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Self-sustained Turbulence and L-mode Confinement in Toroidal Plasmas II

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Abstract

Theory of the anomalous transport coefficient in toroidal helical systems (such as stellarators, torsatron and Heliotron devices) is developed. The theoretical formalism of self-sustained turbulence is applied to the interchange mode turbulence and ballooning mode turbulence. The nonlinear destabilization of microscopic modes by the current diffusivity is the key for the anomalous transport. A general form of the anomalous transport coefficient in toroidal plasmas is derived. The intrinsic importance of the pressure gradient, collisionless skin depth and Alfven transit time is confirmed. The geometrical factors which characterize the magnetic configurations are also obtained. The theory is extended to study the influence of parallel compressibility. The ion viscosities of the perpendicular and parallel momenta, electron viscosity and energy diffusion coefficient are obtained. The comparison with experimental results is also given.

Key word. Anomalous transport coefficient, L-mode, helical plasma, stellarator, magnetic hill, magnetic well, magnetic shear, self-sustained turbulence

1. Introduction

The transport across the magnetic field in toroidal plasmas has long been known as the anomalous transport. Though the phenomenological knowledge has been enriched these days, the fundamental understanding is far from satisfactory. (For a review of present status, see, e.g., Houlberg et al. 1990, Wootton et al. 1990, Ross et al. 1991, Itoh and Itoh 1992, Wagner et al. 1993) One of the working hypothesis to explain the anomalous transport is that it is caused by fluctuations in the confined plasma, and that the fluctuations are sustained by microscopic instabilities. (See e.g., Kadomtsev 1965, Liewer 1985). The instabilities are brought about by the combination of the plasma inhomogeneities and the magnetic field structure. It is therefore meaningful to compare the anomalous transport phenomena in devices of different magnetic configurations. By this comparison, one may be able to extract common natures and to distinguish them from specific features that are characteristic to the particular kind of magnetic configuration. The progress in the transport study in helical systems and stellarators has recently encouraged the comparison studies (Itoh and Itoh1989, Wagner et al. 1993).

The conventional theoretical picture has provided the estimate for the diffusion coefficient as $D = \gamma_L/k_\perp^2$, where γ_L and $k_\perp$ are the linear growth rate and perpendicular wave number of the most unstable mode, respectively (Kadomtsev 1965). Although detailed analysis has given accurate forms of γ_L, this formula was not able to explain the experimental observations on the anomalous transport coefficient (See Connor et al. 1993 for the comparison study). We have recently proposed a new theoretical method to analyze the anomalous transport phenomena in magnetic confinement devices (Itoh et al. 1992, 1993a, b, c, 1994a). In this new theoretical approach, the instabilities are caused by the anomalous transport itself. Hence the fluctuations and transport coefficients are sustaining each other, under the condition of the given gradients of equilibrium plasma parameters. The self-sustained state is realized even below the critical pressure gradient against the ideal magnetohydrodynamic (MHD) instabilities. In this sense, the turbulent state is classified as a kind of subcritical turbulence. In this nonlinear destabilization mechanism, the role of the current diffusivity is essential. The characteristic scale length was found to be the collisionless skin depth, $\delta = c/\omega_p$ (c is the speed of light, and ω_p is the electron plasma oscillation frequency) confirming the idea in the Ohkawa model of anomalous transport (Ohkawa 1978). This method has been applied to helical systems and to tokamaks. Detailed derivation of the method and resultant transport coefficient were discussed for tokamaks in detail (Itoh et al 1994a). The results have shown considerable success in explaining the tokamak L-mode plasmas (Fukuyama et al. 1994a, b).

The purpose of this article is to present the transport coefficient in the toroidal helical plasmas. In this class of magnetic configurations, we choose two typical ones. One is the torsatron/Heliotron configurations (Gouldon et al. 1968, Mohri 1970, Uo 1971). This system is characterized by the magnetic hill and strong magnetic shear. (We may call the torsatron/Heliotron system by the word 'helical system' in this article for abbreviation.) The other one is the conventional (or modular) $\ell = 2$ stellarator (ℓ is the multipolarity of the helical field). This configuration is characterized by the weak shear and magnetic well. The analysis on these toroidal helical plasma complements the study on the toroidal plasmas in general, because the tokamaks are characterized by the magnetic well and strong shear. The torsatron/Heliotron configuration is accompanied by the magnetic hill, the interchange mode turbulence, rather than the ballooning mode turbulence, can occur. By the difference in this character of the modes, the resultant anomalous transport coefficient shows different dependence on the geometrical factors, compared to that for tokamaks. The other purpose of this article is the study of the influence of the parallel compressibility, which was neglected in previous work, on the self-sustained turbulence. It is shown that the effect of this new term is small and does not change the previous conclusion qualitatively. The ion viscosities of the perpendicular and parallel momenta, electron viscosity and energy diffusion coefficients are obtained. These four quantities are found to be of the same order of magnitude. The relative ratio between them is discussed.

The constitution of this article is as follows. In section 2, the basic equation for the case of toroidal helical plasma with magnetic hill is discussed. The renormalization of the turbulence and the mean field approximation are discussed. In section 3, the eigenmode equation for the dressed test mode is obtained. The stability analysis is performed, and the marginal stability condition is obtained. In section 4, the transport coefficient is derived. In section 5, the transport coefficient for the stellarator (the system with magnetic well and weak shear) is discussed. Summary and discussion are given in section 6.

2. Model Equation for Toroidal Helical Plasmas

2.1 Model Equations

We study the high-aspect-ratio, toroidal helical plasma with magnetic hill and strong magnetic shear. The minor and major radii of the torus are given by a and R, respectively, We use the toroidal coordinate (r, θ, ζ). The reduce set of equations for the static potential ϕ, pressure p, current J and parallel velocity v are employed (See Yagi 1989 and Hazeltine 1983). The equation of motion:

$$\frac{\partial \nabla_\perp^2 \varphi}{\partial t} + [\varphi, \nabla_\perp^2 \varphi] = \nabla_\parallel J + (\Omega' \times \hat{\zeta}) \cdot \nabla p + \mu_{\perp c} \nabla_\perp^4 \varphi \tag{1}$$

the Ohm's law:

$$\frac{\partial \Psi}{\partial t} = -\nabla_\parallel \varphi - \frac{1}{\xi}\left(\frac{\partial J}{\partial t} + [\varphi, J] \right) - \eta J + \lambda_c \nabla_\perp^2 J \tag{2}$$

the energy balance equation:

$$\frac{\partial p}{\partial t} + [\varphi, p] = -\beta \nabla_\parallel v + \chi_c \nabla_\perp^2 p \tag{3}$$

and the equation for the parallel motion:

$$\frac{\partial v}{\partial t} + [\varphi, v] = -\nabla_\parallel p + \mu_{\parallel c} \nabla_\perp^2 v \tag{4}$$

constitute the set of basic equations. In these equations, the bracket [f,g] denotes the Poisson bracket,

$$[f, g] = (\nabla f \times \nabla g) \cdot \vec{b}$$

($\vec{b}$ is the unit vector along the field line), Ω' is the average curvature of the magnetic field, Ψ is the vector potential, $1/\xi$ denotes the finite electron inertia, $1/\xi = (\delta/a)^2$, η is the classical resistivity, and β is the ratio of the plasma pressure to the magnetic pressure. The transport coefficients $\mu_{\perp c}$, λ_c, χ_c, $\mu_{\parallel c}$ are the contributions from the collisional diffusion and are viscosity for the perpendicular momentum, current diffusivity, thermal conductivity and viscosity for the parallel momentum, respectively. This set of equations is a generalized form of the one used in previous articles (Itoh et al 1992a, 1993a, b, 1994a, Yagi et al 1993) because the effect of the parallel compressibility is taken into account. The finite gyro-radius effect and the electron pressure terms in Ohm's law are neglected. In writing Eqs.(1)-(4), the normalization for resistive MHD modes are employed:

$$(\varepsilon v_A/a)t \to t, \qquad r/a \to r, \qquad \varphi/(\varepsilon a v_A B_0) \to \varphi, \quad \Psi/(\varepsilon a B_0) \to \Psi$$
$$J(a\mu_0/\varepsilon B_0) \to J, \quad v/(\varepsilon v_A) \to v, \quad p(2\mu_0/\varepsilon B_0^2) \to p, \quad \eta(\tau_{Ap}/\mu_0 a^2) \to \eta$$
$$\mu_\perp(\tau_{Ap}/a^2) \to \mu_\perp, \quad \mu\,(\tau_{Ap}/a^2) \to \mu\,, \quad \chi(\tau_{Ap}/a^2) \to \chi, \quad \lambda(\tau_{Ap}/\mu_0 a^4) \to \lambda$$

where ε is the inverse aspect ratio, a/R, v_A is the Alfven velocity, B_0 is the main magnetic field, and $\tau_{Ap} = R/v_A$.

