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# SUPPRESSION OF PFIRSCH-SCHLÜTER CURRENT BY VERTICAL MAGNETIC FIELD IN STELLARATORS

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## ABSTRACT

This article is devoted to the problem whether it is possible to make stellarator configuration insensitive to plasma pressure using external vertical magnetic field as a control parameter. Conventional stellarators with a planar circular axis are analyzed. It is shown that for these systems the condition of Pfirsch-Schlüter current suppression can be formulated as a two-dimensional equation with all values expressed through the vacuum magnetic field only. The major advantage of this formulation is that it allows to solve the problem without solving equilibrium equations. It is used as a basis to show that complete suppression of Pfirsch-Schlüter current by means of the vertical field is possible, in principle, either in shear-free systems or in stellarators with $\ell \geq 3$. In $\ell=2$ stellarators with a shear it is possible to get significant reduction of the current and to suppress pressure-induced plasma column shift. This can be called an integral independence on $\beta$ in contrast to the true local independence in other cases. In all cases large inward shift of plasma column is necessary. It is out of operational range in all existing devices except Heliotron E. In recent experiments in Heliotron E the state "integrally" independent on $\beta$ was almost achieved. The present study was stimulated by this result.

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**Keywords:** stellarator, plasma equilibrium, Pfirsch-Schlüter current, plasma column shift

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## 1. INTRODUCTION

Recently plasma column shift due to finite $\beta$ has been experimentally studied in the helical device Heliotron E [1]. It was observed that dependence of the shift on $\beta$ changed dramatically when external vertical field was applied to control the position of magnetic axis of the initial vacuum configuration. Inward shift (into the region of stronger toroidal field) of vacuum magnetic axis resulted in the significant decrease of finite-$\beta$ plasma shift. In one case with the largest inward shift magnetic configuration became almost insensitive to $\beta$ [1], which is similar to the effect predicted theoretically in [2,3].

The reason of the weaker dependence of plasma shift on $\beta$ is quite clear: the reduction of Pfirsch-Schlüter current [2]. It is well-known that this reduction is related with a magnetic hill [3-5], which is an inherent property of configurations formed by the superposition of toroidal and helical magnetic fields [2,6].

This knowledge, however, is based only on two numerical examples [2,3], analytical expression for Pfirsch-Schlüter current in a stellarator [4,5], and a discussion in [5] illustrated by a simple analytical example. The problem did not attract much attention because the combination of parameters, where reduction of Pfirsch-Schlüter current could be essential, was somewhat aside from a typical range. Large magnetic hill was necessary as the main element, but the general tendency in stellarator research was completely opposite: configurations with a magnetic well are much more attractive from the viewpoint of plasma stability.

Heliotron E is a unique device in the family of conventional stellarators. It has the largest rotational transform and the largest magnetic hill, as compared with other stellarators. Flexibility of the device allows to scan configurations with different position of vacuum magnetic axis, including those with strongly inward shifted axes. This inward shift is another condition for getting Pfirsch-Schlüter current reduction [2-5] in stellarators with a magnetic hill, and it is also undesirable because of lowering instability threshold.

So, the effect observed in Heliotron E is rare. It hardly could be seen before by one more reason. In experiments with low-$\beta$ plasma the shift of the column is so small, that it remains undetectable if not intendedly measured. The measurements must have high accuracy and good resolution to deal with a shift on the level of 1% of minor radius. Similar measurements are performed in tokamaks, but results [1] from Heliotron E are the first of such kind for helical devices.

Experimental results [1] agree qualitatively with predictions [2-5] of MHD equilibrium theory, which encourages further analysis of the problem in the frame of MHD models. Another motive is the evident insufficiency of known results [2-5] for definite conclusions for existing or realistic stellarators with shear. Because numerical simulation [2,3] was performed for fixed (and rather exotic) set of parameters, and brief discussion in Ref. [5] was supported by an example valid for shear-free stellarators only. Dependence of vertical magnetic field $B_\perp$ necessary for Pfirsch-Schlüter current suppression on the device parameters remained unknown. It was unclear, as well, whether there is any difference between $\ell=2$ and $\ell=3$ stellarators or whether the value of $B_\perp$ depends on plasma pressure profile or not.

The primary goal of our work is to get wider and more detailed understanding of Pfirsch-Schlüter current suppression by vertical magnetic field in stellarators and to reveal major dependencies. Our approach is fully analytical. In the next Section the condition for complete Pfirsch-Schlüter current suppression is formulated in an invariant differential form. Section 3 is devoted to its analysis. It is shown there that, in principle, this condition can be satisfied in all stellarators except $\ell=2$ systems with shear. This latter case, the most interesting from both theoretical and experimental viewpoints, is described in more details in Section 4. Weaker condition of the suppression of pressure-induced plasma column shift is discussed in Section 5. Results are briefly summarized then in the Conclusion.

