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# EFFECT OF SATELLITE HELICAL HARMONICS ON THE STELLARATOR CONFIGURATION

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ABSTRACT

We discuss the problem of self-consistent analytical description of stellarators with a helical field which has, besides the main harmonic, two nearest satellites with the same period in the toroidal angle. The expression for the flux function explicitly incorporating the effect of such satellites on the shape of magnetic surfaces is obtained. It is shown that they produce the shift of magnetic surfaces. The expression for this shift is derived. Two problems are considered where the behavior of $\mathbf{B}^2$ on a magnetic surface is important: Pfirsch-Schlüter current at the presence of satellites and possibility of fulfillment of quasisymmetry condition ($\mathbf{B}^2$ on a magnetic surface does not depend on one of the angular Boozer coordinates) at least at a single magnetic surface. It is shown that effect of satellite harmonics on the magnitude of Pfirsch-Schlüter current turns out to be much smaller than it was predicted earlier on the basis of simplified model where the shift related with satellites was not taken into account. It is shown that quasisymmetry condition in a configuration with two satellites can be fulfilled only in linear approximation in helical field. The analysis is performed for conventional stellarators with planar circular axis making use of stellarator expansion.

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# 1. INTRODUCTION

The spectrum of the helical field in a real stellarator always contains, besides the main harmonic, several satellite ones [1-3]. Even if their amplitudes are small, in numerical calculations sometimes up to several dozens of such satellites are kept [4-7] because numerical results turn out to be sensitive to the choice of the model of magnetic field.

Besides the inevitable necessity of taking account of satellites of a "natural origin" [1] in already constructed devices, it is possible to point out two more factors explaining the interest to stellarators with a mixture of harmonics. First, it is the evident possibility of getting an additional degree of freedom [8-10] which allows to vary the geometry of magnetic configuration by independent control of harmonic amplitudes. Second, predicted by theory [11-24] the possibility of essential improvement of some characteristics of the system at the optimal selection of satellites. Such optimization is one (though, as it was mentioned in [17], not the best) of the generally acknowledged in the theory methods of the reduction of neoclassical transport coefficients, and potentially also a means of plasma stability improvement [12]. Satellite harmonics may be needed also for shear creation when the choice of the main harmonic is restricted by other requirements [21,23].

Physics of plasma confinement in toroidal systems is much determined by the behavior of $\mathbf{B}^2$ on magnetic surfaces. From well-known facts it is suffice to remind that this value enters explicitly the equations for Pfirsch-Schlüter currents, stability criteria (for example, the expression for the magnetic well [25]), drift Hamiltonian for charged particles in a magnetic field [26,27], equations for bootstrap current [28,29]. Adding satellites to the main harmonic of the helical field [11-20,22,24] is often proposed (explicitly or implicitly) for changing $\mathbf{B}^2$ in a proper way. A nontrivial fact, which is the subject of discussion and analysis of the present article, is that appearance of supplementary harmonics in a magnetic field $\mathbf{B}$ leads to the change of geometry of magnetic surfaces. Therefore, in calculation of $\mathbf{B}^2$ as a function of coordinates on a magnetic surface one cannot, generally speaking, to restrict himself by the additive account of supplementary contribution to $\mathbf{B}$ only, but it is necessary also to take account of the accompanying deformation of "coordinate system". The consequence of this, as it will be shown below on a concrete example, may be not simple improving of the accuracy of calculations, but the cardinal change of the result in comparison with that when satellite effect on the geometry of magnetic surfaces is disregarded.

Attracting attention to the principal importance of self-consistent description of stellarator configurations at the presence of satellites, we tried to elucidate this aspect of the problem more completely and consistently. Starting from the most general relationships establishing the relation of the flux function, which describes magnetic surfaces, with given magnetic field of a stellarator. Their brief derivation with short explanations are given in the next section.

These relationships are used then as a basis for analysis of configurations where, besides the main harmonic, there are two nearest satellites with the same period in the toroidal angle. In the third section the expression for vacuum flux function $\psi_V$

in such configurations is obtained, and in the forth section the expression for $\mathbf{B}^2$. The necessity of self-consistent calculation of these two functions is related with the fact that the physically meaningful factor in equilibrium, stability and transport problems is the nonuniformity of $\mathbf{B}^2$ on a magnetic surface.

In the section 5 this necessity is clearly illustrated by analysis of the effect of satellite harmonics on the magnitude of Pfirsch-Schlüter current in stellarators. Without taking account of the shift of vacuum magnetic surfaces under the action of satellite harmonics this current was calculated in [18], where the conclusion was made that this current can be significantly reduced under the proper choice of satellites. We show that in this problem one may not disregard the shift.

In the section 6 we analyze the possibility of such "improvement" of configuration by using satellites which would make $\mathbf{B}^2$ the function of only two Boozer coordinates at least on a single magnetic surface. This problem is closely related with that of transport optimization in stellarators.

The last section is devoted to the summary of main results.

In the brief Appendix the expressions are given for functions which appear in the considered problems.

## 2. FLUX FUNCTION IN A STELLARATOR. GENERAL THEORY

Our goal is such self-consistent description of a configuration when magnetic surface geometry and magnetic field are uniquely related. The first step, therefore, must be the establishment of this relation. This problem is one of the key subjects in the theory of systems with magnetic confinement of a plasma, and because of its importance it is deeply enough studied. Excellent reviews [30,31] are devoted to its analysis and to explanation of basic principles. A great number of articles are devoted to direct numerical calculations of magnetic configurations by given external currents and with account of finite plasma pressure, see [2-7,22,32-34]. Finally, an elegant solution of the problem is known for a stellarator [35,36] (see also [30]) based on the averaging method [37]. This very solution we will use below. The present section gives a brief but exhaustive explanation of the matter. In fact this section can be considered as a summary of general results which we gather here both for completeness and because of their principle importance for subsequent analysis and discussions.

The function $\psi$, by which the magnetic surfaces $\psi = const$ are described, satisfies the equation

$$\mathbf{B}\cdot\nabla\psi = 0 . \qquad (1)$$

Separating fast oscillating components in $\mathbf{B}$ and $\psi$

$$\mathbf{B} = \overline{\mathbf{B}} + \tilde{\mathbf{B}}, \quad \psi = \overline{\psi} + \tilde{\psi} \qquad (2)$$

and making the same operation with Eq. (1), we get from it

$$\overline{\mathbf{B}}\cdot\nabla\overline{\psi} + \langle\tilde{\mathbf{B}}\cdot\nabla\tilde{\psi}\rangle_{\zeta} = 0 , \qquad (3a)$$

$$\overline{\mathbf{B}}\cdot\nabla\tilde{\psi} + \tilde{\mathbf{B}}\cdot\nabla\overline{\psi} = 0 . \qquad (3b)$$

The last equation is linearized over the oscillating components.

Here and in the following the standard notation is used

$$\bar{f} \equiv \langle f \rangle_\zeta = \frac{1}{2\pi}\int_0^{2\pi} f \, d\zeta , \quad \tilde{f} = f - \langle f \rangle_\zeta , \quad \hat{\tilde{f}} = \int \tilde{f} \, d\zeta , \quad (4)$$

where $\zeta$ is the toroidal angle; $(r,\zeta,z)$ is the usual cylindrical coordinates.

In stellarators the rotational transform per one period of the helical field is small. It allows to drop out the quantity $\bar{\mathbf{B}}_p \cdot \nabla\tilde{\psi}$ in the left-hand side of Eq. (3b). Then the explicit expression for $\tilde{\psi}$ can be represented as

$$\tilde{\psi} = - \delta\mathbf{r} \cdot \nabla\bar{\psi} , \quad (5)$$

where

$$\delta\mathbf{r} = \frac{r}{B_t} \hat{\tilde{\mathbf{B}}}_p \cong \frac{r^2}{RB_0} \hat{\tilde{\mathbf{B}}}_p . \quad (6)$$

Subscripts $p$ and $t$ denote the poloidal and toroidal components, $R$ is the radius of stellarator geometrical axis, $B_0$ is the toroidal field at this axis.