2.2 Renormalization

The nonlinear equations (1)-(4) are transformed to the equation for the test mode (denoted by k) in the presence of background fluctuations (denoted by k_1) by employing the renormalization. In the process of the renormalization, the back-interaction of the driven mode (denoted by k_2) to the original test mode is kept. The ExB nonlinearity is taken into account. The detailed procedure was given in the previous article (Itoh et al 1993c). The driven mode is given as

$$U_2 = \frac{1}{K_2}\left\{-N_u - \frac{ik_{\parallel 2}}{\gamma_{j2}}N_j - \frac{iA_2\gamma_{v2}}{\Gamma_{vp2}}N_p + \frac{iA_2\beta k_{\parallel 2}}{\Gamma_{vp2}}N_v\right\} \tag{5}$$

$$J_2 = \frac{1}{K_2}\left\{-\frac{ik_{\parallel 2}\xi}{\gamma_{j2}k_{\perp 2}^2}N_u - \left[\frac{K_2}{\gamma_{j2}} - \frac{\xi k_{\parallel 2}^2}{\gamma_{j2}^2 k_{\perp 2}^2}\right]N_j + \frac{\xi A_2\gamma_{v2}k_{\parallel 2}}{\gamma_{j2}k_{\perp 2}^2\Gamma_{vp2}}N_p - \frac{\xi A_2\beta k_{\parallel 2}^2}{\gamma_{j2}k_{\perp 2}^2\Gamma_{vp2}}N_v\right\} \tag{6}$$

$$p_2 = \frac{1}{K_2}\left\{\frac{i\gamma_{v2}G_2}{\Gamma_{vp2}k_{\perp 2}^2}N_u - \frac{G_2\gamma_{v2}k_{\parallel 2}}{\Gamma_{vp2}\gamma_{j2}k_{\perp 2}^2}N_j\right\}$$
$$+ \frac{1}{K_2}\left\{-\frac{\gamma_{v2}}{\Gamma_{vp2}}\left[K_2 + \frac{G_2A_2\gamma_{v2}}{k_{\perp 2}^2\Gamma_{vp2}}\right]N_p + \frac{\beta k_{\parallel 2}}{\Gamma_{vp2}}\left[\frac{G_2A_2\gamma_{v2}}{k_{\perp 2}^2\Gamma_{vp2}} - iK_2\right]N_v\right\} \tag{7}$$

$$v_2 = \frac{1}{K_2}\left\{\frac{k_{\parallel 2}G_2}{\Gamma_{vp2}k_{\perp 2}^2}N_u + \frac{iG_2k_{\parallel 2}^2}{\Gamma_{vp2}\gamma_{j2}k_{\perp 2}^2}N_j\right\}$$
$$+ \frac{1}{K_2}\left\{\frac{ik_{\parallel 2}}{\Gamma_{vp2}}\left[K_2 + \frac{G_2A_2\gamma_{v2}}{k_{\perp 2}^2\Gamma_{vp2}}\right]N_p - \frac{1}{\Gamma_{vp2}}\left[\frac{iG_2A_2\beta k_{\parallel 2}^2}{k_{\perp 2}^2\Gamma_{vp2}} + \gamma_{p2}K_2\right]N_v\right\} \tag{8}$$

and

$$K_2 = \gamma_{u2} + \frac{k_{\parallel 2}^2}{k_{\perp 2}^2}\frac{\xi}{\gamma_{j2}} - \frac{A_2G_2\gamma_{v2}}{\Gamma_{vp2}k_{\perp 2}^2} \tag{9}$$

where $\gamma_{u2} = \gamma(2) + \Gamma_{u2}$, $\gamma_{j2} = \gamma(2) + \Gamma_{j2}$, $\gamma_{p2} = \gamma(2) + \Gamma_{p2}$, $\gamma_{v2} = \gamma(2) + \Gamma_{v2}$, $\gamma(2)$ is the eigenvalue of the k_2 mode, $\partial\{U_2, J_2, p_2, v_2\}/\partial t = \gamma(2)\{U_2, J_2, p_2, v_2\}$, Γ_{u2}, Γ_{j2}, Γ_{p2} and Γ_{v2} denote the decorrelation rate of U_2, J_2, p_2 and v_2 by the back-ground turbulence, respectively. (Suffix 1 and 2 denotes the abbreviation for k_1 and k_2, respectively.) Other notation is: U is the vorticity, $U = -k_\perp^2\varphi$, $\Gamma_{vp2} = \gamma_{p2}\gamma_{v2} + \beta k_{\parallel 2}^2$, $iA_2 p_2 = (\Omega' \times \zeta)\cdot\nabla p_2$, $G_2 = k_{\theta 2}(dp_0/dr)$, p_0 is the equilibrium pressure profile, and the nonlinear interaction terms are defined as

$$N_u = [\varphi_1, U_k],\ N_j = [\varphi_1, J_k],\ N_p = [\varphi_1, p_k],\ N_v = [\varphi_1, v_k]\,. \tag{10}$$

2.3 Diffusion Approximation and Mean Field Approximation

The nonlinear contribution to the original test wave is obtained by calculating the back-interaction of the driven mode with background turbulence. Such contribution has the form as $\Sigma[\phi_{-1}, U_2]$ and is proportional to $\Sigma[\phi_{-1}, [\phi_1, U_k]]$. Taking the assumption that the wavelength of the turbulence is much shorter than the scale length of the envelope of fluctuations, and that the convective momentum associated with turbulence is small, as in the previous article (Itoh et al 1994a), the nonlinear terms can be expressed in the form of the diffusion matrix. The simplification is taken such that the turbulence is isotropic,

$$\langle|\partial\varphi_1/\partial r|^2\rangle \simeq \langle|\partial\varphi_1/r\partial\theta|^2\rangle \simeq \langle|k_{\perp 1}\varphi_1|^2\rangle/2 \tag{12}$$

Bracket <> means the average. Only the diagonal elements are kept in the following argument. By these approximations and assumptions, Eqs.(1)-(4) deduce to the set of linearlized equations for the dressed test wave as follows.

The equation of motion:

$$\frac{\partial\nabla_\perp^2\varphi}{\partial t} + [\varphi_0, \nabla_\perp^2\varphi] + \mu_{\perp k}\nabla_\perp^4\varphi = \nabla_\parallel J + (\Omega'\times\hat{\zeta})\cdot\nabla_F \tag{13}$$

the Ohm's law:

$$\frac{\partial\Psi}{\partial t} = -\nabla_\parallel\varphi - \frac{1}{\xi}\left(\frac{\partial J}{\partial t} + [\varphi_0, J]\right) - \eta J + \lambda_k\nabla_\perp^2 J \tag{14}$$

the energy balance equation:

$$\frac{\partial p}{\partial t} + [\varphi_0, p] - \chi_k\nabla_\perp^2 p = -[\varphi, p_0] - \beta\nabla_\parallel v \tag{15}$$

and the equation for the parallel motion:

$$\frac{\partial v}{\partial t} + [\varphi_0, v] - \mu_{\parallel k}\nabla_\perp^2 v = -\nabla_\parallel p - [\varphi, v_0] \tag{16}$$

where the suffix 0 denotes the equilibrium distribution, and the suffix k denoting the test wave is suppressed for the simplicity. The diffusion coefficient for the test mode is derived in the appendix.

The mean field approximation is employed to perform the stability analysis and obtain the transport coefficient and turbulence level. We approximate the constants $\{\mu_{\perp k}, \lambda_k, \chi_k, \mu_{\parallel k}\}$ by a set of diffusion coefficients $\{\mu_\perp, \lambda, \chi, \mu_\parallel\}$. We have

$$\mu_\perp = \Sigma \frac{|k_{\perp 1}\varphi_1|^2}{2}\frac{1}{K_1} \tag{17}$$

$$\lambda = (\delta/a)^2\mu_e \tag{18}$$

$$\mu_e = \Sigma \frac{|k_{\perp 1}\varphi_1|^2}{2}\frac{1}{K_1}\frac{\gamma_{u1}}{\gamma_{j1}}\left[1 - \frac{\gamma_{v1}}{\gamma_{u1}}\frac{A_1 G_1}{\Gamma_{vp1}k_{\perp 1}^2}\right] \tag{19}$$

$$\chi = \Sigma \frac{|k_{\perp 1}\varphi_1|^2}{2}\frac{1}{K_1}\frac{\gamma_{u1}\gamma_{v1}}{\Gamma_{vp1}}\left[1 + \frac{\xi k_{\parallel 1}^2}{k_{\perp 1}^2\gamma_{u1}\gamma_{j1}}\right] \tag{20}$$

$$\mu_\parallel = \Sigma \frac{|k_{\perp 1}\varphi_1|^2}{2}\frac{1}{K_1}\frac{1}{\Gamma_{vp1}}\left[\gamma_{p1}K_1 + \frac{iA_1 G_1 \beta k_{\parallel 1}^2}{\Gamma_{vp1}k_{\perp 1}^2}\right] \tag{21}$$

and

$$K_1 = \gamma_{u1} + \frac{k_{\parallel 1}^2}{k_{\perp 1}^2}\frac{\xi}{\gamma_{j1}} - \frac{A_1 G_1 \gamma_{v1}}{\Gamma_{vp1}k_{\perp 1}^2} \tag{22}$$

In these expressions summation is taken over background fluctuations, k_1. The coefficient μ_e is the electron viscosity. This result is the generalization for the three field calculation in the previous article (Itoh et al 1994a) and distinguishes the ion viscosity of the perpendicular momentum ($\mu_\perp$) from that of the parallel momentum ($\mu_\parallel$).