## 2. BASIC EQUATIONS

General expression for Pfirsch-Schlüter current in conventional stellarators with planar circular axis and helical magnetic field can be written as [7,8]

$$j_\zeta = 2\pi R p'(\psi)(\Omega - \langle\Omega\rangle). \tag{1}$$

Here $R$ is the major radius, $p$ is the plasma pressure, $\psi$ is the poloidal flux, $\Omega$ is a value characterizing the inhomogeneity of the magnetic field,

$$\Omega = \frac{<\tilde{B}^2>_\zeta}{B_0^2} + 1 - \frac{R^2}{r^2} \cong \frac{<\tilde{B}^2>_\zeta}{B_0^2} - \frac{2\rho}{R} cos u, \tag{2}$$

$\tilde{\boldsymbol{B}}$ is the helical field, $B_0$ is the toroidal field at geometrical axis, $\rho, u, \zeta$ are quasi-cylindrical coordinates related with geometrical axis so that $r = R - \rho cos u$, brackets $\langle ... \rangle$ denote volume averaging and $\langle ... \rangle_\zeta$ stands for averaging over toroidal angle $\zeta$:

$$\langle X \rangle \equiv \frac{d}{dV}\int_V X d\tau, \qquad \langle X \rangle_\zeta \equiv \frac{1}{2\pi}\int_0^{2\pi} X d\zeta.$$

Expression (1) is valid for any shape of magnetic surfaces. In a simplest case when cross-sections of averaged magnetic surfaces are shifted circles

$$r = R - \rho cos u = R + \Delta - a cos\theta, \quad z = \rho sin u = a sin\theta, \tag{3}$$

it is reduced to [3-5]

$$j_\zeta = \frac{2p'(a)}{\mu B_0}\left[1 + B_0^2 V_0''(\Phi)\frac{\Delta}{2}\right] cos\theta. \tag{4}$$

Here $\mu$ is the rotational transform, $V$ is the volume inside a magnetic surface, $\Phi$ is the toroidal magnetic flux, subscript "0" at $V$ shows that derivative is taken at $\beta = 0$ and toroidal corrections are disregarded. It is supposed also in deriving (4) that there is no shift in the initial vacuum configuration, so that effect of a vertical field should be included through $\Delta$. For such configurations with a single dominating harmonic of a helical field $\propto sin(\ell u - m\zeta)$ "magnetic hill" $V_0''(\Phi)$ is given by [6]

$$B_0^2 \frac{d^2 V_0}{d\Phi^2} = \frac{R}{a}\frac{d}{da}\frac{<\tilde{B}^2>_\zeta}{B_0^2} = \frac{m\left(a^4 \mu_h\right)'}{\ell R a^3}. \quad (5)$$

Here $\mu_h$ is the vacuum rotational transform produced by the helical field $\tilde{\boldsymbol{B}}$, which may differ from the total rotational transform $\mu$ even in the absence of net toroidal current. The quantity $V_0''(\Phi)$ is positive, therefore to get a reduction of Pfirsch-Schlüter current one has to have inward shift, $\Delta < 0$ [2-5].

It was pointed out in Ref. [5] that this current vanishes if, according to (4),

$$B_0^2 V_0''(\Phi)\Delta = -2. \quad (6)$$

Very simple example was proposed in [5] to show the possibility to fulfill (6): $\ell = 2$ shearless stellarator. If $\mu = const$, then $RB_0^2 V_0''(\Phi) = 2m\mu$, and one gets for $\Delta$ from (6):

$$\frac{m\Delta}{R} = -\frac{1}{\mu}. \quad (7)$$

The shift must satisfy the equilibrium equation [3-5,9]

$$\left[a^3(\mu\Delta)'\right]' + \Delta\left[a^3(\mu_h - \mu)'\right]' = \frac{2p'(a)a^2 R}{\mu B_0^2}\left(1 + \frac{1}{2}B_0^2 V_0''(\Phi)\Delta\right). \quad (8)$$

At $\mu = \mu_h = const$ the left-hand side of this equation vanishes if $\Delta = const$, which means that (7) is compatible with (8). Thus, (7) can be considered as a solution of the problem.

This conclusion of Ref. [5] could be true, if it would not contradict to the initial assumption that $\mu = const$. In $\ell = 2$ stellarators with a single harmonic of the helical field

$$\mu_h \cong \mu_0\left[1 + \frac{1}{2}\left(\frac{m\rho}{R}\right)^2\right]. \quad (9)$$

There is no shear if $mb/R \ll 1$, where $b$ is the minor radius of the boundary cross-section. The same inequality must be valid for the left-hand side of (7), if shift $\Delta$ is not larger than $b$. If so, rotational transform must be very large to make small the right-hand side of (7). But usually $\mu$ is of the order of unity (larger in Heliotron E, but it has very large shear), so condition (7) of the complete Pfirsch-Schlüter current compensation in shear-free stellarators is far from being realistic at present.