Thus, the solution of the equation $\mathbf{B}\cdot\nabla\psi = 0$ is

$$\psi = \bar{\psi} + \tilde{\psi} = \bar{\psi} - \delta\mathbf{r}\cdot\nabla\bar{\psi} = \bar{\psi}(\mathbf{r} - \delta\mathbf{r}) . \quad (7)$$

Three-dimensional nature of configuration is reflected here in the linear approximation in $|\tilde{\mathbf{B}}|/B_0$. And two-dimensional function $\bar{\psi}$ itself, through which the full three-dimensional solution is expressed, satisfies equation (3a). When function $\tilde{\psi}$ is substituted into this equation, it is reduced to

$$\left[ \bar{\mathbf{B}}_p + \frac{1}{2\pi} \left[ \nabla\psi_V \; \nabla\zeta \right] \right] \cdot \nabla\bar{\psi} = 0 \, . \tag{8}$$

We have used here the identity (see [38], page 313)

$$- \left\langle \tilde{\mathbf{B}} \cdot \nabla(\delta\mathbf{r} \cdot \nabla\bar{f}) \right\rangle_\zeta = \frac{1}{2\pi} \left[ \nabla\psi_V \; \nabla\zeta \right] \cdot \nabla\bar{f} \; , \tag{9}$$

where

$$\psi_V = \frac{\pi \, r^2}{B_t} \left\langle \left[ \tilde{\mathbf{B}} \, \hat{\tilde{\mathbf{B}}} \right] \cdot \frac{\nabla\zeta}{|\nabla\zeta|} \right\rangle_\zeta = \frac{2 \, \pi \, r^2}{B_t} \left\langle \tilde{B}_z \, \hat{\tilde{B}}_r \right\rangle_\zeta \, . \tag{10}$$

In (8), $\bar{\mathbf{B}}_p$ is the axially symmetric field which, obviously, due to the condition $div \; \bar{\mathbf{B}}_p = 0$ can be represented in the form

$$\bar{\mathbf{B}}_p = rot \; \mathbf{A}_t = \left[ \nabla(rA_t) \; \nabla\zeta \right] , \tag{11}$$

where $A_t$ is the toroidal component of vector potential. Thus, we get for $\bar{\psi}$ finally

$$\left[ \nabla(2\pi rA_t + \psi_V) \; \nabla\zeta \right] \nabla\bar{\psi} = 0 \; . \tag{12}$$

The equality $\left[ \nabla f \; \nabla g \right] \nabla\zeta = 0$, where $f = f(r,z)$ and $g = g(r,z)$, means that $\nabla f$ and $\nabla g$ are parallel and, consequently, $f = f(g)$. Conditions $f = const$ and $g = const$ in such a case are equivalent. Therefore, if we are trying to find the surfaces $f = const$, then without loss of generality we can put $f = g$.

It follows from the stated above that the equality

$$\bar{\psi} = 2\pi rA_t + \psi_V \tag{13}$$

not only satisfies Eq. (12), but also completely concludes the problem of finding the flux function $\psi$ by given magnetic field. Appeared at the final stage of calculations, the freedom in the choice of the specific form of the solution is related with the fact that initial equation (1) is satisfied on the whole family of functions $f(\psi)$, where $\psi$ is any particular solution of (1).

The choice of the solution of Eq. (12) exactly in the form (13) gives a concrete physical meaning to the function $\bar{\psi}$. It is easy to show that it is the external (relative to the magnetic surface) poloidal magnetic flux. Indeed, conditions $\mathbf{B}\cdot\nabla\psi = 0$ and $div\ \mathbf{B} = 0$ allow to use for calculation of $\psi_{pol}$ a horizontal membrane bounded by wavy contour lying on the magnetic surface. Then

$$\psi_{pol} = \int B_z\, dS_\perp = \int B_z\, rdr\, d\zeta = 2\pi \left\langle \int_0^{\bar{r}+\delta r} rB_z\, dr \right\rangle_\zeta \qquad (14)$$

because, according to (7), coordinates $\mathbf{r}_s$ of any point on magnetic surface can be represented as

$$\mathbf{r}_s - \delta\mathbf{r} = \bar{\mathbf{r}}\ . \qquad (15)$$

After corresponding change of variables in (14)

$$\int_0^{\bar{r}+\delta r} f(\mathbf{r})\, dr = \int_0^{\bar{r}} \left[ f(\bar{\mathbf{r}}) + \frac{\partial}{\partial\bar{r}}(f\delta r) \right] d\bar{r}\ ,$$

where only linear in $|\tilde{\mathbf{B}}|/B_0$ small corrections are retained, the integration of $\bar{B}_z$ in the right-hand side of (14) gives $2\pi r A_t$, and integration of $\underline{\tilde{B}_z}$ gives the quantity $\psi_V$, see (10).

For calculating $\psi$ by formulae (10), (11), (13) one needs to know the magnetic field. To find a field due to equilibrium plasma currents (additive contribution to $A_t$), it is necessary to solve two-dimensional equilibrium equation [35,36,38]. But for a vacuum stellarator configuration, when the magnetic field $\bar{\mathbf{B}}$ is known, the desired relation between $\psi$ and $\mathbf{B}$ is completely established by general relationships written above. Very often in analytical calculations they are disregarded, and magnetic field and geometry of magnetic surfaces are prescribed independently of each other. Due to such a "tradition" the equality (13) remained practically unrequired in the theory. In numerical calculations it was replaced partly by boundary conditions. In our analysis its key role will be evident.

## 3. FLUX FUNCTION IN A STELLARATOR AT THE PRESENCE OF SATELLITES

General relationships from the preceding section give a relation of flux function $\psi$ with a magnetic field. The contribution to $\psi$ from the helical field is described by function $\psi_V$. At low $\beta$ the helical field is almost the same as the vacuum one. Assuming $\tilde{\mathbf{B}} = \nabla\tilde{\varphi}$ in (10) and representing potential $\tilde{\varphi}$ in the form

$$\tilde{\varphi} = \sum_{m>0,k} \varphi_{km}(\rho)\ \sin(ku - m\zeta + \alpha_{km}) \ , \tag{16}$$

we get for the function $\psi_V$

$$\psi_V = -\pi R^2 B_0 \sum_{\substack{m>0,\\ k,n}} \frac{\psi_{kn}(\rho)}{m} \cos\left[(k-n)u + \alpha_{km} - \alpha_{nm})\right], \quad (17)$$

where

$$\psi_{kn} \equiv \frac{1}{2\rho B_0^2}\left[ k\varphi_{km}\frac{d\varphi_{nm}}{d\rho} + n\varphi_{nm}\frac{d\varphi_{km}}{d\rho}\right], \quad (18)$$

$(\rho, u, \zeta)$ are quasicylindrical coordinates related with geometrical axis $r = R$ of a stellarator, $\alpha_{km}$ are the phase constants. By definition $\psi_{kn} = \psi_{nk}$. Let us note that in the right-hand side of (18) there are harmonics with the same period in $\zeta$. Only at the same $m$ harmonics with different multipolarity $(k \neq n)$ give "off-diagonal" contribution to $\psi_V$. Due to this, probably, they are called sometimes interacting. In the absence of such harmonics $\psi_V = \psi_V(\rho)$, and contribution of harmonics with different $m$ into this function is additive.