3. Stability Analysis

3.1 Eigenmode Equation

The renormalized equations are given in a form of the linear form of the dressed test wave with diffusion coefficients $(\mu_\perp, \lambda, \chi, \mu_\parallel)$. Eliminating J and v from the set of equations, we have

$$k_{\parallel}\frac{k_{\perp}^{2}}{\gamma\left(1+\xi^{-1}k_{\perp}^{2}\right)+\lambda k_{\perp}^{4}}k_{\parallel}\,\tilde{\varphi}+\left(\gamma+\mu_{\perp}k_{\perp}^{2}\right)k_{\perp}^{2}\tilde{\varphi}-iA_{k}\,\tilde{p}=0 \tag{23}$$

and

$$\left(\gamma+\chi k_{\perp}^{2}+\beta k_{\parallel}\frac{1}{\gamma+\mu_{\parallel}k_{\perp}^{2}}k_{\parallel}\right)\tilde{p}-iG_{k}\,\tilde{\varphi}=0 \tag{24}$$

(The tilde denotes the dressed test wave in this set of equation.) The term which is proportional to β (the last term in the bracket of the left hand side of Eq.(24)) denotes the effect of the parallel compressibility. If this term is neglected, the result in the previous article is recovered.

The effect of the parallel compressibility is treated perturbatively. It is shown *a posteriori* that the correction is of the higher order in the inverse aspect ratio, and this expansion is validated. Expanding the pressure perturbation with respect to β, Eq.(24) gives p in terms of ϕ as

$$\tilde{p}=\frac{iG_{k}}{\gamma+\chi k_{\perp}^{2}}\left\{1-\beta k_{\parallel}\frac{1}{\gamma+\mu_{\parallel}k_{\perp}^{2}}k_{\parallel}\frac{1}{\gamma+\chi k_{\perp}^{2}}\right\}\tilde{\varphi} \tag{25}$$

Substituting this expression into Eq.(23), we have

$$\begin{aligned} &k_{\parallel}\frac{k_{\perp}^{2}}{\gamma\left(1+\xi^{-1}k_{\perp}^{2}\right)+\lambda k_{\perp}^{4}}k_{\parallel}\,\tilde{\varphi}+\left(\gamma+\mu_{\perp}k_{\perp}^{2}\right)k_{\perp}^{2}\tilde{\varphi}\\ &\quad-\frac{k_{\theta}^{2}D_{0}}{\gamma+\chi k_{\perp}^{2}}\left\{1-\beta k_{\parallel}\frac{1}{\gamma+\mu_{\parallel}k_{\perp}^{2}}k_{\parallel}\frac{1}{\gamma+\chi k_{\perp}^{2}}\right\}\tilde{\varphi}=0 \end{aligned} \tag{26}$$

The term D_0 denotes the driving term of the interchange instability ,

$$D_{0}=-\Omega'\frac{dp_{0}}{dr} \tag{27}$$

showing that the combination of the bad magnetic curvature and pressure gradient causes the instability (Rosenbluth and Longmire). Equation (26) is the eigenvalue equation for the dressed test wave. It is found that, compared to the case of three field model, the driving term is decreased due to the influence of the parallel compressibility. If the term of $O(\beta)$ is neglected in Eq.(26), the model equation in the preceding article (Itoh et al 1992a) is recovered.

It has been shown that the mode can be strongly destabilized by the current diffusivity. The asymptotic form of the growth rate, in the small but finite nonlinear interactions, was given as $\gamma \propto \lambda^{1/5}$ (Itoh et al 1993a, Yagi et al 1993). The low level of turbulence easily influence the growth rate. The schematic picture of the growth rate as a function of the level of the background fluctuations are shown in Fig.1. The marginal stability condition is determined by the balance between the nonlinear destabilization and nonlinear stabilization. The nonlinear stationary state is different from the conventional picture in which the linear growth balances with the nonlinear damping.

3.2 Marginal Stability Condition

The marginal stability condition for the least stable mode dictates the anomalous transport coefficient. We here obtain the marginal stability condition by setting $\gamma = 0$ in Eq.(26).

We solve this equation by the Fourier transformation,

$$\tilde{\varphi}(r,\theta,\zeta) = \sum_{m,n} \exp\left(im\theta - in\zeta\right) \int_{-\infty}^{\infty} dk\,\varphi_{mn}(k)\exp(ikx) \tag{28}$$

The mode is microscopic, and the (m,n) component is localized near the relevant rational surface $r = r_s$ (x denotes the distance from the rational surface, $x = r - r_s$). The (m,n) modes are treated separately in Eq.(26), because Eq.(26) is linearlized for the dressed test wave. We solve each (m,n) component and suppress the suffix (m,n) unless necessary. The argument k is the radial mode number in this subsection.

In the vicinity of the rational surface, the parallel mode number is expressed as $k_\parallel = k_\theta s x$ where s is the shear parameter $q^{-2}r(dq/dr)$ and q is the safety factor. By employing the Fourier transformation, x is replaced by the operator i(d/dk). The eigenvalue equation is then given as

$$k_\theta^2 s^2 \frac{d}{dk}\frac{1}{\lambda k_\perp^2}\frac{d}{dk}\,\varphi(k) + \mu_\perp k_\perp^4 \varphi(k) - \frac{k_\theta^2 D_0}{\chi k_\perp^2}\left\{1 - \beta k_\theta^2 s^2 \frac{d}{dk}\frac{1}{\mu_\parallel k_\perp^2}\frac{d}{dk}\frac{1}{\chi k_\perp^2}\right\}\varphi(k) = 0 \tag{29}$$

where the perpendicular wave number is given as

$$k_\perp^2 = k_\theta^2 + k^2 \tag{30}$$

The equation is now expressed in terms of the second order ordinary differential equation with respect to the radial mode number k.

The purpose of this article is to present the analytic insight of the problem. As was examined in the study of the ballooning mode turbulence in tokamaks (Itoh et al. 1994a, Yagi et al. 1993), we employ the similar approximation so as to neglect the first derivative $d\phi/dx$, This approximation yields

$$\left(1 + \beta\frac{\lambda}{\chi}\frac{D_0 k_\theta^2}{\mu_{\parallel}\chi k_\perp^4}\right)\frac{d^2}{dk^2}\varphi - \frac{\lambda\mu_\perp k_\perp^6}{k_\theta^2 s^2}\varphi + \frac{\lambda}{\chi}\frac{D_0}{s^2}\varphi = 0 \tag{31}$$

from Eq.(29). As was shown in previous article, we write χ in the form as

$$\chi = h\,\frac{D_0^{3/2}}{s^2}\frac{\lambda}{\chi}\left(\frac{\chi}{\mu_\perp}\right)^{1/2} \tag{32}$$

where h is the numerical coefficient to be determined by the eigenvalue equation. For the simplicity, we introduce the normalized wave number as

$$b = k_\theta^2\left(\frac{\lambda\mu_\perp}{s^2}\right)^{1/3} \tag{33}$$

and

$$z^2 = k^2\left(\frac{\lambda\mu_\perp}{s^2}\right)^{1/3} \tag{34}$$

By using this normalization, the eigenvalue equation is rewritten as

$$\left(1 + g_0 b(b+z^2)^{-2}\right)\frac{d^2\varphi}{dz^2} + \left\{(H - b^2) - g_1 z^2 - g_2 z^4 - g_3 z^6\right\}\varphi = 0 \tag{35}$$

where the coefficients $\{g_j; j=0\text{-}3\}$ are given as

$$g_0 = \frac{\beta}{D_0}s^2 h^{-4/3} \tag{36}$$

$g_1 = 3b$, $g_2 = 3$ and $g_3 = 1/b$. The term of g_0 is the contribution of the parallel compressibility. This term is the higher order correction with respect to the inverse aspect ratio. Equation (35) is solved in a manner of perturbation, and the term $g_0 b(b+z^2)^{-2}$ is replaced by a constant

$$g = g_0 \langle b(b+z^2)^{-2} \rangle \quad , \tag{37}$$

The bracket $\langle\rangle$ indicates the average

$$\langle f \rangle = \int_{-\infty}^{\infty} f\,|\varphi|^2 dk \left(\int_{-\infty}^{\infty} |\varphi|^2 dk \right)^{-1} \tag{38}$$

the constant g and coefficient g_0 has a similar order of magnitude.