Analysis of Ref. [5] is not much informative for stellarators with shear. Because at $\mu' \neq 0$ functions $V_0''(\Phi)$ and $\Delta$ have different dependence on minor radius $a$, and condition (6) can be satisfied on a single magnetic surface only. More than that, the method based on Eqs. (4) and (8) cannot be directly applied to $\ell = 3$ stellarator with $\mu_h = \mu_b \rho^2 / b^2$, which was the first known example to demonstrate the effect numerically [2]. In this case Eq. (8) fails to describe configuration properly in the central region, because its solution $\mu\Delta = const$ at $p' = 0$ gives unlimited $\Delta$ at the magnetic axis. If we will try to avoid this singularity in the most natural way by substituting some constant instead of $\Delta(0)$ into Eq. (6), its left-hand side will vanish at the axis, because in this case $V_0''(\Phi) \propto a^2$, see (5). In addition, there is another singularity in Eq. (4) due to the presence of $\mu$ in the denominator. And all this leads to definitely pessimistic conclusion, which is in absolute disagreement with the result of Ref. [2].

This reflects well-known difficulty (and, sometimes, potential unreliability) of analytical description of $\ell = 3$ stellarators, which is related with vanishing of $\mu_h$ at the axis. Similar difficulties remain also in the case of large-shear $\ell = 2$ stellarators.

Fortunately, the problem is rather associated with drawbacks of standard theoretical approaches than physics. It can be elegantly overcome by avoiding the use of the traditional analytical model (3), which certainly turns out to be inadequate for stellarators with $\ell \geq 3$ and in some cases for $\ell$=2 also.

To show this, let us turn again to the initial general expression (1) for

Pfirsch-Schlüter current. Its right-hand side could be identically zero if $\Omega = \langle\Omega\rangle$. This is equivalent to the condition

$$\Omega \;=\; \Omega(\psi). \tag{10}$$

It looks rather simple, but function $\psi$ is an integral characteristic, which considerably complicates the equation. This is the reason why simplified geometrical model (3) is widely used in the analytical theory. It allows not to worry about true dependence of $\psi$ function on the magnetic field.

In conventional stellarators this dependence looks like [9,10]

$$\overline{\boldsymbol{B}}_p \;=\; \frac{1}{2\pi}\nabla(\psi \;-\; \psi_V)\times\nabla\zeta, \tag{11}$$

where $\overline{\boldsymbol{B}}_p$ is the axisymmetric component of the poloidal field, and $\psi_V$ is the poloidal flux of the helical field $\tilde{\boldsymbol{B}}$:

$$\psi_V \;=\; \frac{2\pi r^3}{RB_0} < \tilde{B}_z \int \tilde{B}_r\, d\zeta >_\zeta. \tag{12}$$

In all expressions here function $\psi$ does not depend on $\zeta$. This allows to rewrite the condition (10) as $\nabla\psi\times\nabla\zeta\cdot\nabla\Omega = 0$. And, finally, using (11) we get instead of (10) an equivalent differential formulation of the condition:

$$\left(\overline{\boldsymbol{B}}_p \;+\; \frac{\nabla\psi_V\times\nabla\zeta}{2\pi}\right)\cdot\nabla\Omega \;=\; 0. \tag{13}$$

The advantages of this new differential formulation can be seen easily. First of all, it is not necessary to calculate $\psi$ function for analyzing the condition of Pfirsch-Schlüter current compensation. Then, to analyze this condition, it is sufficient to substitute vacuum field for $\overline{\boldsymbol{B}}_p$ into (13), because there is no additional poloidal field at the absence of Pfirsch-Schlüter current. And, finally, equation (13) is written in an invariant form which is the best for starting any analytical or numerical calculations.

## 3. GENERAL SOLUTION OF EQUATION (13)

To prove these statements, let us consider the problem of interest using this equation as a starting point. For the beginning it is necessary to know two basic functions characterizing vacuum configuration of a stellarator: $\psi_V$ and $\Omega_0 \equiv \langle \tilde{B}^2 \rangle_\zeta / B_0^2$. They can be calculated with a desired accuracy if helical field $\tilde{B}$ is known. But in the case when there is no shift of vacuum magnetic surfaces without an additional external vertical field, both of them can be expressed through the vacuum rotational transform $\mu_h$: $\Omega_0$ is given by (5), and (for more details see [9])

$$\psi_V = -2\pi B_0 \int \rho \mu_h(\rho) d\rho. \tag{14}$$

In this case $\psi_V = \psi_V(\rho)$, $\Omega_0 = \Omega_0(\rho)$, and because of this

$$\nabla \psi_V \times \nabla \zeta \cdot \nabla \Omega_0 = 0. \tag{15}$$

It is clear that $\Omega_0$ is the asymptotic value of $\Omega$ in the limit of infinite aspect ratio, $R/b \to \infty$. Therefore, the last relationship shows very natural result: the absence of Pfirsch-Schlüter current in a straight stellarator. As for real devices, there is an additional contribution to $\Omega$ due to toroidicity, see (2). As a result we get

$$\frac{\nabla \psi_V \times \nabla \zeta}{2\pi} \cdot \nabla \Omega = 2 B_0 \mu_h \frac{\rho}{R^2} \sin u. \tag{16}$$

