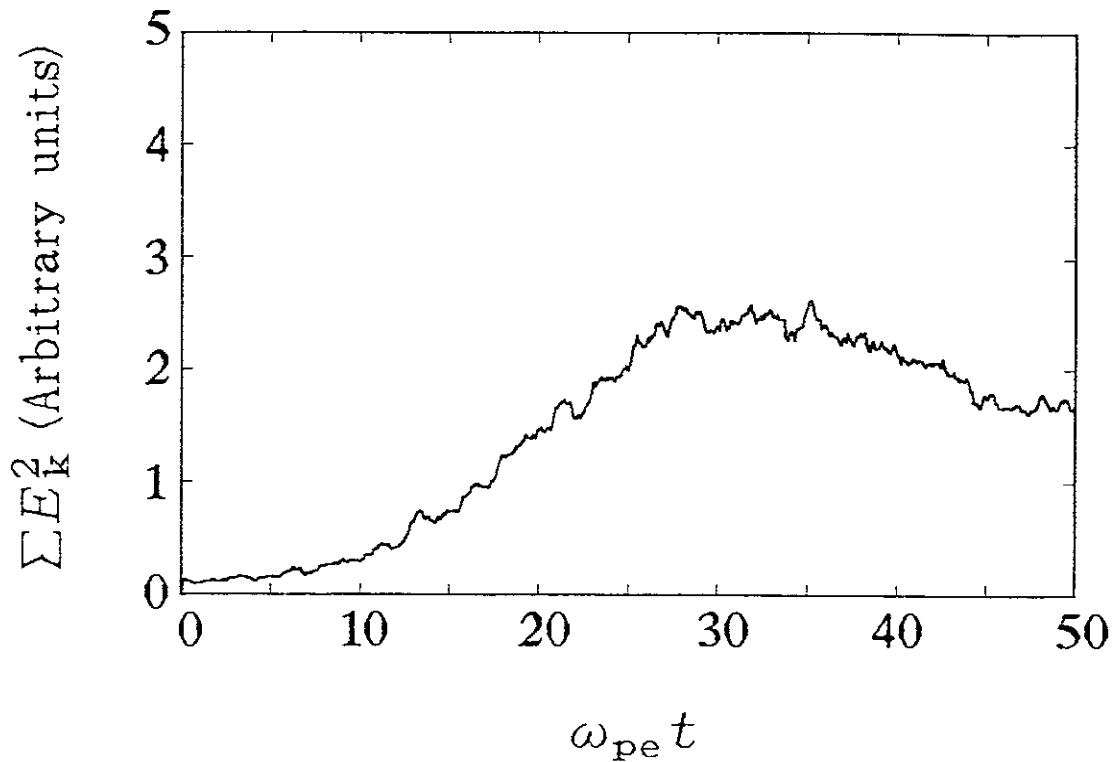


Figure 1

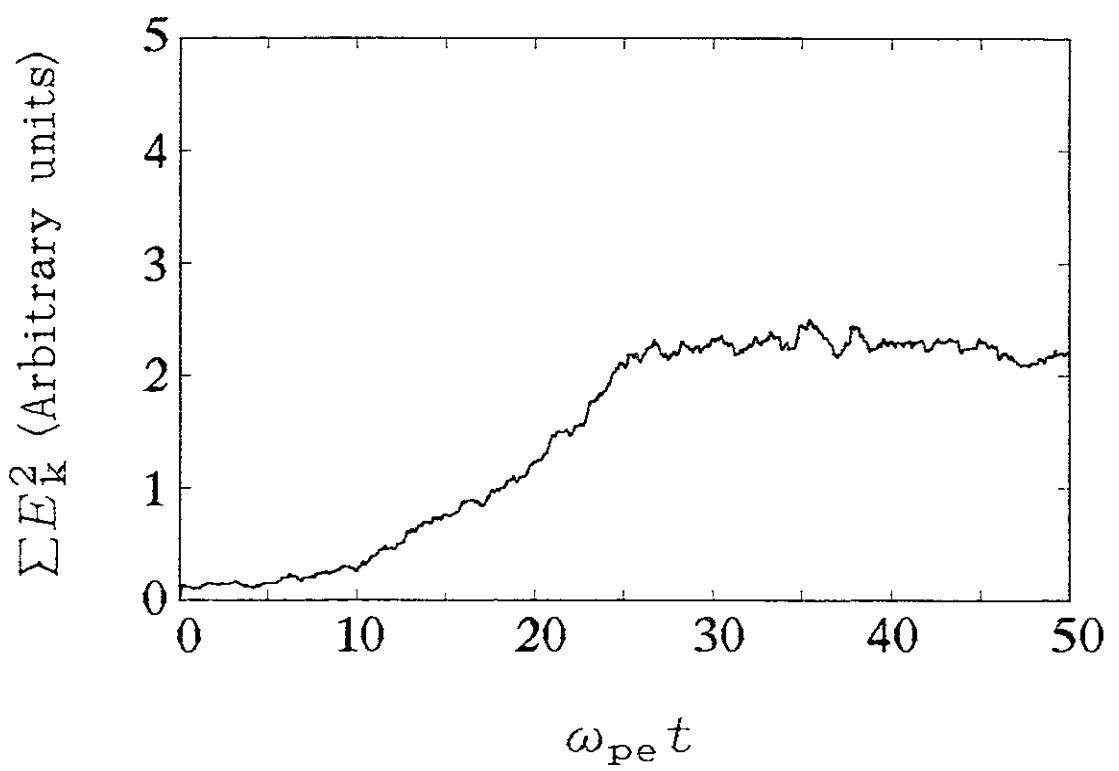


Figure 2