Neglecting g_1 and g_2 as in Yagi et al (1993) and replacing the term g_0 by g, the eigenvalue equation Eq.(35) is solved by the WKB method as

$$\left(\frac{H - b^2}{1 + g} \right)^{2/3} \{b(1+g)\}^{1/6} \int_0^1 \sqrt{1-y^6}\, dy = \frac{\pi}{4} \tag{39}$$

The eigenvalue H is a function of b (i.e., normalized poloidal mode number) as

$$H = b^2 + C b^{-1/4} \tag{40}$$

where the coefficient C is given as

$$C = (1+g)^{3/4} \left(\frac{4}{\pi} \int_0^1 \sqrt{1-y^6}\, dy \right)^{-3/2} \tag{41}$$

The eigenvalue H takes the minimum H* of

$$H^* = 9(C/8)^{8/9} \tag{42}$$

at the mode number satisfying

$$b = b^* = (C/8)^{4/9} \tag{43}$$

This mode number specifies the least stable mode. The coefficient h is given as

$$h = H^{*-3/2}. \tag{44}$$

We here confirm that the correction due to the compressibility is of the higher order in the inverse aspect ratio. In case of g = 0, we have the estimations $b^* \cong 0.43$, $\langle b(b+z^2)^{-2} \rangle \cong 1.4$ and $h \cong 0.8$ (Itoh et al 1993d). By using these numbers, we have

$$g \cong 1.9 \frac{\beta}{D_0} s^2 \tag{46}$$

We here note the relation $D_0 \approx \varepsilon^{-1}(d\beta/dr)\Omega'$ and $\Omega' \approx O(1)$ in the form of normalization. This relation shows that g is of the order ε for the standard plasma profile, $\beta^{-1}(d\beta/dr) \approx O(1)$. This result confirms that the contribution of the parallel compression is small, and the treatment by the perturbation is validated.

4. Transport Coefficient

4.1 Transport Coefficient

Using the result of the stability boundary for the least stable mode, we have the transport coefficient as

$$\chi = \frac{0.8}{1+g} \frac{D_0^{3/2}}{s^2} \frac{\lambda}{\chi} \left(\frac{\chi}{\mu_\perp} \right)^{1/2} \tag{47}$$

This result shows that the transport coefficient is the function of the Prandtl numbers, $\mu_e/\mu_\perp$ and $\chi/\mu_\perp$. The Prandtl numbers are found to be close to unity. For the case of interchange mode, they were given as

$$\mu_e/\mu_\perp \simeq 2.3, \; \chi/\mu_\perp \simeq 2.0 \tag{48}$$

in the limit of g = 0 (Itoh et al 1993d). Since the term with the coefficient g_0 is the correction of the order ε, we substitute these numbers of $\mu_e/\mu_\perp$ and $\chi/\mu_\perp$ into Eq.(47) and have the transport coefficient, in the normalized form , as

$$\chi \simeq \frac{1.3}{1+g} \frac{D_0^{3/2}}{s^2} \tag{49}$$

This result confirms the previous theoretical estimate on the thermal conductivity. The numerical coefficient, 1.3/(1+g), is slightly smaller than the previous article (Itoh et al 1992) in which it was given as 3. It is because that only an order estimate was given analytically in the preceding article. The careful numerical study of the variational equation has also shown the reduction in the numerical coefficient (Miller 1992).

The relation between the various transport coefficients are also obtained. In the stationary turbulent state, the relation

$$\mu_\perp^2 + \mu_\parallel^2 = \mu_e^2 + \chi^2 \qquad (50)$$

holds in the limit of the large aspect ratio. The derivation is given in the appendix B. Using Eqs.(48) and (50), we have

$$\frac{\mu_e}{\mu_\perp} \cong 2.3, \quad \frac{\chi}{\mu_\perp} \cong 2.0, \quad \frac{\mu_\parallel}{\mu_\perp} \cong 2.9 \qquad (51)$$

The transport coefficient is expressed in the physics quantity by substituting the expression of the normalization. The thermal conductivity is given as

$$\chi = F(\hat{r}) \left(\frac{d\beta}{d\hat{r}}\right)^{3/2} \left(\frac{c}{\omega_p}\right)^2 \frac{v_A}{R} \qquad (52)$$

where $\hat{r}$ is the normalized radius, r/a, illustrating that the term $\delta^2 v_A/R$ has the dimension of the diffusivity. The coefficient F is the numerical coefficients representing the geometrical factor and is given as

$$F(\hat{r}) = \frac{1.3}{1+g} \left(\frac{q(\hat{r})}{dq/d\hat{r}}\right)^2 \left(\frac{R^2}{2a^2} \frac{d\Omega}{d\hat{r}}\right)^{3/2} \qquad (53)$$

In the limit of high-aspect-ratio torsatron/Heliotron configuration, the magnetic structure is approximately expressed by use of the Bessel function (Solov'ev and Shafranov 1970). In such a case, an analytic estimate yields

$$\frac{d\Omega}{d\hat{r}} \cong \frac{m}{\ell} \left(\frac{a}{R}\right)^2 \frac{1}{\hat{r}^2} \frac{d}{d\hat{r}} \left(\frac{\hat{r}^4}{q}\right) \qquad (54)$$

where m is the toroidal pitch number and ℓ is the polarity of the helical windings, respectively. This gives an analytic expression of the geometrical factor F as

$$F(\hat{r}) = 1.3\left(\frac{q(\hat{r})^2}{dq/d\hat{r}}\right)^2\left(\frac{m}{2\ell}\frac{1}{\hat{r}^2}\frac{d}{d\hat{r}}\left(\frac{\hat{r}^4}{q}\right)\right)^{3/2} \tag{55}$$

where the coefficient 1.3/(1+g) is replaced by 1.3, since g is the correction of the order of ε.

It is noted that this formula of the thermal conductivity χ has the same β' dependence as that obtained for tokamaks. The geometrical dependence appears in the term F. The important role of the magnetic shear is shown. As has been known widely, the magnetic shear is inevitable in the system with magnetic hill. The anomalous transport coefficient is predicted to depend like s^{-2}. When the magnetic shear is weak, the transport coefficient becomes very large. This is in contrast to the case of tokamaks and stellarators (Itoh et al 1994a, Fukuyama et al. 1994a).

4.2 Comparison with Experiments

The prediction of the theory is discussed, comparing to the experimental results. Firstly, the dimensional dependence of χ is

$$[\chi] \propto [T]^{1.5}[R]^{-1}[B]^{-2} \tag{56}$$

and is independent of that of density [n]. Secondly, the formula of χ includes the radial dependence of $(\beta'/n)^{3/2}$, not $T^{3/2}$, and predicts a larger transport coefficient near edge. The radial profile of F is slightly decreasing towards edge, but the overall radial dependence can be increasing toward edge, because the profile of $(\beta'/n)^{3/2}$ is an increasing function towards the edge. Figure 2 illustrates a typical example of the radial profile of the predicted thermal conductivity. The theoretical predictions of χ, (i) that χ is larger near edge and (ii) that it is larger for higher temperature at fixed radial location, are consistent with the results of experiments (Sano et al. 1990, Wagner et al 1993).

Third, the thermal conductivity which is deduced from heat pulse propagation, χ_{HP}, can be larger than the thermal conductivity at the steady state. If the density profile is much flatter than that of the temperature, and if only temperature is modulated at the heat pulse, the theoretical formula of χ gives the relation

$$\chi_{HP} \simeq 2.5\,\chi \tag{57}$$

as was discussed in the case of tokamaks. A larger thermal conductivity for the heat pulse, that the power balance analysis, was observed in Heliotron-E experiments (Zushi 1991).

The theory predicts that the ion viscosity is also enhanced to the level of thermal conductivity. Anomalous ion shear viscosity is observed recently in CHS device, and the relation $\mu_{\parallel} \simeq \chi$ was observed (Ida et al 1992). This observation is consistent with theory. The dependence of the ion viscosity on various plasma parameters would be studied in future and then more detailed comparison would be possible.

The geometrical dependence explains couples for differences, compared to tokamaks, in helical systems.

The point model analysis gives the energy transport scaling law as

$$\tau_E \propto A_i^{0.2} B^{0.8} n^{0.6} a^2 R P^{-0.6} \langle F \rangle^{-0.4} \tag{58}$$

where A_i is the ion mass number, P is the heating power and $\langle F \rangle$ is the average of F near the boundary. (The reason to choose the edge value is discussed in Itoh et al 1991.) By comparing the various configurations with different pitch number and aspect ratio, we find that the improvement of the confinement by the increase of the shear (s^{-2} term in F) is almost completely offset by the increment of the magnetic hill Ω'. The coefficient F weakly depends on geometrical parameters such as pitch numbers. It is concluded that the energy confinement time τ_E depends on the toroidal magnetic field, not on the poloidal magnetic field. This result may explain the fact that τ_E seems to depend only weakly on the rotational transform or on magnetic shear in experimental data. The predicted indices to B, n, a, R and P, as a whole, are consistent with the experimental scaling law (Sudo et al 1990).