If there is an additional vertical field $B_\perp e_z$ (as was first proposed in [2]), then

$$B_\perp e_z \cdot \nabla \Omega = B_\perp e_z \cdot \nabla \Omega_0 = B_\perp \frac{m\left(\rho^4 \mu_h\right)'}{\ell R^2 \rho^2} \sin u. \tag{17}$$

Equation (13) becomes now

$$2 B_0 \rho \mu_h + B_\perp \frac{m\left(\rho^4 \mu_h\right)'}{\ell \rho^2} = 0. \tag{18}$$

It looks like $\rho y' + C y = 0$ for unknown function $y = \rho^4 \mu_h$, and its solution is

$$\mu_h = \mu_b x^{-(C+4)}, \tag{19}$$

where $\mu_b$ is a constant, $x = \rho / b$ is the dimensionless radius, and $C = 2\ell B_0 / (m B_\perp)$.

This formal solution of Eq.(13) could be a true solution of the problem if it would coincide with a real profile of the vacuum rotational transform $\mu_h$.

First, we must conclude that it is not a case for $\ell = 2$ stellarators with a shear, where $\mu_h$ can be approximated by (9) or similar dependence with terms of higher orders.

But for all other stellarators with a single harmonic of the helical field very often the expression

$$\mu_h = \mu_b x^{2(\ell-2)} \tag{20}$$

is used to describe $\mu_h$. It coincides with (19) when $C + 4 = -2(\ell - 2)$, which leads to

$$B_\perp \;=\; -B_0 / m. \tag{21}$$

The negative sign here shows that necessary vertical field must be oriented opposite to $\boldsymbol{e}_z$: $\boldsymbol{B}_\perp = -\boldsymbol{e}_z |B_\perp|$. This our result given by (21) should be valid for $\ell = 2$ stellarators without shear and for stellarators with $\ell \geq 3$. We can check it in "two points" in a parameter space by comparing (21) with predictions of [2] and [5] for two particular examples.

In [2] equilibrium equation was numerically solved with prescribed plasma pressure profile for $\ell$=3 large-aspect-ratio ($R/b = 16$) stellarator with given numbers for its parameters. It was shown there that at $\beta$=2.5% magnetic configuration was geometrically the same as the vacuum one if vertical field with amplitude $|B_\perp| / B_0 = R^{-1} = 0.05$ was applied to shift magnetic axis inward. Some dimensionless units were used for distances. In the example considered $m / R = 1$, which corresponds to $m$=20. With this number our formula (21) gives exactly the same result for $B_\perp$ as that obtained in [2].

This absolute agreement seems unbelievable. We get the result by essentially much more simple way, without solving equilibrium problem, using as an input vacuum magnetic field only. At the same time we do not lose accuracy. It could be expected, however, because no restrictive simplifying assumptions were made in our analysis. The result reflects the basic intrinsic property of vacuum magnetic field only, and our approach is just the most direct way to analyze vacuum configuration avoiding unnecessary equilibrium calculations.

It follows naturally from our consideration that complete suppression of Pfirsch-Schlüter current in $\ell$=3 stellarators under condition (21) is a feature which does not depend on plasma pressure profile. It is not related also with the particular choice of its parameters in [2]. For completeness, the same can be said about nonexisting stellarators with $\ell > 3$.

To compare (21) with the result [5] represented by (7), we need a relation between shift $\Delta$ and vertical field $B_{\perp}$. Again only vacuum dependence is necessary, which is $\mu\Delta = RB_{\perp} / B_0$ for a stellarator with $\mu = const$. It makes (7) and (21) equivalent.

We have shown that known results [2] and [5] are reproduced by the general solution (21). Relationship (21) is a consequence of the local condition (13). When it is fulfilled, there is no Pfirsch-Schlüter current at all. But in the most interesting case, for $\ell$=2 stellarators with a shear, Pfirsch-Schlüter current cannot vanish at every point over the minor radius. Because of this configuration cannot be made completely insensitive to plasma pressure. We can conclude that, despite seeming similarity of the results obtained in [2] and [3] for $\ell$=3 and $\ell$=2 stellarators, there is a great difference of underlying physics in these cases.

This conclusion could not be made on the basis of previous results [2,3], because only dependence of plasma column shift on $\beta$ was given in [3] for $\ell$=2 stellarator, which is an integral characteristic. And there was no local analysis for comparing magnetic surfaces in vacuum and at finite $\beta$ or for getting Pfirsch-Schlüter current distribution.