Satellites with the same $m$ as that of the main harmonic are known to be for example, in helical devices ATF [39,5], Heliotron E [5], Uragan-2M [24]; according to calculations they should be in LHD [6], Liven-2/4 [21,23]. And in those cases when satellites are discussed as a means of optimization of magnetic configuration [11-20,22,24], always harmonics with the same $m$ are meant. At that, inevitably, $\psi_V = \psi_V(\rho, u)$. Indeed, let us assume that helical field is a superposition of the main harmonic $(m, \ell)$ and two satellites $(m, \ell \pm 1)$:

$$\tilde{\varphi} = \sum_{\ell-1}^{\ell+1} \varphi_k(\rho) \sin(ku - m\zeta) . \quad (19)$$

Then from (17) we get:

$$\psi_V = -\frac{\pi R^2 B_0}{m}\left[\psi_0 + \psi_1 \cos u + \psi_2 \cos 2u\right], \qquad (20)$$

where

$$\psi_0 = \psi_{\ell\ell} + \psi_{\ell-1,\ell-1} + \psi_{\ell+1,\ell+1}, \qquad (21)$$

$$\psi_1 = 2\left[\psi_{\ell,\ell-1} + \psi_{\ell,\ell+1}\right], \qquad (22)$$

$$\psi_2 = 2\,\psi_{\ell-1,\ell+1}. \qquad (23)$$

We restricted ourselves in Eq. (16) by two nearest satellites, $\ell \pm 1$, as by the most important in all above-mentioned cases [6,11-24,39]. Owing to such satellites the term $\psi_1 cosu$ appears in $\psi_V$, which corresponds to the shift of magnetic surfaces $\psi_V = const$. The terms proportional to $cos2u$ describe the elongation of surfaces, the effect similar to the action of quadrupole field [40]. Usually satellite amplitudes are smaller than that of the main harmonic, and what's more sometimes they are significantly smaller. In such a situation the function $\psi_2$ quadratic in $\varphi_{\ell\pm1}$ is small, and the main effect of satellites is described in (20) by the function $\psi_1$. In the following we will consider that very case which is typical for existing installations and is of undoubted interest for theory. Let us note that restricting ourselves by two satellites, we excluded from consideration the stellatron-type [41,42] exotic configurations.

Starting from Eq. (17) the toroidal corrections of the order $\rho/R$ are dropped in $\psi_V$. In the large-aspect-ratio approximation one can use for $\varphi_k$ the expression

$$\varphi_k = B_0 \, \gamma_k \frac{R}{m} I_k(x), \qquad x \equiv m\rho/R . \tag{24}$$

Here $\gamma_k$ are the constants characterizing the amplitudes of harmonics, $I_k$ are the modified Bessel functions. Substitution of $\varphi_k$ in such a form into (18) gives

$$\psi_0 = \sum_{\ell-1}^{\ell+1} \gamma_k^2 f_{kk} , \tag{25}$$

$$\psi_1 = \gamma_\ell \left[ \gamma_{\ell-1} f_{\ell,\ell-1} + \gamma_{\ell+1} f_{\ell,\ell+1} \right] , \tag{26}$$

where

$$f_{kk} = \frac{k I_k I_k'}{x} = \frac{1}{4} \left[ I_{k-1}^2 - I_{k+1}^2 \right] , \tag{27}$$

$$f_{\ell,\ell-1} = I_{\ell-1} I_{\ell-1}' - I_\ell I_\ell' = \frac{\ell - 1}{x} I_{\ell-1}^2 + \frac{\ell}{x} I_\ell^2 , \tag{28}$$

$$f_{\ell,\ell+1} = I_\ell I_\ell' - I_{\ell+1} I_{\ell+1}' = \frac{\ell}{x} I_\ell^2 + \frac{\ell + 1}{x} I_{\ell+1}^2 . \tag{29}$$

In all three expressions those functions are placed at the first position which give the lowest term in expansion in $x$; prime denotes the derivative over $x$. Let us note that

$$f_{\ell,\ell-1} + f_{\ell,\ell+1} = 2 \frac{d}{dx} f_{\ell\ell} . \tag{30}$$

Therefore the expression for $\psi_1$ can be written as

$$\psi_1 = 2 \frac{\gamma_{\ell-1}}{\gamma_\ell} \psi'_{\ell\ell}(x) + \gamma_\ell \left[ \gamma_{\ell+1} - \gamma_{\ell-1} \right] f_{\ell,\ell+1} \, . \tag{31}$$

Thus, the presence of satellites $(\ell \pm 1)$ shows itself in the asymmetry of function $\psi_V$. It is clear that nonzero "cosine" harmonic of $\psi$ is related with shift of magnetic surfaces. Contribution to this harmonic due to helical fields is described by function $\psi_1$ in $\psi_V$. Additional independent contribution can arise also due to homogeneous transverse (vertical) field $B_\perp \mathbf{e}_z$, for which

$$2\pi r A_t = \pi r^2 B_\perp \cong \pi R^2 B_\perp - \frac{\pi R^2 B_0}{m} \psi_\perp \cos u \, , \tag{32}$$

where

$$\psi_\perp \equiv 2 \frac{m \rho}{R} \frac{B_\perp}{B_0} \, , \tag{33}$$

and the second harmonic is dropped out, which corresponds to large-aspect-ratio approximation. At finite plasma pressure the field of equilibrium plasma currents must be also included in $A_t$. Accurate description of vacuum configuration is a necessary basis for their correct calculation.

Now equation of averaged vacuum magnetic surfaces, where besides satellites also external vertical field is taken into account, can be written in the form

$$\psi_0 + \left[ \psi_1 + \psi_\perp \right] \cos u = const \, . \tag{34}$$

For surfaces which shift $\Delta$ is small as compared with their averaged radius $a$, $|\Delta|/a \ll 1$, we get from here

$$\rho \cong a - \Delta \cos u , \qquad \frac{\Delta}{a} = \frac{\psi_1 + \psi_\perp}{x\psi_0'(x)} . \tag{35}$$

If both satellites are small, then $\psi_0 \cong \gamma_\ell^2 f_{\ell\ell}$. In such a case substitution of the explicit expressions for functions $\psi_1$ and $\psi_\perp$, which are found earlier, into (35) gives

$$\Delta = 2 \frac{R}{m} \frac{\gamma_{\ell-1}}{\gamma_\ell} + \frac{\gamma_{\ell+1} - \gamma_{\ell-1}}{\gamma_\ell} \frac{R}{m} \frac{f_{\ell,\ell+1}}{f_{\ell\ell}'(x)} + \Delta_\perp , \tag{36}$$

where $\Delta_\perp$ is the shift due to vertical field:

$$\Delta_\perp = \frac{R}{\mu} \frac{B_\perp}{B_0} . \tag{37}$$

Here $\mu$ is the rotational transform which at $|\Delta|/a \ll 1$ is related with function $\psi_0$ by the relationship

$$\psi_0'(x) = 2 \mu \frac{x}{m} . \tag{38}$$

Let us notice one particular case: $\gamma_{\ell-1} = \gamma_{\ell+1}$. At such condition and at the absence of vertical field the shift is constant: $\Delta = const.$

The fact that magnetic surfaces are shifted under the action of satellites and the derived expressions for the shift will be used below in consideration of two particular problems: in calculation of Pfirsch-Schlüter current in configurations with satellites and in analysis of the quasisymmetry condition. In both cases we will need also the explicit expression for $\mathbf{B}^2$.