The geometrical dependence of the confinement time is recently studied in Heliotron-E and CHS devices by changing the location of the magnetic axis. When the magnetic axis is shifted inward or outward, by applying the vertical field, the magnetic shear and well change. The competition between the change in shear and well is shown in Fig 3 for the case of CHS device. When the magnetic axis is shifted inward, the increment of the shear dominates than the increment of hill, so that the minimum of the factor F appears. When the axis is shifted far outside, the coefficient is predicted again to decrease. This is due to the reduction of the magnetic hill. This reduction of hill has been discussed in relation with the MHD instability in torsatron/Heliotron configurations (See, e.g., Wakatani et al 1992). The trend of τ_E was observed in experiments that τ_E shows optimum when the axis is shifted inward (Obiki et al. 1991, Kaneko et al. 1991), supporting, qualitatively, the theoretical result. However, τ_E continuously decreases when the axis is shifted outward, and no turn-over has been observed in Heliotron-E and CHS experiments. This bad confinement in the case of outer shift would be partly attributed to the poor heating efficiency (see e.g., Hanatani et al. 1992, Itoh et al 1991b). Recently, it was also found that the particle pinch changes sign (i.e., there is outward flow) when the plasma is placed outside (Iguchi et al. 1994). It is possible that there is other additional loss mechanisms than the thermal diffusion in the case of the outward shift.

We finally note the characteristics of fluctuations. The theory predicts the relation between the relative density perturbation to the potential perturbation. It was calculated from the relation $\tilde{n}/n \simeq (\omega_*/\gamma)e\varphi/T$, giving

$$\frac{\tilde{n}}{n} \simeq \left(\frac{3|q'|}{q\bar{D}_0}\frac{\beta(a)R}{L_n}\right)\frac{e\varphi}{T} \tag{59}$$

where L_n is defined as $n'/n = -1/L_n$ and $\bar{D}_0 = -(R^2/2a^2)\,(d\beta/d\hat{r})\,(d\Omega/d\hat{r})$. For the case of Heliotron-E plasma, $\bar{D}_0 \simeq 60a\beta(0)/L_p$ and $q'/q \approx 4$ hold (L_p being the pressure gradient scale length), and we have

$$\frac{\tilde{n}}{n} \simeq \left(\frac{2L_p}{L_n}\frac{\beta(a)}{\beta(0)}\right)\frac{e\varphi}{T} \tag{60}$$

The term in the bracket is order 1/10, showing that density fluctuation is smaller than the potential fluctuations. (This means that the correction remains small, supporting the neglect of ω_* term in the equation of motion.) The ratio of the density fluctuation to potential fluctuation is predicted to be larger if the pressure profile becomes broader. Fluctuation measurements in high power heating experiments have shown that $\tilde{n}/n$ is smaller than $e\phi/T$ (Zushi et al 1988, Ritz et al 1991), which is consistent with our theory. The result of the broader profile is also consistent with the observation by Zushi.

5. Transport Coefficient in Stellarators

The transport analysis in the system with magnetic well was derived in the previous article (Itoh et al 1994a). We here characterize the magnetic configuration of the stellarator plasma only by the magnetic well and magnetic shear. There is a possibility that the helical deformation influences the anomalous transport. However, we here neglect the influence of the helical deformation of the magnetic structure on the microscopic instabilities.

The theory of the self-sustained turbulence in the case of finite magnetic well was developed. The thermal transport coefficient for the system with weak magnetic shear was given as

$$\chi = F_s(\hat{r})\, q^2 \left(R\frac{d\beta}{dr}\right)^{3/2} \left(\frac{c}{\omega_p}\right)^2 \frac{v_A}{R} \tag{61}$$

where F_S denotes the geometrical factor for stellarators. This coefficient was obtained, by considering the anisotropy of the fluctuations, which is noticeable when the shear is weak. (See appendix in Itoh et al 1993c for the detailed derivation.) We have

$$F_s = \frac{1}{\sqrt{(1-2\hat{s})\left[(2+4\hat{s}^2 s^{-2})(1-2\hat{s})+2\hat{s}^2\right]}} \tag{62}$$

where $\hat{s} = r(dq/dr)q^{-1}$ and $\hat{s} = s - \alpha$ and α is defined as $\alpha = q^2R(d\beta/dr)$. This result shows that the thermal conductivity remains small numbers even though the shear is weak. It is noted that the anisotropy of the turbulence is characterized as

$$\frac{\langle k_\theta^2 \rangle}{\langle k_r^2 \rangle} = \frac{1-2\hat{s}}{s^2} \frac{\left(1+4\hat{s}^2 s^{-2}\right) + 2\hat{s}^2}{\left(1+2\hat{s}^2 s^{-2}\right) + \hat{s}^2} \tag{63}$$

The result shows that the wave number is dominated by the poloidal component when the magnetic shear is weak. This also implies that, in the weak shear limit, the radial wave length becomes longer, having the characteristics of the global mode. As the typical poloidal mode number is O(10) to O(100), the analytic theory which assumes the localization of the mode may be invalid when the magnitude of the shear parameter $|s|$ is O(1/100). The case of $|\hat{s}| \to 0$ is given by the extrapolation of the results of $|\hat{s}| \neq 0$.

We compare the transport coefficient, Eqs.(61) and (62) to those of tokamaks and helical systems. Generally speaking, the similarity to tokamaks is prominent, rather than to helical systems. This is due to the fact that the magnetic well suppresses the interchange instability and the ballooning mode is dictating the anomalous transport in stellarators and tokamaks.

The prediction of the theory for stellarators is as follows.

(i) The dimensional dependence is

$$[\chi] \propto [T]^{1.5}[R]^{-1}[B]^{-2} \tag{64}$$

as in the case of helical systems.

(ii) The gradient of pressure generates χ. Therefore χ can have a large value near edge where temperature is low. The same q-dependence is obtained as in the case of tokamaks. However, the radial profile of the safety factor is much flatter compared to tokamaks, and so is the geometrical factor associated with the magnetic configuration, $F_s q^2$. The radial shape of χ is governed by the dependence $(\beta'/n)^{3/2}$. For usual plasma profiles, this quantity is weakly increasing towards edge. The thermal transport coefficient is increasing towards edge in stellarators, but is much flatter than in the case of tokamaks.

(iii) The estimate of the energy confinement time is given as

$$\tau_E \propto A_i^{0.2} B^{0.8} q^{-0.8} n^{0.6} a^2 R^{0.4} P^{-0.6} L_p^{0.6} \tag{65}$$

This result predicts a positive dependence on the rotational transform. The power degradation is also recovered.

(iv) The peaking parameter of temperature, $T_e(0)/\langle T_e\rangle$, is predicted to show only weak dependence on the edge safety factor. This is in contrast to the theoretical result of tokamaks. This is because the profile of the safety factor is almost flat in stellarators, and the shear parameter does not depend on the parameter q(a) strongly. [In tokamaks, the shear parameter is a strong function of the edge q value, so that the theory predicts the peaking of temperature at high edge q-value. See Itoh et al 1994a and Fukuyama 1994b.]

(v) The similar argument to helical systems is given for the thermal conductivity deduced from the heat pulse propagation. If only the temperature is perturbed, enhancement of χ_{HP} over χ is predicted from the theory.

(vi) The typical perpendicular wave number of the most unstable mode is given as $k_\perp \delta \simeq 1/\sqrt{\alpha}$. Due to the weak shear, radial correlation length is predicted much longer than the poloidal correlation length. The estimate of the fluctuation level was given as

$$\frac{e\varphi}{T} \simeq \chi \frac{eB}{T} \tag{66}$$

This relation implies that the level of turbulence is predicted in the range of strong turbulence. Compared to χ, there appear an additional dependence of eB/T. Since the temperature T is a decreasing function of radius, the level of fluctuations are much larger towards edge. This radial increment is much stronger than χ itself.

These predictions are compared to experimental observations. First, the dependence of χ was studied. The dependence like $[\chi] \propto [T]^{1.5}[R]^{-1}[B]^{-2}$ seems to be consistent with observations. Detailed information on the radial profile is published (Wagner et al 1993). The experimental observation is characterized that the radial profile is weakly increasing towards edge; this is clearly different from the prediction of the neoclassical theory. The radial shape is closer to flat than in the case of tokamaks. This nature is in accord with theoretical result. The parameter dependence of the energy confinement time is also compared. The dependencies on the power, density and magnetic field seem to show satisfactory agreement.

The comparison of the dependence on the edge safety factor is not straightforward. This is because the strong corrugation in the q(a) dependence of τ_E has been observed in experiments on W7-A and W7-AS. One of the reason for the sharp reduction of τ_E near the lower order rational number of q (i.e., when q(a) is rational number and is written as m_1/n_1, n_1 is a small integer) would be the destruction of the magnetic surfaces (See for instance, Wobig et al. 1987 and Jaenicke et al 1993). Therefore our theory may not reproduce experiments well when q(a) is a lower order rational number. If the q(a) dependence of τ_E is deduced from experiments by choosing the local peaks of τ_E, the dependence such as $\tau_E \propto q(a)^{-0.6}$ is obtained (Brakel et al 1992). This observation is consistent with the prediction by the theory.

The experimental observations have shown that the q-dependence of the peaking parameter is not prominent in stellarators (Wagner et al 1993). It seems to be consistent with theoretical prediction. This would support that the difference in the shear profile causes the distinction of the temperature profile in tokamaks and stellarators. More careful study will be performed in future.