## 4. LOCAL ANALYSIS FOR $\ell=2$ STELLARATOR

It can be easily seen that in stellarators with parabolic profile of vacuum rotational transform

$$\mu_h = \mu_0 + (\mu_b - \mu_0)x^2, \tag{22}$$

where $\mu_0$ is the value of $\mu_h$ at the axis and $\mu_b$ is its value at the boundary, condition (18) is reduced to

$$(\mu_b - \mu_0)\left(B_0 + \frac{3m}{\ell}B_\perp\right)x^2 + \mu_0\left(B_0 + \frac{2m}{\ell}B_\perp\right) = 0. \tag{23}$$

For $\ell$=3 and shear-free $\ell$=2 stellarators, which are included here as two end limits $\mu_0$=0 and $\mu_b$=$\mu_0$, respectively, Eq. (23) is satisfied by (21) at any radius. But in the intermediate case, $\ell$=2 stellarator with a shear, it can be satisfied at a single point only over minor radius:

$$x_0^2 = -\frac{\mu_0}{\mu_b - \mu_0}\frac{1 + mB_\perp / B_0}{1 + 1.5mB_\perp / B_0}. \tag{24}$$

Right-hand side of (24) must be positive, at the same time it should not exceed unity because, by definition, $x = \rho / b$ where $b$ is the radius of the boundary. These two natural demands are met when

$$-1 \le \frac{mB_\perp}{B_0} \le -\frac{2\mu_b}{3\mu_b - \mu_0}. \tag{25}$$

At the lower limit for $B_\perp$, which is the same as the value (21), $x_0$=0. At the upper limit, which gives smaller value of $|B_\perp|$, $x_0$=1. In the former case $\Omega = \Omega(\psi)$ at the axis, and in the latter one $\Omega = \Omega(\psi)$ at the boundary. At intermediate values of the vertical field Pfirsch-Schlüter current vanishes at a single magnetic surface inside the plasma. The current changes its phase when crossing this surface.

To consider in more details this phenomena, let us rewrite expression (4 ) in the form

$$j_\zeta = j_0 C_{PS}, \tag{26}$$

where $j_0$ is the value of Pfirsch-Schlüter current for the case without shift, and $C_{PS}$ is a coefficient showing increase or decrease of the current when $\Delta \neq 0$:

$$j_0 \equiv \frac{2p'(a)}{\mu B_0}\cos\theta, \qquad C_{PS} \equiv 1 + \frac{m\mu_h\Delta}{2\ell R}\frac{\left(a^4\mu_h\right)'}{a^3\mu_h}. \qquad (27)$$

In $C_{PS}$ magnetic hill is replaced with account of (5), and terms are combined to use the fact that $\mu_h\Delta \cong const$, when configuration does not depend strongly on plasma pressure.

If there is no shift, $C_{PS}$=1. At negative (inward) shift $C_{PS}$ becomes smaller. If $\Delta < 0$, then $C_{PS}' < 0$ also. Therefore, when inward shift becomes larger, at first $C_{PS}$ may turn to zero at the edge. It corresponds to the right limit in (25). At more stronger shift $C_{PS}$<0 at the edge, but positive at the axis, vanishing between them at the point (24). With increasing shift this point moves inside and, finally, comes to the axis. It happens when the left limit of (25) is reached. Then $C_{PS}$=0 at the axis and negative everywhere. This behaviour of $C_{PS}$ is illustrated by Fig. 1 for the case $\mu_0 / \mu_b = 1/3$.

Shift $\Delta$ is a convenient characteristic to describe geometry of magnetic surfaces. But for practical purposes it could be useful to express results in terms of $B_\perp$, which is used as a real external control parameter in experiments. It is not easy in a general case, because, as one can see from Eq. (8), $\Delta$ depends on $\beta$ and pressure profile. Fortunately, with decreasing $|C_{PS}|$ this dependence becomes weaker and weaker. If disregarded, then

$$C_{PS} = 1 + \frac{1}{2\ell}\left(4 + \frac{a\mu_h'}{\mu_h}\right)\frac{mB_\perp}{B_0}. \qquad (28)$$

To transform (27) into (28), one needs the "vacuum" solution $\mu_h\Delta = const$ of Eq. (8) at $\mu=\mu_h$ supplemented by the boundary conditions

$$\Delta_b = \Delta_\perp + \Delta_\beta, \qquad (29)$$

where $\Delta_b$ is the shift of the plasma boundary (which was measured in Heliotron

E [1]), $\Delta_\perp$ is the contribution to $\Delta_b$ due to the vertical field $B_\perp$, and $\Delta_\beta$ is the shift due to plasma pressure:

$$\frac{\Delta_\perp}{b} = \frac{B_\perp}{B^*}, \qquad \frac{\Delta_\beta}{b} = -\frac{\beta_0}{2\beta_{eq}^0}\int_0^b \frac{p'(a)}{p_0}\frac{\mu_b}{\mu(a)}\frac{a^2}{b^2}C_{PS}da. \tag{30}$$

Two constants here are

$$B^* \equiv \mu_b \frac{b}{R} B_0, \qquad \beta_{eq}^0 \equiv \mu_b^2 \frac{b}{R}. \tag{31}$$

The first one can be considered as the "effective" poloidal field at the plasma boundary, and the second one is a scale for measuring $\beta$, often used for rough estimate of equilibrium $\beta$ limit. If weak dependence of $\Delta$ on $\beta$ is assumed, then $\Delta_\beta$ must be small, and

$$\mu_h \Delta = \mu_b \Delta_b = R\frac{B_\perp}{B_0}. \tag{32}$$

With this relationship $C_{PS}$ from (27) turns into (28).