## 4. $\mathbf{B}^2$ AT THE PRESENCE OF SATELLITES

Stellarator magnetic field is a superposition of the toroidal field $B_t\mathbf{e}_\zeta$, axially symmetric poloidal field $\bar{\mathbf{B}}_p$ and helical field $\tilde{\mathbf{B}}$, therefore

$$\mathbf{B}^2 = B_0^2 \left[ 1 + b_1 + b_2 \right], \tag{39}$$

where $b_1$ and $b_2$ are small quantities as compared with unity which describe the helical and toroidal inhomogeneity of $\mathbf{B}^2$:

$$b_1 = 2 \frac{R}{r} \frac{\tilde{B}_\zeta}{B_0} + 2 \frac{\bar{\mathbf{B}}_p \cdot \tilde{\mathbf{B}}}{B_0^2}, \tag{40}$$

$$b_2 = \frac{R^2}{r^2} - 1 + \frac{\tilde{\mathbf{B}}^2}{B_0^2} + \frac{\bar{\mathbf{B}}_p^2}{B_0^2}. \tag{41}$$

The main fast-oscillating in $\zeta$ term in (39) is the quantity $b_1$, and because of this from $\tilde{\mathbf{B}}^2$ only the "axially symmetric" contribution $< \tilde{\mathbf{B}}^2 >_\zeta$ turns out to be essential in $b_2$. For a field with a potential described by (16) it is not difficult to obtain a general formula similar to (17) (for simplicity here $\alpha_{ik} = 0$)

$$\Omega^0 \equiv \left\langle \frac{\tilde{\mathbf{B}}^2}{B_0^2} \right\rangle_\zeta = \frac{1}{2B_0^2} \sum_{\substack{m>0 \\ k,n}} \frac{m^2}{R^2} \left[ \varphi_k' \varphi_n' + \left( \frac{kn}{x^2} + 1 \right) \varphi_k \varphi_n \right] \cos (k - n)u, \tag{42}$$

which, when the main harmonic $(m, \ell)$ has only two nearest satellites $(m, \ell \pm 1)$, takes the form

$$\Omega^0 = \Omega_0^0(x) + \Omega_1^0(x)\cos u + \Omega_2^0(x)\cos 2u , \qquad (43)$$

where

$$\Omega_0^0 = \sum_{\ell-1}^{\ell+1} \gamma_k^2 g_{kk} , \qquad (44)$$

$$\Omega_1^0 = \gamma_\ell \left[ \gamma_{\ell-1} g_{\ell,\ell-1} + \gamma_{\ell+1} g_{\ell,\ell+1} \right] . \qquad (45)$$

The explicit form of $\Omega_2^0$ will not be needed below. If both satellites are small, the quantity $\Omega_2^0$ in (43) can be disregarded because it is quadratic in $\varphi_{\ell\pm 1}$, but at the same time $\Omega_1^0$ is linear.

For the functions entering (44) and (45) we get in the cylindrical approximation, when $\varphi_k$ is expressed through Bessel functions (24),

$$g_{kk} = \frac{1}{2}\left[ I_k'^2 + \left[ \frac{k^2}{x^2} + 1 \right] I_k^2 \right] , \qquad (46)$$

$$g_{\ell,\ell-1} = I_\ell' I_{\ell-1}' + \left[ \frac{\ell(\ell-1)}{x^2} + 1 \right] I_\ell I_{\ell-1} = I_{\ell-1} I_{\ell-1}' + I_\ell I_\ell' , \qquad (47)$$

$$g_{\ell,\ell+1} = I_\ell' I_{\ell+1}' + \left[ \frac{\ell(\ell+1)}{x^2} + 1 \right] I_\ell I_{\ell+1} = I_\ell I_\ell' + I_{\ell+1} I_{\ell+1}' . \qquad (48)$$

It is not difficult to check that they are related by the equality similar to (30):

$$g_{\ell,\ell-1} + g_{\ell,\ell+1} = f_{\ell,\ell-1} - f_{\ell,\ell+1} + 4\frac{x}{\ell} f_{\ell\ell} = 2g'_{\ell\ell} . \quad (49)$$

Here we show also their relations with earlier introduced functions $f_{ik}$. Besides, it is possible to give another two useful relationships:

$$g_{\ell,\ell-1} - g_{\ell,\ell+1} = f_{\ell,\ell-1} + f_{\ell,\ell+1} = 2f'_{\ell\ell} , \quad (50)$$

$$g_{\ell\ell} = \frac{1}{2\ell x} \frac{d}{dx} \left[ x^2 f_{\ell\ell} \right] . \quad (51)$$

Together with (30) and (49) they allow (when there are two satellites) to reduce the number of independent functions in $\psi_V$ and $\Omega^0$ to two only: $f_{\ell\ell}$ and, for example, $f_{\ell,\ell+1}$.

Now, after getting the explicit expressions for $\psi_V$ and $\Omega^0$, where the presence of satellites manifests itself in asymmetrical (in poloidal direction) terms looking like $f(\rho)\ cosu$, we can turn to consideration of concrete tasks. Let us consider first the problem of reduction of Pfirsch-Schlüter currents which has been studied earlier analytically in [18] and then numerically in [43].

## 5. PFIRSCH-SCHLÜTER CURRENT AT THE PRESENCE OF SATELLITES

It is known that in stellarators with a planar axis Pfirsch-Schlüter current is described by the expression

$$\bar{j}_\zeta = 2\pi R p'(\bar{\psi}) \left[ \Omega - < \Omega > \right] , \quad (52)$$

where

$$\Omega = \Omega^0 + 1 - \frac{R^2}{r^2} \cong \Omega^0 - 2\frac{\rho}{R} \cos u . \qquad (53)$$

The latter approximate equality corresponds to large-aspect-ratio approximation. Brackets <...> in (52) denote the averaging over the volume between adjacent surfaces $\bar{\psi} = const$:

$$< f > \equiv \frac{d}{dV} \int_V f \, d\bar{\mathbf{r}} . \qquad (54)$$

If transverse cross-sections of these surfaces are circular, then $\bar{\psi}' \cong -2\pi a\mu B_0$, where $a$ is the averaged radius of cross-sections, and formula (52) is reduced to

$$\bar{j}_\zeta = \frac{p'(a)}{\mu B_0} \frac{R}{a} \Big[ < \Omega > - \Omega \Big] . \qquad (55)$$

Let us note that small corrections quadratic in $\Delta'$ were dropped out at mentioned substitution of $\bar{\psi}'$. Implicitly the dependence on the shift $\Delta$ remains in $< \Omega > - \Omega$.

At circular "in average" nonshifted magnetic surfaces, when there are no satellites, $\Omega^0 = \Omega^0(\psi_V)$ (both functions depend on $\rho$ only), and, respectively, $\Omega^0 - < \Omega^0 > = 0$. In the right-hand side of (55) there remains only "toroidal" term from $\Omega$, and, besides, in this case $< cos u > = 0$. As a result, for $\bar{j}_\zeta$ one gets from (55) the well-known simple expression:

$$\bar{j}_\zeta = \frac{2p'(\rho)}{\mu B_0} \cos u . \qquad (56)$$

If at the absence of satellites $\Delta \neq 0$, then [45,46,12]

$$\bar{j}_\zeta = \frac{2p'(a)}{\mu B_0} \left[ 1 + B_0^2 V_0''(\Phi) \frac{\Delta}{2} \right] \cos\theta , \qquad (57)$$

where $\theta$ is the poloidal angle along the perimeter of transverse cross-section of the surface $a = const$ (at the inner side $\theta = 0$, and at the outer side $\theta = \pi$), $V_0''(\Phi)$ is the quantity characterizing the vacuum magnetic hill, $\Delta > 0$ corresponds to the outward shift.

As it is seen from (57), when magnetic surfaces are shifted into the region of stronger toroidal field, Pfirsch-Schlüter current can be reduced and, as a consequence, configuration can be made weakly dependent on the pressure [12,45]. But at parameters, which are typical to stellarators, this effect is weak, and its enhancement by the increase of magnetic hill is disadvantageous because of the danger of sharp deterioration of plasma stability.

According to Ref. [18] the more radical means of the reduction of Pfirsch-Schlüter current could be the addition of one or two nearest satellite harmonics to the main one. In [18] it was shown that this leads to the appearance of additional terms in the expression for Pfirsch-Schlüter current, which are proportional to the amplitudes of the satellites, and depending on their sign the Pfirsch-Schlüter current can be increased or decreased.