A noticeable difference is seen in W7-As experiment in comparison with tokamaks. The thermal conductivity deduced from the heat pulse propagation was found to be almost equal to χ of the power balance. One may consider that this seems to be a contradiction to the theoretical prediction. It is noted, however, the relation $\chi_{HP} \approx 2.5\chi$ is obtained in theory by the assumption that only the temperature is perturbed. In the real experiments, not only the temperature but also the radial electric field inhomogeneity could be modulated. As was shown in Itoh et al 1994b, the influence of the radial electric field inhomogeneity is much effective in stellarators than in tokamaks. (This is due to the weak shear in stellarators.) The mechanism, that the associated perturbation in the radial electric field inhomogeneity suppresses the increment of χ_{HP} over χ, will be discussed in a separate article.

6. Summary And Discussion

In this article we develop the theory of self-sustained turbulence and the associate anomalous transport in the toroidal helical plasmas. The study is made for the torsatron/Heliotron plasmas and for the stellarator plasmas. The former configuration is characterized by the magnetic hill and strong magnetic shear. In such a case, the interchange mode turbulence can develop. In the latter configuration, the magnetic well exists and the turbulence is composed of the ballooning modes. The results for theses configurations, combined with that for tokamaks, clarifies the role of configurations on the anomalous transport coefficient.

The theory of self-sustained turbulence is extended in such a manner that the effect of plasma compression along the magnetic field line is taken into account. The nonlinear theory for the four field model of the plasma is developed. The ion viscosity for the parallel velocity is newly obtained by this extension. The similar procedure like the previous article is developed. By the help of the renormalization and mean-field approximation, the model equation is derived for the dressed test waves. The equation is linearlized for the dressed test wave; all the nonlinearlities are renormalized in a form of the diffusion coefficient. The main conclusion of this extension is summarized as follows. The effect of the parallel compressibility causes the reduction in the anomalous transport coefficient. However, the correction is higher order with respect to the inverse aspect ratio. This confirms the validity of the previous work in which the parallel compression was neglected. The diffusivity of the parallel momentum is found to be close to that of the cross-field momentum. The simplified expression is obtained as

$$\mu_\perp : \mu_e : \chi : \mu_\parallel \simeq 1 : 2.3 : 2 : 3 \quad (67)$$

This result provides a basis for the study of the dynamics associated with the L-H transition, where the diffusion of the parallel and perpendicular momenta play important roles (Itoh et al 1988, 1992b).

The formula of thermal conductivity is obtained for helical systems and stellarators. The thermal conductivity in toroidal systems can be generally given as

$$\chi = \bar{Q}\left(\frac{d\beta}{d\hat{r}}\right)^{3/2}\left(\frac{c}{\omega_p}\right)^2\frac{v_A}{R} \tag{68}$$

and the influence of the magnetic configuration (tokamak, stellarator, helical system) can be summarized in the coefficient $\bar{Q}$. In the helical systems, the balance between the shear and magnetic hill determines the geometrical factor. For the system with $\ell = 2$, the shear may be increased by increasing the pitch number m. However, the increment in m is associated with the enhancement in the magnetic hill. The gain by the shear is almost offset by the increment in the hill. In the stellarator plasmas, the dependence on the rotational transform is predicted. This implies that the energy confinement time will be enhanced by increasing the rotational transform. An additional method to improve the confinement is predicted. The equation (62) shows that if the shear is negative, (i.e., $dq/dr<0$) the thermal conductivity is further reduced. If the central rotational transform is kept smaller than the edge value in stellarators (for instance by driving small current near the axis), the energy confinement time will be improved. The role of the radial electric field was analyzed in Itoh et al (1994b,c). Combining these analyses, it is expected to progress the understanding of the anomalous transport phenomena.

The theoretical framework of the self-sustained turbulence is enlarged in this article. However, there are still lot of tasks which requires future theoretical research. One is the influence of the finite gyro radius effect and coupling with the drift waves. For the case of helical systems, we have the estimates ω_*/γ is O(1/10), showing that these effect are small. This relation may not, however, hold for all cases and the analysis on the finite-gyro radius effect is necessary. Also important would be the effect of the incoherent convection of the turbulence. It was pointed out that the turbulence level is affected by this effect, and the method of two-scale direct interaction approximation has been developed (Yoshizawa 1985). The kinetic treatment of the problem needs further research. Comparing the result of Ohkawa with that of the kinetic treatments in the slab plasmas (Kadomtsev et al 1985), we see that the generic feature is not affected by the kinetic plasma response. However, there can appear a difference on the parameter v_e/v_A as was shown by Connor (1993). Finally, the basic picture of the nonlinear destabilization by the convective nonlinearlity of the current must be studied by nonlinear simulation. Preliminary study has been performed by Yagi et al 1994. It was confirmed that when the turbulence level reaches a certain threshold, the nonlinear destabilization occur. Due to this nonlinear destabilization, the level of turbulence grows much larger than the value which is predicted by the balance between the linear growth and nonlinear ion viscosity and thermal conductivity. The study for the anomalous transport still requires much theoretical efforts.

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Appendix A: Equation for the Test Wave

Taking the back interaction with the driven mode (k2), the equation for the test mode (denoted in this appendix by tilde as $\tilde{\varphi}$) is given as

$$\frac{\partial \tilde{U}}{\partial t} + [\varphi_0, \tilde{U}] + \mu_c \nabla_\perp^2 \tilde{U} - \nabla_\parallel \tilde{J} - (\Omega' \times \hat{\zeta}) \cdot \nabla \tilde{p} = -\sum_{k_1} [\varphi_{-1}, U_2] \quad \text{(A1)}$$

$$\frac{\partial \tilde{\Psi}}{\partial t} + \nabla_\parallel \tilde{\varphi} - \frac{1}{\xi}\left(\frac{\partial \tilde{J}}{\partial t} + [\varphi_0, \tilde{J}]\right) + \eta \tilde{J} - \lambda_c \nabla_\perp^2 \tilde{J} = \frac{1}{\xi} \sum_{k_1} [\varphi_{-1}, J_2] \quad \text{(A2)}$$

$$\frac{\partial \tilde{p}}{\partial t} + [\varphi_0, \tilde{p}] + \beta \nabla_\parallel \tilde{v} - \chi_c \nabla_\perp^2 \tilde{p} = -\sum_{k_1} [\varphi_{-1}, p_2] \quad \text{(A3)}$$

$$\frac{\partial \tilde{v}}{\partial t} + [\varphi_0, \tilde{v}] + \nabla_\parallel \tilde{p} - \mu_{\parallel c} \nabla_\perp^2 \tilde{v} = -\sum_{k_1} [\varphi_{-1}, v_2] \quad \text{(A4)}$$

where the suffix 0 denotes the equilibrium distribution, -1 denotes the mode $-k_1$, and 2 denotes the driven mode k_2.

The right hand side of Eqs.(A1)-(A4) shows the contribution of the back ground fluctuations. The result of the driven mode is expressed, symbolically, as

$$\begin{pmatrix} U_2 \\ J_2 \\ p_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{pmatrix} \begin{pmatrix} [\varphi_1, \tilde{U}] \\ [\varphi_1, \tilde{J}] \\ [\varphi_1, \tilde{p}] \\ [\varphi_1, \tilde{v}] \end{pmatrix} \quad \text{(A5)}$$

Explicit form of the matrix elements $\{H_{ij}\ (i,j:1\text{-}4)\}$ is seen from Eqs.(5)-(9). We have

$$H_{11} = \frac{-1}{K_2}, \quad H_{12} = -\frac{ik_{\parallel 2}}{K_2 \gamma_{j2}}, \quad H_{13} = -\frac{iA_2 \gamma_{v2}}{K_2 \Gamma_{vp2}}, \quad H_{14} = \frac{iA_2 \beta k_{\parallel 2}}{K_2 \Gamma_{vp2}}$$

$$H_{21} = -\frac{ik_{\parallel 2} \xi}{K_2 \gamma_{j2} k_{\perp 2}^2}, \quad H_{22} = -\left[\frac{1}{\gamma_{j2}} - \frac{\xi k_{\parallel 2}^2}{K_2 \gamma_{j2}^2 k_{\perp 2}^2}\right]$$

$$H_{23} = \frac{\xi A_2 \gamma_{v2} k_{\parallel 2}}{K_2 \gamma_{j2} k_{\perp 2}^2 \Gamma_{vp2}}, \quad H_{24} = -\frac{\xi A_2 \beta k_{\parallel 2}^2}{K_2 \gamma_{j2} k_{\perp 2}^2 \Gamma_{vp2}}$$

$$H_{31} = \frac{i\gamma_{v2} G_2}{K_2 \Gamma_{vp2} k_{\perp 2}^2}, \quad H_{32} = -\frac{G_2 \gamma_{v2} k_{\parallel 2}}{K_2 \Gamma_{vp2} \gamma_{j2} k_{\perp}^2}$$

$$H_{33} = -\frac{\gamma_{v2}}{\Gamma_{vp2}}\left[1 + \frac{G_2 A_2 \gamma_{v2}}{K_2 k_{\perp 2}^2 \Gamma_{vp2}}\right] \quad H_{34} = \frac{\beta k_{\parallel 2}}{\Gamma_{vp2}}\left[\frac{G_2 A_2 \gamma_{v2}}{K_2 k_{\perp 2}^2 \Gamma_{vp2}} - i\right]$$

$$H_{41} = \frac{k_{\parallel 2} G_2}{K_2 \Gamma_{vp2} k_{\perp 2}^2}, \qquad H_{42} = \frac{i G_2 k_{\parallel 2}^2}{K_2 \Gamma_{vp2} \gamma_{jv2} k_{\perp 2}^2},$$

$$H_{43} = \frac{i k_{\parallel 2}}{\Gamma_{vp2}} \left[1 + \frac{G_2 A_2 \gamma_{v2}}{K_2 k_{\perp 2}^2 \Gamma_{vp2}} \right], \qquad H_{44} = -\frac{1}{\Gamma_{vp2}} \left[\frac{i G_2 A_2 \beta k_{\parallel 2}^2}{K_2 k_{\perp 2}^2 \Gamma_{vp2}} + \gamma_{p2} \right] \qquad \text{(A6)}$$