It is worth to note that in the case of large shear Eq. (32) may be not good enough to describe near-axis region, where $\mu_h << \mu_b$. Fortunately, this central region does not give essential contribution when integration (30) is performed (because of $a^2$ there).

For $\ell$=2 stellarators with shear $C_{PS}$ is not a constant, and, therefore, in this case Pfirsch-Schlüter current cannot be completely suppressed by shifting plasma column inward. But $C_{PS}$, first, can be made much smaller than unity, and, second, it is possible to force $C_{PS}$ even to change its sign.

The smaller $C_{PS}$, the smaller Pfirsch-Schlüter current. $C_{PS}$ can be reduced by a factor of 4÷7 and, locally, even larger, as it is shown in Fig. 1. Corresponding significant reduction of Pfirsch-Schlüter current shows itself in smaller shift $\Delta_\beta$, see (30). This is the effect experimentally observed in Heliotron E [1].

One can see from (30) that at $C_{PS}> 0$ shift $\Delta_\beta$ is positive. It is a natural outward shift due to finite $\beta$. But at $C_{PS}< 0$ shift $\Delta_\beta$ becomes negative. During continuous transition from positive to negative with proper increase of $|B_{\perp}|$ shift $\Delta_\beta$ must pass through zero point. It can happen when $C_{PS}$ changes its sign somewhere inside the plasma, similar to that shown by solid line in Fig. 1. The visible effect will be that plasma column will remain in a fixed position, which will not change with increase of $\beta$. This position is determined by the external vertical field only. From this point of view configuration can be considered as insensitive to $\beta$. One numerical example of such a kind for $\ell$=2 stellarators with shear is known from [3]. Our analysis explains this result now and reveals a principal difference between cases considered in [2], [3] and [5].

The main physical difference between $\ell$=2 systems with shear and other stellarators is that in the first case complete suppression of Pfirsch-Schlüter current cannot be achieved by means of external vertical field, but in the second case the current can be made identically zero (in the frame of our theoretical model) all over the plasma cross-section. Without Pfirsch-Schlüter current the geometry of magnetic surfaces is not affected even locally by plasma pressure. This cannot be realized in $\ell$=2 stellarators with shear. However, it is possible to suppress pressure-induced shift of a plasma column. This can be called an integral compensation of Pfirsch-Schlüter current effect.

## 5. INTEGRAL INDEPENDENCE ON $\beta$

This integral compensation occurs at some $B_{\perp}$ from the interval (25). It is clear that for peaked pressure profiles stronger vertical field is necessary to get $\Delta_\beta$=0, because in this case zero point of $C_{PS}$ must be close to the axis. And for flat pressure distribution this field should be smaller. To find the exact value of $B_{\perp}$ necessary for suppressing $\Delta_\beta$, one must calculate the integral (30) with given pressure distribution $p(a)$.

Let us assume that

$$p = p_0(1 - \psi / \psi_b), \tag{33}$$

where $\psi_b$ is the value of $\psi$ at the edge. This pressure profile was used in numerical calculations in Ref. [3]. By definition, $\mu = -\psi' / \Phi'$, hence

$$-\frac{p'(a)}{\mu(a)} = \frac{dp}{d\psi}\Phi' \cong 2\pi a B_0 \frac{dp}{d\psi}. \tag{34}$$

Expression (30) in this case looks like

$$\frac{\Delta_\beta}{b} = -\frac{\beta_0}{4\beta_{eq}^0}\frac{\pi b^2 B_0 \mu_b}{\psi_b} I, \tag{35}$$

where $I$ is the integral

$$I = \int_0^1 4x^3\left[1 + \left(2 + \frac{(\gamma - 1)x^2}{1 + (\gamma - 1)x^2}\right)\frac{mB_\perp}{\ell B_0}\right]dx, \tag{36}$$

and $\gamma = \mu_b / \mu_0$. For $C_{PS}$ Eq. (28) was used here with $\mu_h$ given by (22). Integration in (36) gives

$$I = 1 + \left[3 - \frac{2}{\gamma - 1}\left(1 - \frac{ln\,\gamma}{\gamma - 1}\right)\right]\frac{mB_\perp}{\ell B_0}. \tag{37}$$