Analytical calculations have been done in [18] under assumption that averaged magnetic surfaces are nonshifted torii $\rho = const$. Strictly speaking, as it was shown above, this assumption contradicts the main condition of the problem [18], namely the presence of two satellites with the same period in $\zeta$ as that of the main harmonic. For this case we got here expressions for functions $\psi_V$ and $\Omega^0$. It is a sufficient basis to remove this contradiction. From the formal point of view the main difference of our analysis from that of [18] is the account of the shift of magnetic surfaces (36). In other respects we will try to follow [18].

As in Ref. [18], we will assume that amplitudes of satellites are small and will retain in $\bar{j}_\zeta$ only terms linear in $\varphi_{\ell\pm1}/\varphi_\ell$. At that $\psi_0 \cong \gamma_\ell^2 f_{\ell\ell}$, $\Omega^0 \cong \gamma_\ell^2 g_{\ell\ell}$, satellites are present in $\psi_1$ and $\Omega_1^0$ only. In the expression for $B = |\mathbf{B}|$, which in accordance with (39)-(41) can be written as

$$B = B_0 \left[ 1 + \frac{\rho}{R} \cos u + \sum_{\ell-1}^{\ell+1} b_k \cos(ku - m\zeta) \right] , \qquad (58)$$

harmonic amplitudes $b_k$ was prescribed in [18] as power dependencies: $b_k = C_k \rho^k$. Following the same way, we must replace everywhere Bessel functions $I_k$ by the first term of their expansions:

$$b_k = - \gamma_k I_k(x) \cong - \gamma_k \frac{1}{k!} \left[ \frac{x}{2} \right]^k . \qquad (59)$$

At small $x$ this approximation is quite justified. It entails the following simplifications:

$$\frac{\ell - 1}{\ell + 1} \frac{g_{\ell,\ell+1}}{I_{\ell+1}} \cong \frac{g_{\ell,\ell-1}}{I_{\ell-1}} \cong I'_{\ell-1} \cong \frac{1}{2} \frac{1}{(\ell - 2)!} \left[ \frac{x}{2} \right]^{\ell-2} . \qquad (60)$$

Besides that,

$$\frac{\mu}{b_\ell}\left[\frac{x}{2}\right]^2 \cong -\frac{m\gamma_\ell^2}{16}\frac{\ell}{(\ell-2)!}\left[\frac{x}{2}\right]^{\ell-2}, \tag{61}$$

because at $|x| \ll 1$

$$\mu \cong \frac{m\gamma_\ell^2}{16}\frac{1}{(\ell-2)!(\ell-1)!}\left[\frac{x}{2}\right]^{2\ell-4}. \tag{62}$$

Replacing the right-hand side of (60) with the help of (61) and substituting $g_{ik}$ into (45), we get with account of (59)

$$\Omega_1^0 = \frac{2}{m\ell}x^2\frac{\mu}{b_\ell}\left[b_{\ell-1} + \frac{\ell+1}{\ell-1}b_{\ell+1}\right]. \tag{63}$$

This quantity enters $\bar{j}_\zeta$ in combination $\Omega = \Omega_0^0 + [\Omega_1^0 - 2(\rho/R)]\cos u$. In an expanded form:

$$\bar{j}_\zeta = \frac{2p'(a)}{B_0}\left[\frac{1}{\mu} - \frac{m}{\ell}\frac{\rho}{R}\frac{1}{b_\ell}\left[b_{\ell-1} + \frac{\ell+1}{\ell-1}b_{\ell+1}\right]\right]\frac{\rho}{a}\cos u +$$

$$+ \frac{p'(a)}{\mu B_0}\frac{R}{a}\left[<\Omega> - \Omega_0^0\right]. \tag{64}$$

If there would be no shift of magnetic surfaces, then with $a(\mathbf{r}) = \rho$ the quantity $<\Omega> - \Omega_0^0$ would vanish, and (64) would exactly correspond to the result of [18] obtained for $\ell = 2$ and $\ell = 3$ (to make the transition to [18], $m$ and $\cos u$ should be replaces by $-m$ and $-\cos\theta$).

But in fact under the action of satellite harmonics all magnetic surfaces become shifted. Remaining in the frame of approximation $x \ll 1$, $\varphi_{\ell\pm1}/\varphi_{\ell} \ll 1$, we get for this shift from (36) in the notations (59) (which were introduced here only for convenience of comparison with [18])

$$\Delta = \frac{a}{\ell} \left[ \frac{b_{\ell-1}}{b_{\ell}} + \frac{\ell+1}{\ell-1} \frac{b_{\ell+1}}{b_{\ell}} \right] . \qquad (65)$$

Let us remind that for surfaces with such a shift (far from the magnetic axis) $\rho \cong a - \Delta \cos u$, so that

$$\Omega_0^0(\rho) \cong \Omega_0^0(a) - \frac{d\Omega_0^0}{d\rho} \Delta \cos u . \qquad (66)$$

At $\Delta \neq 0$, but without satellites the last term leads to the appearance of the term with $V_0^{''}(\Phi)\Delta$ in the right-hand side of (57). In our case, as it follows from (44) and (51),

$$\Omega_0^0 = \gamma_\ell^2 g_{\ell\ell} = \frac{1}{2\ell x} \frac{d}{dx} \left[ x^2 \psi_0 \right] = \psi_0 , \qquad (67)$$

because $\psi_0 = Cx^{2\ell-2}$. Thus

$$\Omega_0^0 \cong \Omega_0^0(a) - 2\mu \frac{ma^2}{\ell R^2} \left[ \frac{b_{\ell-1}}{b_{\ell}} + \frac{\ell+1}{\ell-1} \frac{b_{\ell+1}}{b_{\ell}} \right] \cos u . \qquad (68)$$

Here $\mu$ appeared after taking derivative of $\psi_0$, see (38).

Substitution of the obtained expression for $\Omega_0^0$ into (64) leads to unexpected result: the term with $b_{\ell\pm1}$, with which the possibility of the reduction of equilibrium currents was related in [18], is completely compensated by the term of the opposite sign which appeared because of the shift of magnetic surfaces. In other words, even at the presence of satellites one get for

$\bar{j}_\zeta$ the expression similar to (56), the effect predicted in [18] is absent.

One should remember, of course, that we came to this conclusion under some specific restrictions explained in details above. Nevertheless, it is a serious warning that the change of the shape of magnetic surfaces under the action of satellites is the factor which may not be disregarded. In the considered case its contribution turned out to be exactly equal (but with another sign) to the expected result.

It was stated in [43] that numerical results was in a good (qualitative) agreement with [18]. Probably, it is an accidental coincidence caused by two different reasons. The first one is purely methodical: discrepancy of models. In [43], for example, $B_\perp/B_0 = 2\%$, but in [18] there is no vertical field in equations. Besides, in [43] harmonic content of the field is not shown, but it is undoubtedly different from that adopted in [18]. The second one is the physical reason: strong dependence of $\bar{j}_\zeta$ on the shape of magnetic surfaces. The dependence which has shown itself, in particular, in just now considered example with account of shift due to satellites. As an another example let us show the expression

$$\bar{j}_\zeta = \frac{2p'(a)}{\mu B_0}\frac{K^2+1}{2K^2}\left[\cos\theta + \frac{\Delta'}{2} + \frac{R}{2a}\left[<\Omega^0> - \Omega^0\right]\right] \quad (69)$$

for the case when $\mu = const$ and surfaces $\bar{\psi} = const$ are shifted ellipses:

$$r = R + \Delta - a\cos\theta , \qquad z = Ka\sin\theta . \qquad (70)$$

It is seen from (69) that $\bar{j}_\zeta$ strongly depends on surface ellipticity $K$: elongation of surfaces in the vertical direction ($K > 1$) leads to the reduction of $\bar{j}_\zeta$, but "flattening" ($K < 1$) leads to the increase. Even at so moderate elongation as $K = 1.5$ the magnitude of the current is already $1.4$ times smaller, but at $K = 0.75$ it is $1.4$ times larger than that for circular surfaces ($K = 1$).