The right hand side of Eqs.(A1)-(A4) are given as

$$\begin{pmatrix} [\varphi_{-1}, U_2] \\ [\varphi_{-1}, J_2] \\ [\varphi_{-1}, p_2] \\ [\varphi_{-1}, v_2] \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{pmatrix} \begin{pmatrix} [\varphi_{-1}, [\varphi_1, \tilde{U}]] \\ [\varphi_{-1}, [\varphi_1, \tilde{J}]] \\ [\varphi_{-1}, [\varphi_1, \tilde{p}]] \\ [\varphi_{-1}, [\varphi_1, \tilde{v}]] \end{pmatrix} \qquad \text{(A7)}$$

As in the previous article, we assume that the typical wave length is much shorter than the radial scale length of the envelop of the turbulence. It is also assumed that the convective transport of the wave is not important. Under these circumstances, we have the estimate (Itoh et al. 1993c)

$$[\varphi_{-1}, [\varphi_1, \tilde{Y}]] = \left(\left| \frac{\partial \varphi_1}{\partial r} \right|^2 \frac{\partial^2}{r^2 \partial \theta^2} + \left| \frac{\partial \varphi_1}{r \partial \theta} \right|^2 \frac{\partial^2}{\partial r^2} \right) \tilde{Y} \qquad \text{(A8)}$$

The assumption of the isotropic turbulence is employed as

$$\left| \frac{\partial \varphi_1}{\partial r} \right|^2 \simeq \left| \frac{\partial \varphi_1}{r \partial \theta} \right|^2 \simeq \frac{|k_{\perp 1} \varphi_1|^2}{2} \qquad \text{(A9)}$$

and the operator $[\phi_{-1}, [\phi_1, \tilde{Y}\ ]]$ is replaced by the diffusion operator. The nonlinear terms in Eqs.(A1)-(A4) are given as

$$\sum_{k_1} \begin{pmatrix} [\varphi_{-1}, U_2] \\ [\varphi_{-1}, J_2] \\ [\varphi_{-1}, p_2] \\ [\varphi_{-1}, v_2] \end{pmatrix} = \sum_{k_1} \frac{|k_{\perp 1} \varphi_1|^2}{2} \begin{pmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{pmatrix} \nabla_\perp^2 \begin{pmatrix} \tilde{U} \\ \tilde{J} \\ \tilde{p} \\ \tilde{v} \end{pmatrix} \qquad \text{(A10)}$$

As in the previous article (Itoh et al 1993c), we take the diagonal terms. Substituting Eq.(A10) into Eqs.(A1)-(A4), we have

$$\frac{\partial \tilde{U}}{\partial t} + [\varphi_0, \tilde{U}] + \mu_c \nabla_\perp^2 \tilde{U} - \nabla_\parallel \tilde{J} - (\Omega' \times \zeta) \cdot \nabla \tilde{p} = \mu_{\perp a} \nabla_\perp^2 \tilde{U} \qquad \text{(A11)}$$

$$\frac{\partial \tilde{\Psi}}{\partial t} + \nabla_{\parallel} \tilde{\varphi} - \frac{1}{\xi}\left(\frac{\partial \tilde{J}}{\partial t} + [\varphi_0, \tilde{J}]\right) + \eta \tilde{J} - \lambda_c \nabla_{\perp}^2 \tilde{J} = \frac{1}{\xi}\mu_e \nabla_{\perp}^2 \tilde{J} \qquad \text{(A12)}$$

$$\frac{\partial \tilde{p}}{\partial t} + [\varphi_0, \tilde{p}] + \beta \nabla_{\parallel} \tilde{v} - \chi_c \nabla_{\perp}^2 \tilde{p} = \chi_a \nabla_{\perp}^2 \tilde{p} \qquad \text{(A13)}$$

$$\frac{\partial \tilde{v}}{\partial t} + [\varphi_0, \tilde{v}] + \nabla_{\parallel} \tilde{p} - \mu_{\parallel c} \nabla_{\perp}^2 \tilde{v} = \mu_{\parallel a} \nabla_{\perp}^2 \tilde{v} \qquad \text{(A14)}$$

where the suffix a denotes the contribution of the ExB nonlinearlity. The coefficients are given as

$$\mu_{\perp a} = \sum_{k_1} \frac{|k_{\perp 1}\varphi_1|^2}{2} H_{11} \qquad \text{(A15-1)}$$

$$\mu_{ea} = \sum_{k_1} \frac{|k_{\perp 1}\varphi_1|^2}{2} H_{22} \qquad \text{(A15-2)}$$

$$\chi_a = \sum_{k_1} \frac{|k_{\perp 1}\varphi_1|^2}{2} H_{33} \qquad \text{(A15-3)}$$

and

$$\mu_{\parallel a} = \sum_{k_1} \frac{|k_{\perp 1}\varphi_1|^2}{2} H_{44} \qquad \text{(A15-4)}$$

The transport coefficients $\{\mu_{\perp a}, \mu_{ea}, \chi_a, \mu_{\parallel a}\}$, in principle, depend on the wave number of the test wave, since the coefficients H_{ij} depends on the wave number of the beat mode, $k_2 = k + k_1$.

Summing up the classical term and ExB nonlinear term, the total diffusion coefficients are obtained.

Appendix B: Ratio of Transport Coefficients

It is noted that the anomalous terms $\{\mu_{\perp}, \mu_e, \chi, \mu_{\parallel}\}$ are in the same order of magnitude. The Prandtl number has been calculated for three field models (Itoh et al

1993d). The ratios between ion viscosity of parallel motion and perpendicular motion is derived from the four filed models discussed here.

The transport coefficients are derived as

$$\mu_{\perp} = \sum_{k_1} \frac{|k_{\perp 1}\varphi_1|^2}{2K_1} \tag{B1}$$

$$\mu_e = \sum_{k_1} \frac{|k_{\perp 1}\varphi_1|^2}{2K_1} \frac{\gamma_{u1}}{\gamma_{j1}} \left[1 - \frac{\gamma_{v1}}{\gamma_{u1}} \frac{A_1 G_1}{\Gamma_{vp1} k_{\perp 1}^2}\right], \tag{B2}$$

$$\chi = \sum_{k_1} \frac{|k_{\perp 1}\varphi_1|^2}{2K_1} \frac{\gamma_{u1}\gamma_{v1}}{\Gamma_{vp1}} \left[1 + \frac{\xi k_{\parallel 1}^2}{k_{\perp 1}^2 \gamma_{u1}\gamma_{j1}}\right] \tag{B3}$$

and

$$\mu_{\parallel} = \sum_{k_1} \frac{|k_{\perp 1}\varphi_1|^2}{2} \frac{1}{\Gamma_{vp1}} \left[\gamma_{p1} - \frac{i G_1 A_1 \beta k_{\parallel 1}^2}{K_1 k_{\perp 1}^2 \Gamma_{vp1}}\right] \tag{B4}$$