This value is unity when $B_\perp = 0$. It linearly decreases with $B_\perp$ for negative values of $B_\perp$, and so does the shift $\Delta_\beta$. When $I$=0, there is no shift due to plasma pressure. From (37) one gets for this case very simple expression for necessary $B_\perp / B_0$, which depends on $m / \ell$ and $\gamma$ only. For one particular set of parameters we can compare this result with that numerically obtained in [3]. Calculations in [3] were performed for $\ell$=2 stellarator with $m = 100$ and extremely large aspect ratio $R / b = 100$. Instead of rotational transform the relative amplitude $\varepsilon_h$ of $\ell$=2 helical harmonic was given in [3], which allows to get for this case $\gamma$=1.53. For these parameters we get $-B_\perp / B_0$=0.00887. In [3] this number was found to be 0.0088. We can see again that our analytical model has excellent accuracy. It is worth to note that we could use Eq. (9) to find $\gamma = \mu_b / \mu_0$ even without knowing $\varepsilon_h$. It follows from (9) that $\gamma = 1 + 0.5mb / R$, which is 1.5 for the same choice of $m$ and $R/b$ as in Ref. [3]. Correspondingly, $-B_{\perp}/B_0$=0.00891, which differs by 1.25% only from the numerical result [3].

The ratio $|B_{\perp}|/B_0$ in this example is small, because at $mb/R=1$, which is a reasonable value, $m$ is enormously large. Among existing helical devices, Heliotron E has the largest $m$. It allows to see the effect of reduction of the pressure-induced plasma shift in the range of relatively moderate $|B_{\perp}|/B_0$. Other stellarators have smaller $m$, so larger $|B_{\perp}|/B_0$ are needed. The feasibility of complete suppression of Pfirsch-Schlüter current effect `in conventional stellarators is limited also by another practical restriction. When vertical field is applied, plasma column is shifted as a whole, which is described by $\Delta_{\perp}$ in (29). At $m|B_{\perp}|/B_0=1$, which gives the order of necessary vertical field for all cases considered here, this shift can be estimated as

$$\frac{|\Delta_{\perp}|}{b} = \frac{R}{mb\mu_b}. \tag{38}$$

For parameters of Heliotron E [1] this value is, approximately, 0.2. It is rather large, but tolerable for experiment. For other stellarators this value is much larger. For example, it is 0.6 for CHS ($R/b\cong 5$, $m=8$, $\mu_b\cong 1$) and 0.65 for LHD ($R/b\cong 6.5$, $m=10$, $\mu_b\cong 1$). It is out of the acceptable range, because plasma-wall clearance is not so wide in the devices.

## 6. CONCLUSION

We have shown that condition of the suppression of Pfirsch-Schlüter current in conventional stellarators is equivalent to two-dimensional differential equation (13). It contains characteristics of vacuum magnetic field only, which considerably simplifies the problem. A posteriori, it seems very natural: if configuration is not affected by plasma pressure, its properties must be completely determined by the vacuum field.

Our model is simple enough to perform calculations analytically, and at the same time it allows to reproduce with a striking accuracy both known numerical results [2] and [3]. These results were obtained for fixed parameters, therefore they could not show relevant functional dependencies. And even a difference between $\ell$=2 and $\ell$=3 stellarators could not be seen from [2] and [3].

However, it follows from our analysis that complete suppression of Pfirsch-Schlüter current by external vertical field in stellarators is possible, in principle, in the case only when vacuum rotational transform profile can be approximated by power dependence (20). This family includes shear-free stellarators and stellarators with $\ell \geq 3$. In these systems Pfirsch-Schlüter current could vanish all over the radius at $B_{\perp} = -B_0 / m$, and configuration would be then unaffected by plasma pressure. It can be called a local independence on $\beta$. This is the property of vacuum configuration itself and it is not related with plasma parameters.

In $\ell$=2 stellarators with shear, at the contrary, only integral independence can be achieved when $B_{\perp}$ is in the range (25). Ends of this interval correspond to vanishing of Pfirsch-Schlüter current at the axis or at the edge of plasma column. At intermediate values of $B_{\perp}$ Pfirsch-Schlüter current vanishes at some inner magnetic surface, being oppositely directed inside and outside, as shown by solid curve in Fig. 1. Cancelation of the magnetic fields of these opposite currents can result in the suppression of $\beta$-induced plasma shift $\Delta_{\beta}$. The value of $B_{\perp}$ necessary for getting $\Delta_{\beta}$=0 depends in this case on pressure profile: for more peaked $p(a)$ larger $|B_{\perp}|$ are needed. But interval (25) is not too wide, which shows that this dependence is rather weak. It should allow to get strong integral effect even without careful adjusting of $B_{\perp}$ inside limits (25). If not a complete suppression of Pfirsch-Schlüter current and associated pressure-induced shift $\Delta_{\beta}$, at least it must be their significant reduction.