Both examples show that calculating $\bar{j}_\zeta$ one should avoid rough models, and it is necessary to be cautious in interpretation of numerical results.

We talked above about the shift, so it is pertinent here to attract attention to quantity $\Delta'$ in formula (69), which at $K = 1$ is valid at any $\mu(a)$. The presence of $\Delta'$ in expressions for $\bar{j}_\zeta$ (including also the implicit one in (64)) contradicts to generally accepted opinion that current must be purely "cosine". However it is just a case when the conclusion based on a simplified model is incorrect. Indeed, the square element of the transverse cross-section between adjacent circular ($K = 1$) magnetic surfaces is equal to $dS_\perp = a(1 - \Delta' \cos\theta) da d\theta$. Without $\Delta'$ in (69) the integral $\int \bar{j}_\zeta dS_\perp$ would not vanish, though it must vanish, which is the case with (69). If the shift $\Delta$ is caused by finite plasma pressure, then $\Delta' < 0$. At that, according to (69), the value of Pfirsch-Schlüter current must be larger at the outer side of the torus, $\theta = \pi$, and smaller at the inner side, $\theta = 0$. At $\Delta' = 1/2$, which is yet far from the equilibrium limit, we get for the ratio of these values $5/3$ (instead of traditionally expected unity). The effect, as we can see, turns out to be strong enough, and with increasing $\beta$ it must further increase. It is not surprising therefore that attempts to find somewhere a direct confirmation of this consequence of (69) turn out to be successful: results of numerical calculations [36,47-49], show the evident asymmetry of $\bar{j}_\zeta$ over $u$ at finite $\beta$, see, for example, Fig. 1 copied from Ref. [47]. We can add to this that at the presence of interacting satellites there is a shift $\Delta \neq 0$ in the vacuum configuration itself, and similar effect must be seen even at small $\beta$.

Detailed analytical calculations given above are illustrative and can be easily compared in main elements with those of [18]. Now we may give another, more brief, but formal proof of the conclusion about much smaller effect of satellites on Pfirsch-Schlüter current than it was predicted in [18]. This proof can be considered as a short explanation of mathematical aspect of the problem.

If in formulae for functions $f_{ik}$ and $g_{ik}$ obtained by the expansion in $x$ from (27) - (29) and (46) - (48), see Appendix, we would retain only the first terms of the expansion, then we would come to the conclusion that $f_{ik} = g_{ik}$. At that, as it is seen from comparison of (25), (26) and (44), (45), $\Omega^0/\psi_V = const$. In the case considered above $\bar{\psi} = \psi_V$ and, consequently, $\Omega^0 - < \Omega^0 > = 0$ at any $\Omega^0(\psi_V)$. In this approximation the function $\Omega^0$ just drops out from the general expression (52) for $\bar{j}_\zeta$ and, correspondingly, from (69). If, following the tradition, we would drop out also $\Delta'/2$ in (69), then there would remain no trace of satellites in $\bar{j}_\zeta$. A weak dependence of $\bar{j}_\zeta$ on satellites can appear in the next order of the expansion in $x$ due to small difference between $f_{ik}$ and $g_{ik}$, which is shown explicitly in the Appendix.

## 6. SATELLITES AND QUASISYMMETRY

Let us turn now to one more problem where the presence of satellite is dictated by the requirement of quasisymmetry.

Concept of quasisymmetry [50,51] was discussed in details in recently published review-like articles [52,53]. One can find there exhaustive explanations and more extensive bibliography. Formally, the matter is reduced to finding such configurations where $\mathbf{B}^2$ is a functions of two variables only in Boozer coordinates $(a,\theta_B,\zeta_B)$:

$$\mathbf{B}^2 = \mathbf{B}^2(a,\theta_B - N\zeta_B) \ . \tag{71}$$

In conventional toroidal stellarators with a planar circular axis this condition is not fulfilled because toroidal and helical fields have a symmetry of different type. But we may try, as it was proposed in [52,53], to satisfy it at a single magnetic surface. Let us analyze this possibility.

In the frame of stellarator approximation the quasisymmetry condition (71) for conventional stellarators can be reduced to two equations [54]: one for the helical and another for the nonoscillating in $\zeta$ component of $\mathbf{B}^2$. If cross-sections of averaged magnetic surfaces are shifted circles, see (70), the first from these equations takes the form

$$\frac{1}{N}\frac{\partial\tilde{\varphi}}{\partial\zeta} + \left[1 + \left[\frac{\Delta}{\rho} + \Delta'\right]\cos u\right]\frac{\partial\tilde{\varphi}}{\partial u} + \Delta\sin u\,\frac{\partial\tilde{\varphi}}{\partial\rho} = 0. \tag{72}$$

At the absence of the shift $(\Delta = 0)$ this equation at $N = m/\ell$ could be satisfied by any single harmonic $(\ell, m)$. But the second quasisymmetry equation, see below, certainly cannot be satisfied without shift. Because of this we must assume from the very beginning that $\Delta \neq 0$.

In this more general case the solution of (72) in linear in $\Delta/\rho$ approximation will be the potential similar to (19) with the main harmonic and such two nearest satellites that

$$2\varphi_{\ell-1} = \varphi_\ell'\Delta + \ell\varphi_\ell \left[ \frac{\Delta}{\rho} + \Delta' \right] , \qquad (73)$$

$$2\varphi_{\ell+1} = \varphi_\ell'\Delta - \ell\varphi_\ell \left[ \frac{\Delta}{\rho} + \Delta' \right] . \qquad (74)$$

At the condition $\rho/R \ll 1$, which is typical for stellarators, substitution of expressions (24) for $\varphi_k$ and recurrence formulae for Bessel functions

$$xI_\ell'(x) + \ell I_\ell = xI_{\ell-1} , \qquad xI_\ell'(x) - \ell I_\ell = xI_{\ell+1}$$

allow to reduce these relationships to the form

$$2\varphi_{\ell-1} = \frac{m\ \Delta}{R} \frac{\gamma_\ell}{\gamma_{\ell-1}} \varphi_{\ell-1} + \ell\varphi_\ell\Delta' , \qquad (75)$$

$$2\varphi_{\ell+1} = \frac{m\ \Delta}{R} \frac{\gamma_\ell}{\gamma_{\ell+1}} \varphi_{\ell+1} - \ell\varphi_\ell\Delta' , \qquad (76)$$

where shift $\Delta$, let us remind, depends itself on functions $\varphi_k$. In principle, equations (73) and (74) can be satisfied at any shift. We will consider below a case when this shift is created by satellites and by external vertical field only.

According to (36), for a vacuum configuration

$$\Delta = 2 \frac{R}{m} \frac{\gamma_{\ell-1}}{\gamma_\ell} + \Delta_1 , \qquad (77)$$

where the first term is a constant independent on minor radius. Quasisymmetry equations (75), (76) will reduce with such a shift to requirements on $\Delta_1$:

$$\Delta_1 = - \frac{\ell I_\ell}{I_{\ell-1}} \Delta_1'(x) = - \frac{x}{2} \left[ 1 - \frac{I_{\ell+1}}{I_{\ell-1}} \right] \Delta_1'(x) , \tag{78}$$

$$\frac{m \ \Delta_1}{R} = \frac{\gamma_{\ell+1} - \gamma_{\ell-1}}{\gamma_\ell} \frac{I_{\ell+1}}{I_\ell'(x)} . \tag{79}$$