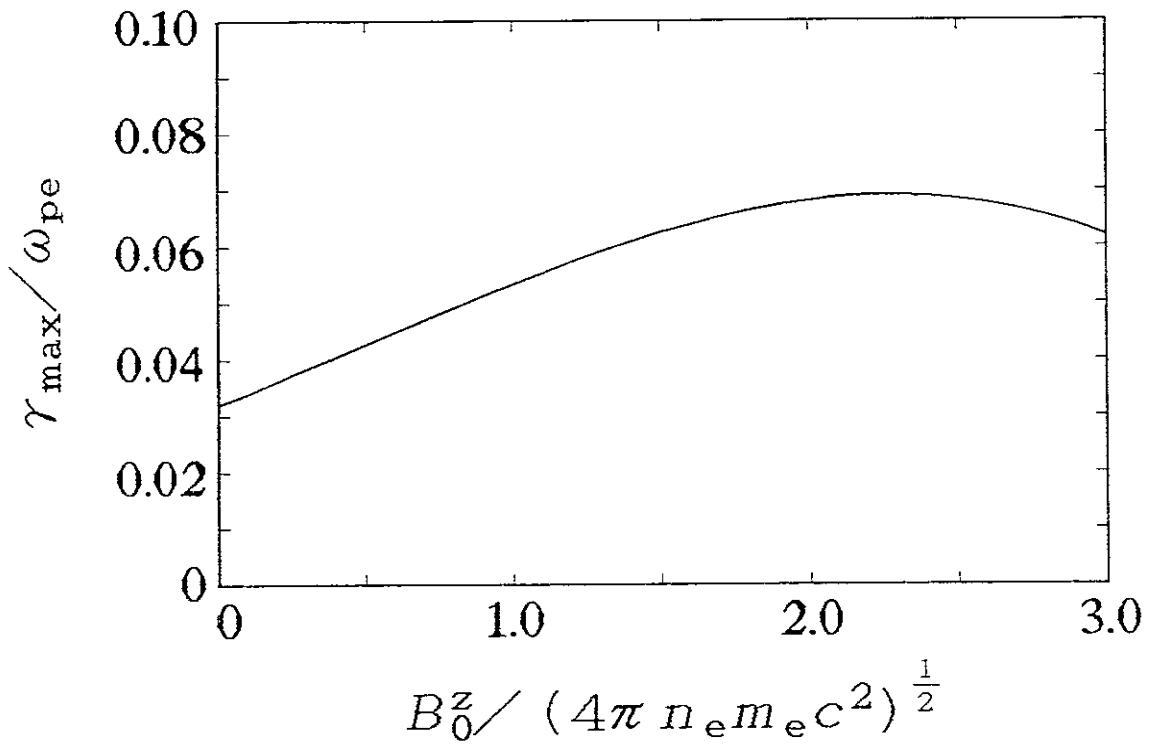


Figure 3

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