In all cases to reach the state with $\Delta_{\beta}$=0, rather large inward shift of plasma column is necessary, which can be approximated as $R/(m\mu_b)$. At typical parameters it lies beyond the admissible limits. Heliotron E is a unique device where this value is in the range of practical accessibility. Results [1] show clear tendency to the state with $\Delta_\beta$=0, which was already very near, but still was not achieved. If achieved, then at larger $|B_\perp|$ "overcompensation" should occur with $\Delta_\beta$<0. It seems that even such exotic regimes can be realized in Heliotron E. It does not promise any practical advantage, however, except getting new knowledge in plasma physics and another more detailed verification of MHD equilibrium theory.

Comparing experimental results [1] with theoretical predictions, we can see their very good agreement. To make this comparison more straightforward and informative, it is necessary to establish exactly which Heliotron E configuration is the most similar to the model (14). Parameters of this configuration are used as a reference basis in our analysis, and all results are expressed through these values. The "standard" configuration of Heliotron E [1] with $R$=220 cm is a little bit asymmetric: its axis is shifted outward with respect to the boundary, see [11]. It should be attributed to some initial $\Delta_\perp$, which could be easily found from known maps of Heliotron E magnetic surfaces at different $B_\perp$.

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## *P-S coefficient*

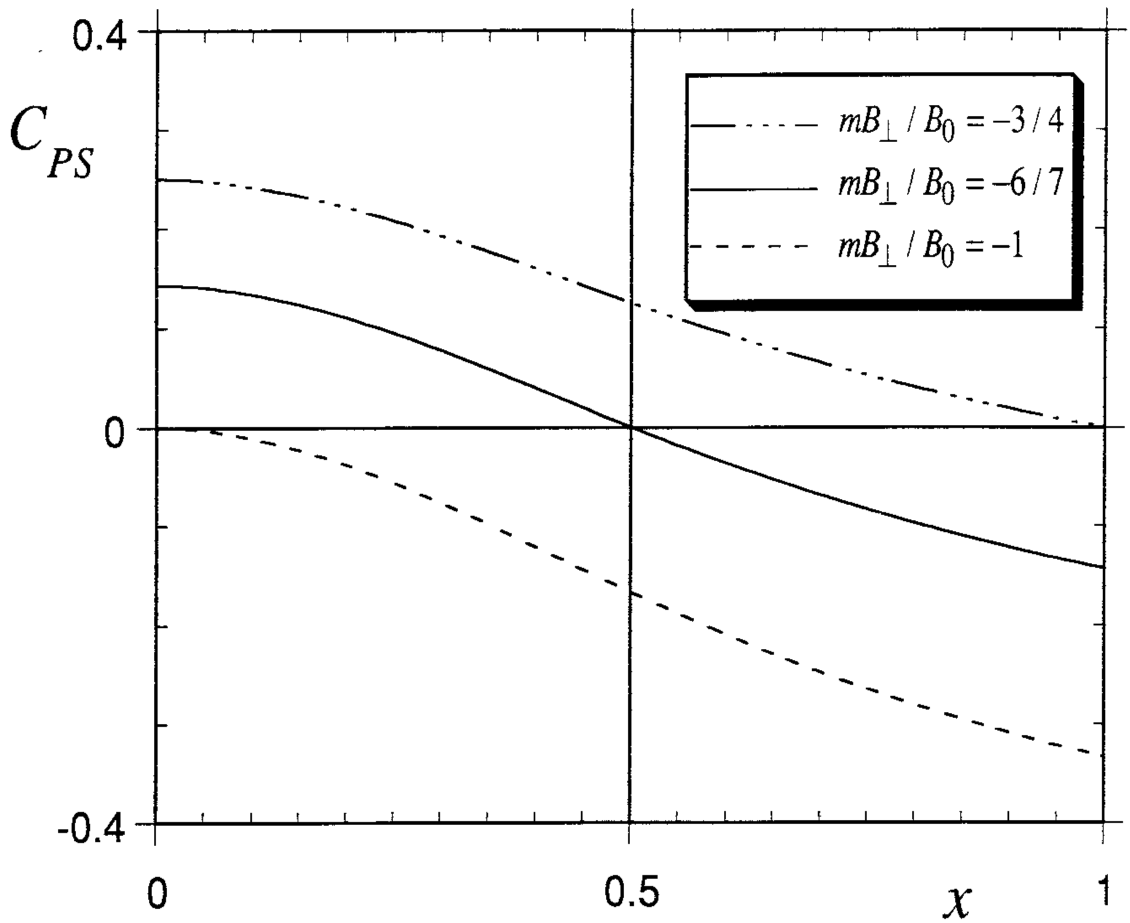


Fig. 1. Amplitude coefficient $C_{PS}$ for Pfirsch-Schlüter current in $\ell$=2 stellarator with $\mu_0 / \mu_b = 1/3$ at different values of $B_\perp / B_0$. The upper curve corresponds to the right end of the interval (25), and the lower one to the left end. Intermediate solid line shows $C_{PS}$ for the case when $C_{PS}$ vanishes at $x = 0.5$.

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