It is obvious that the simplest solution of this system is $\Delta_1 = 0$, which corresponds to the case $\gamma_{\ell+1} = \gamma_{\ell-1}$, $B_\perp = 0$. Let us show that system (78), (79) has no other, nontrivial solution. The most simple way to do it is to use near-axis expansions of Bessel functions (expansions in $x$). At that

$$(\ell + 1) \frac{I_{\ell+1}}{I_\ell'(x)} \cong (\ell - 1) \frac{f_{\ell,\ell+1}}{f_{\ell\ell}'} \cong \frac{x^2}{2\ell} , \tag{80}$$

so in the expression

$$\Delta_1 = \Delta_h + \Delta_\perp \tag{81}$$

one can use for $\Delta_h$ at $x \leq 1$ the formula

$$\Delta_h \cong \frac{R}{m} \frac{\gamma_{\ell+1} - \gamma_{\ell-1}}{\gamma_\ell} \frac{x^2}{2\ell(\ell - 1)} . \tag{82}$$

In this case equation (78) can be written as

$$\Delta_h + \frac{x}{2}\Delta_h' + \Delta_\perp \left[ 1 - \frac{x\mu'}{2\mu} \right] = 0 , \tag{83}$$

and equation (79), with account of (80) and (82), takes the form

$$\Delta_1 = \frac{\ell - 1}{\ell + 1} \Delta_h . \tag{84}$$

As a result, (78) and (79) are reduced to the system of two equations

$$2\Delta_h + \Delta_\perp \left[ 1 - \frac{x\mu'}{2\mu} \right] = 0 , \tag{85}$$

$$2\Delta_h + \Delta_\perp \left[ \ell + 1 \right] = 0 \tag{86}$$

with nonzero denominator, which is equal to $2\ell + x\mu'/\mu$. Thus, there is no other solution besides $\Delta_h = \Delta_\perp = 0$.

If instead of (80) we would use more accurate relationships

$$\frac{f_{\ell,\ell+1}}{f_{\ell\ell}'} = \frac{2f_{\ell,\ell+1}}{f_{\ell,\ell-1} + f_{\ell,\ell+1}} \cong \frac{x^2}{2\ell} \frac{\ell + 1 + x^2/(4\ell)}{\ell^2 - 1 + x^2} , \tag{87}$$

$$\frac{I_{\ell+1}}{I_\ell'(x)} \cong \frac{x^2}{2\ell} \frac{1}{\ell + 1 + x^2/(2\ell)} , \tag{88}$$

giving reliable approximations up to $x = 2$, the result would not change: the system of equations (78), (79) has only trivial solution $\Delta_h = 0$, $\Delta_\perp = 0$.

So, the first quasisymmetry equation is satisfied at the presence of two satellites when $\gamma_{\ell-1} = \gamma_{\ell+1}$, $B_\perp = 0$. In this case configuration preserves a helical symmetry in approximation linear in $|\tilde{\mathbf{B}}|/B_0$. Inhomogeneity of the toroidal field is not compatible with this type of symmetry. The necessity to suppress this inhomogeneity at least on a single magnetic surface leads to the equation [54]

$$\left[ \mathbf{e}_\zeta \; \nabla\bar{\psi} \right]\cdot\nabla \left\{ \frac{R^2}{r^2} - H \right\} = 0 . \tag{89}$$

where

$$H \equiv \left\langle \frac{\tilde{\mathbf{B}}^2}{B_0^2} - 4\,\frac{\tilde{B}_\zeta^2}{B_0^2} \right\rangle_\zeta . \tag{90}$$

All calculations with function $H$ are similar to those made in the forth section with function $\Omega^0$. Expression for $H$ in configuration with two satellites turns out to be similar to (43):

$$H = H_0(\rho) + H_1(\rho)\cos u + H_2(\rho)\cos 2u . \tag{91}$$

Here

$$H_0 = \gamma_\ell^2 \left[ g_{\ell\ell} - 2I_\ell^2 \right] , \tag{92}$$

$$H_1 = \gamma_\ell \left[ \gamma_{\ell-1}\left[g_{\ell,\ell-1} - 4I_\ell I_{\ell-1}\right] + \gamma_{\ell+1}\left[g_{\ell,\ell+1} - 4I_\ell I_{\ell+1}\right] \right], \tag{93}$$

$g_{ik}$ are the functions (46) - (48). In $H_0$ only contribution from the main harmonic is retained. Expressions (49) allow to express $H_1$ through $H_0$:

$$H_1 = 2\frac{\gamma_{\ell-1}}{\gamma_\ell} H_0' + \gamma_\ell \left[ \gamma_{\ell+1} - \gamma_{\ell-1} \right] \left[ g_{\ell,\ell+1} - 4I_\ell I_{\ell+1} \right]. \quad (94)$$

At $\gamma_{\ell-1} = \gamma_{\ell+1}$, $B_\perp = 0$, which is necessary for maintaining a helical symmetry in the lowest approximation, this expression for $H_1$ takes the form

$$H_1 = \Delta \frac{dH_0}{d\rho}, \quad (95)$$

where, as before, $\Delta$ is the shift (36) of magnetic surfaces.

At the same time equation (89) requires, when magnetic surfaces are shifted circles, that

$$H_1 - \Delta \frac{dH_0}{d\rho} = 2\frac{\rho}{R}, \quad (96)$$

which does not conform with (95). Thus, at the presence of two satellites and at the absence of vertical field the condition of quasisymmetry can be satisfied in linear in $|\tilde{\mathbf{B}}|/B_0$ approximation only, but in the quadratic one it is violated. Violated because in the considered case $m\Delta/R = 2\gamma_{\ell-1}/\gamma_{\ell+1}$. The left-hand side of (96) could be positive at smaller, and all the more at negative shifts. Formally, as it is seen from (94), the increase of $H_1$ at $\gamma_{\ell+1} > \gamma_{\ell-1}$ also could improve the situation. These favorable factors could play their role if there would be no rigid relation (36) or (65) between $\Delta$ and amplitudes of satellites.

For the analysis of this possibility it is necessary to consider more general case. With account, for example, of quadrupole field which reduces Pfirsch-Schlüter currents (69) and, correspondingly, the shift of magnetic surfaces; of finite plasma pressure or longitudinal current.

The considered example is interesting as showing the possibility to create in conventional stellarators the whole family of configurations with arbitrary shift of magnetic surfaces, but at the same time "helically symmetric" in linear in $|\tilde{\mathbf{B}}|/B_0$ approximation. Despite the fact that sign and magnitude of the shift can be arbitrary, the helical field drops out from the second quasisymmetry condition (96) because of (95).

## 7. SUMMARY

The possibility to improve stellarators in respects of equilibrium, stability and confinement of a plasma at some optimal choice of satellite helical harmonics may be considered now as reliably proved by the theory [22]. Interest to such configurations is maintained already for more than ten years, and, undoubtedly, it will only further increase because to create reactor-stellarator more better characteristics are needed than those of the contemporary devices.

Analysis of plasma behavior in stellarators needs a reliable description of vacuum configuration. As a minimum, it is necessary to know two functions: $\psi_V$ and $<\tilde{\mathbf{B}}^2>_\zeta$. As it is shown in the present article, in configurations with satellites, which have the same period in $\zeta$ as that of the main harmonic, $\psi_V$ and

$<\tilde{\mathbf{B}}^2>_\zeta$ are asymmetrical in a poloidal angle. Asymmetry of $\psi_V$ corresponds to the shift of magnetic surfaces under the action of satellites. As for $<\tilde{\mathbf{B}}^2>_\zeta$ in paraxial approximation this function does not differ from $\psi_V$. In this case $<\tilde{\mathbf{B}}^2>_\zeta = const$ at surfaces $\psi_V = const$, and because of this even at the presence of satellites Pfirsch-Schlüter current remains the same as in stellarators with a single harmonic of helical field.

Calculation of Pfirsch-Schlüter current was made here in the same approximation as that used in [18]: it was assumed that amplitudes of satellites were small, system was supposed to be long-period (to justify expansion in $\rho$), and $\beta \ll \beta_{eq}$. The new element here, in comparison with [18], was the account of the magnetic surface shift at the presence of satellites. Our analysis shows that disregard of this shift leads to physically incorrect result. The principal necessity of self-consistent description of stellarators in cases when effects are related with inhomogeneity of $\mathbf{B}^2$ on magnetic surfaces is confirmed thereby by clear and concrete example.