It is shown that the compressibility effect is the higher order in inverse aspect ratio. We therefore approximate $\Gamma_{pv} = \gamma_p \gamma_v$ and neglect β terms in Eq.(B4) and K_1. The terms $\{\gamma_u, \gamma_j, \gamma_p, \gamma_v\}$ are given by $\{\mu_{\perp} k_{\perp 1}^2, \mu_e k_{\perp 1}^2, \chi k_{\perp 1}^2, \mu_{\parallel} k_{\perp 1}^2\}$ in the stationary turbulence which is realized by the marginal stability condition. Substituting these relations into Eqs.(B1)-(B4), we have

$$\frac{\mu_e^2}{\mu_{\perp}} = \sum_{k_1} \frac{|k_{\perp 1}\varphi_1|^2}{2K_1} \left[1 - \frac{A_1 G_1}{\gamma_{u1}\gamma_{j2} k_{\perp 2}^2}\right], \tag{B5}$$

$$\frac{\chi^2}{\mu_{\perp}} = \sum_{k_1} \frac{|k_{\perp 1}\varphi_1|^2}{2K_1} \left[1 + \frac{\xi k_{\parallel 1}^2}{\gamma_{u1}\gamma_{j1} k_{\perp 1}^2}\right] \tag{B6}$$

$$\frac{\mu_{\parallel}^2}{\mu_{\perp}} = \sum_{k_1} \frac{|k_{\perp 1}\varphi_1|^2}{2K_1} \left[1 + \frac{k_{\parallel 1}^2}{k_{\perp 1}^2} \frac{\xi}{\gamma_{u1}\gamma_{j1}} - \frac{1}{k_{\perp 1}^2} \frac{A_1 G_1}{\gamma_{u1}\gamma_{p1}}\right] \tag{B7}$$

where we neglect the $O(\beta)$ term in Eq.(B4), which was found to be higher order correction of ε(see eq.(46)). From these relations we see that a sum rule holds as

$$\frac{\mu_e^2}{\mu_\perp} + \frac{\chi^2}{\mu_\perp} = \frac{\mu_\parallel^2}{\mu_\perp} + \mu_\perp \tag{B8}$$

or

$$\mu_e^2 + \chi^2 = \mu_\perp^2 + \mu_\parallel^2 \tag{B9}$$

The Prandtl numbers, μ_e/μ and χ/μ, were obtained for the case of interchange mode. They were given as (Itoh et al 1993d)

$$\mu_e/\mu_\perp \simeq 2.3,\ \chi/\mu_\perp \simeq 2.0 \tag{B10}$$

Substituting Eq.(B10) into Eq.(B9), we have the relation between various transport coefficients as

$$\frac{\mu_e}{\mu_\perp} \cong 2.3,\ \ \frac{\chi}{\mu_\perp} \cong 2.0,\ \ \frac{\mu_\parallel}{\mu_\perp} \cong 2.9 \tag{B11}$$

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Figure Captions

Fig.1 Schematic drawing of the growth rate of the test mode as a function of the fluctuation level. Solid line indicates the nonlinear instability leading to the self-sustained turbulence state. The dashed line denotes the conventional model.

Fig.2 Radial profile of the thermal conductivity for the case of Heliotron-E plasma. Profiles are chosen such that $p_0(\hat{r}) = p_0(0)[1 - \hat{r}^2 + \Delta]$, $T(\hat{r})/T(0) = n(\hat{r})/n(0)$, $\Delta = 0.05$, $T(0) = 500$ eV, $B = 2$T, $n(0) = 5\times10^{19}\,m^{-3}$, and $\varepsilon = 0.1$. The hatched region indicates the experimental observation. The solid line shows 2.2 times of the theoretical result. Dotted line employs a factor 9.

Fig.3 Geometrical form factor F of the thermal conductivity at r/a=0.9 is shown as a function of the location of the major axis, R_{ax}, for CHS plasma (in an arbitrary unit). Arrows on the abscissa indicate the condition of the maximum plasma volume (a) and that the bumpiness of the toroidal field vanishes at the axis (b), respectively. The quality of the confinement becomes optimum at $R_{ax} \simeq 0.93$ m.

Fig. 1

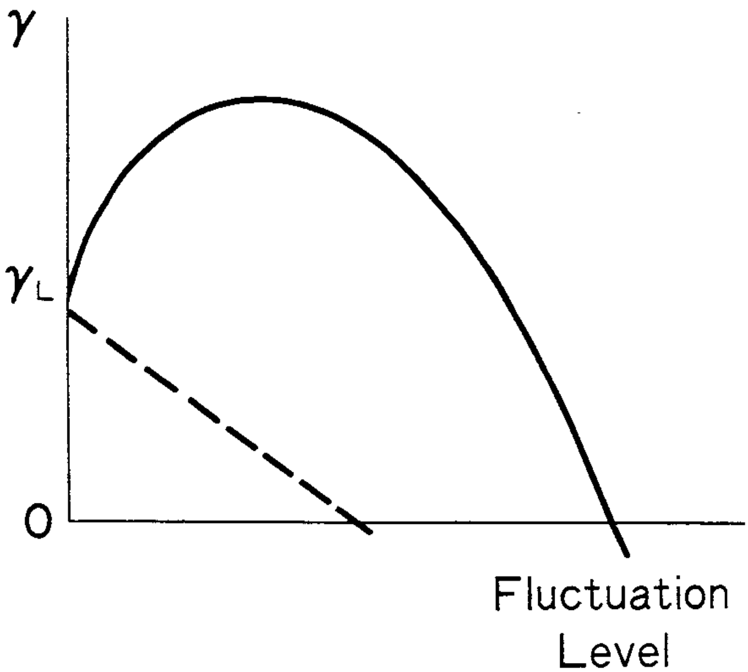


Fig.2

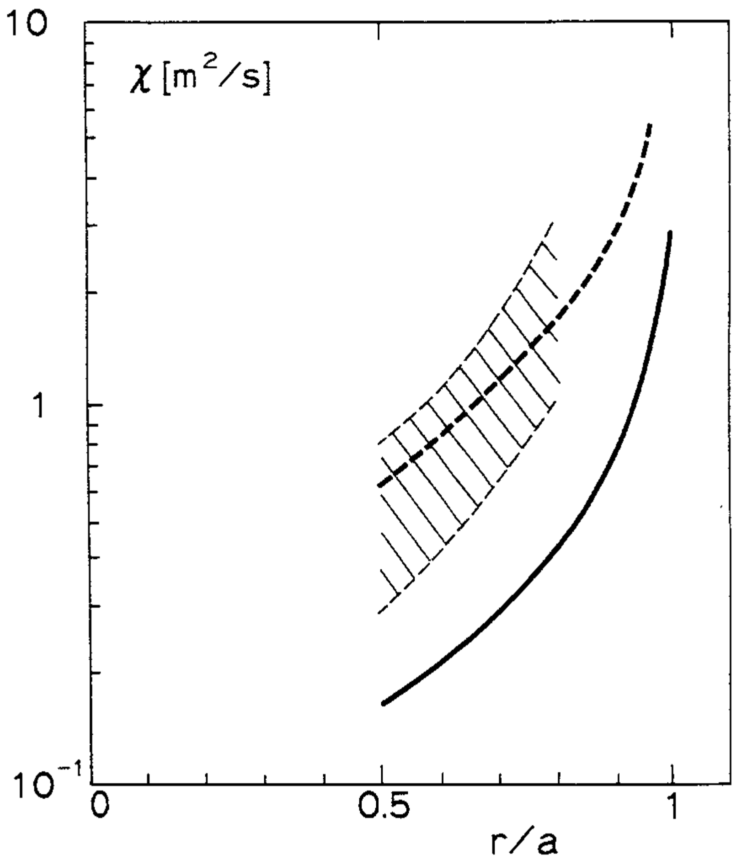
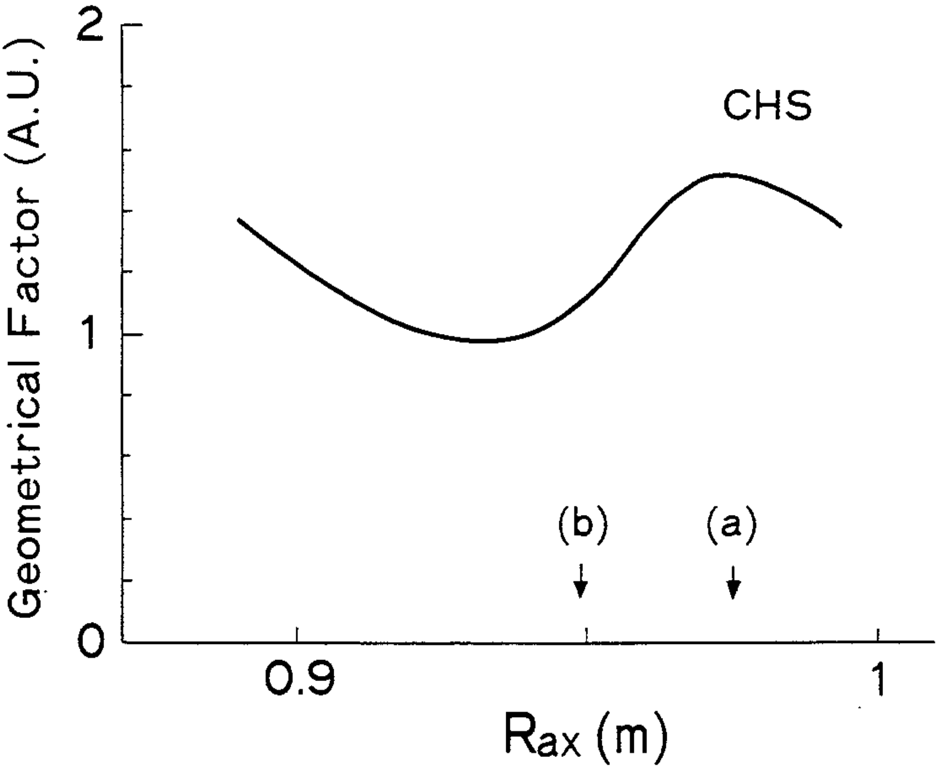


Fig.3



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