Given above explicit expressions for functions $\psi_V$ and $\mathbf{B}^2$ in stellarator with satellites were used also for the analysis of quasisymmetry condition (71). It was shown that in linear approximation in $|\tilde{\mathbf{B}}|/B_0$ this condition can be easily satisfied at proper choice of satellites, see (73), (74), in configuration with shifted magnetic surfaces.

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## APPENDIX

Here we give the expressions for functions $f_{ik}$ and $g_{ik}$, obtained from the general formulae by expansion in $x$ with keeping the first and second terms of the expansion series. For convenience of comparison both functions $f_{ik}$ and $g_{ik}$ with the same indexes are given side by side.

At the absence of satellites the only, and at small amplitudes of satellites the main contribution to $\psi_V$ and $\Omega^0$ is described by $f_{\ell\ell}$ and $g_{\ell\ell}$:

$$f_{\ell\ell} \cong \frac{1}{4}\frac{1}{\ell!}\frac{1}{(\ell-1)!}\left[\frac{x}{2}\right]^{2\ell-2}\left[\ell + 2\left[\frac{x}{2}\right]^2\right], \qquad (A.1)$$

$$g_{\ell\ell} \cong \frac{1}{4}\frac{1}{\ell!}\frac{1}{(\ell-1)!}\left[\frac{x}{2}\right]^{2\ell-2}\left[\ell + 2\left[1+\frac{2}{\ell}\right]\left[\frac{x}{2}\right]^2\right]. \qquad (A.2)$$

Functions which appear in $\psi_V$ and $\Omega^0$ at the presence of lower satellite $(\ell - 1, m)$ to the main harmonic $(\ell, m)$:

$$f_{\ell,\ell-1} \cong \frac{1}{2}\frac{1}{\ell!}\frac{1}{(\ell-1)!}\left[\frac{x}{2}\right]^{2\ell-3}\left[\ell(\ell-1) + (2\ell-1)\left[\frac{x}{2}\right]^2\right], \qquad (A.3)$$

$$g_{\ell,\ell-1} \cong \frac{1}{2}\frac{1}{\ell!}\frac{1}{(\ell-1)!}\left[\frac{x}{2}\right]^{2\ell-3}\left[\ell(\ell-1) + (2\ell+1)\left[\frac{x}{2}\right]^2\right]. \qquad (A.4)$$

For functions $f_{\ell,\ell+1}$ and $g_{\ell,\ell+1}$, which appear in $\psi_V$ and $\Omega^0$ at the presence of higher satellite $(\ell + 1, m)$, the similar expressions are obtained if we replace here $\ell$ by $\ell + 1$:

$$f_{\ell,\ell+1} \cong \frac{1}{2}\frac{1}{\ell!}\frac{1}{(\ell+1)!}\left[\frac{x}{2}\right]^{2\ell-1}\left[\ell(\ell+1) + (2\ell+1)\left[\frac{x}{2}\right]^{2}\right], \tag{A.5}$$

$$g_{\ell,\ell+1} \cong \frac{1}{2}\frac{1}{\ell!}\frac{1}{(\ell+1)!}\left[\frac{x}{2}\right]^{2\ell-1}\left[\ell(\ell+1) + (2\ell+3)\left[\frac{x}{2}\right]^{2}\right]. \tag{A.6}$$

In the lowest approximation, when the second term in expansion in $x$ can be disregarded, $f_{ik} = g_{ik}$. In this case $\Omega^{0} = F(\psi_V)$, and expressions like $\left[\mathbf{a}\ \nabla\psi_V\right]\nabla\Omega^{0}$ vanish. In fact such a degeneracy is not complete, it disappears if terms of the next order in $x$ are taken into account in $f_{ik}$ and $g_{ik}$. Just because of this, to emphasize their difference, we give these functions here in such a form.

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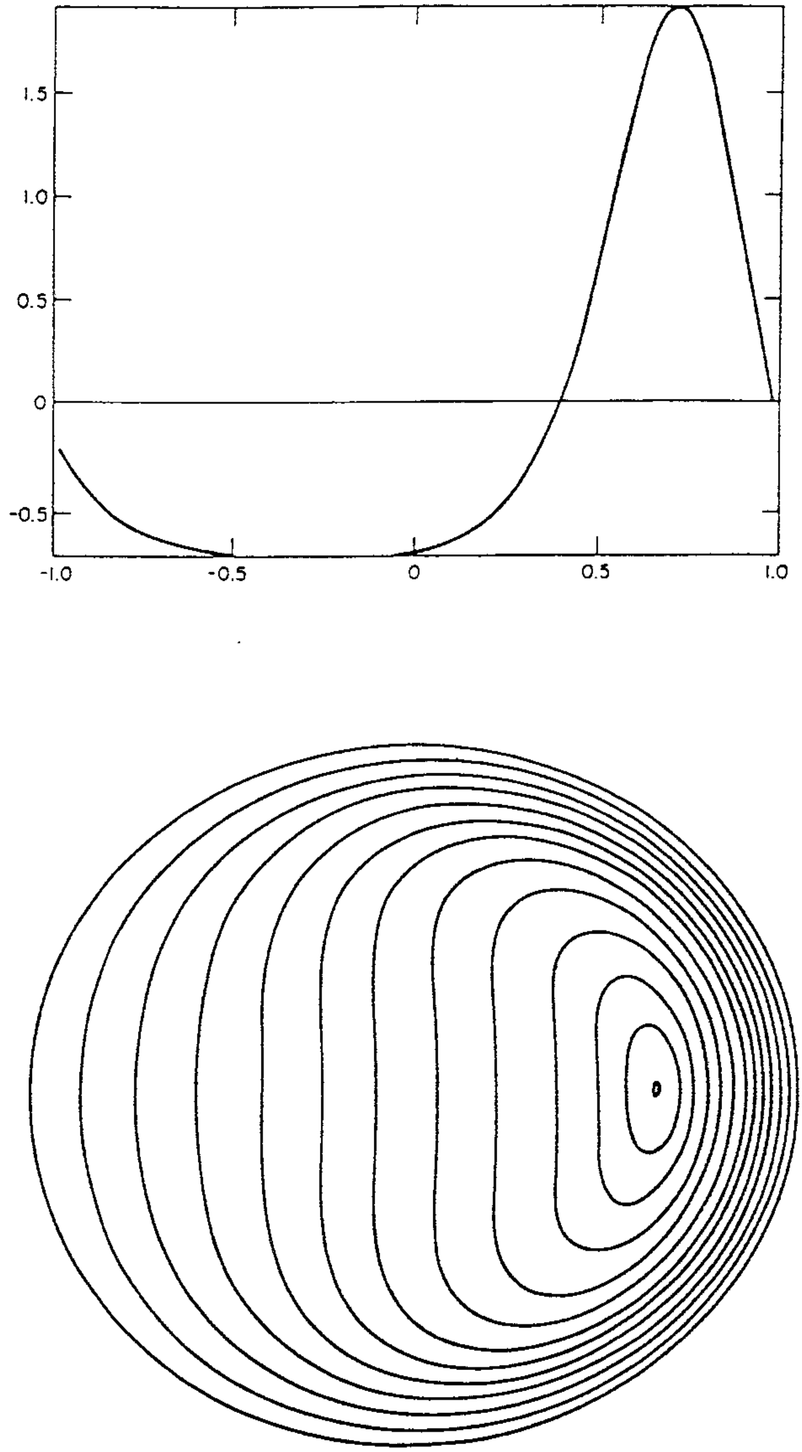


Fig. 1. Pfirsch-Schlüter current as a function of major radius (above) for equilibrium configuration with a strong shift of magnetic surfaces (below) due to finite $\beta$, Ref. [47]

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