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# COUPLING OF TILTING GAUSSIAN BEAM WITH HYBRID MODE IN THE CORRUGATED WAVEGUIDE

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## Abstract

The mode-conversion loss in the matching between the gaussian beam emanated from the gyrotron and the hybrid mode in the circular corrugated waveguide with the diameter of  $2a$  is discussed. By numerical calculation, it is found that the loss considerably increases and optimum waist size  $w_0$  changes when TEM<sub>00</sub> mode with the wavelength  $\lambda$  is injected with offset or tilt. By fitting numerical data to the polynomial function, it becomes evident that the scaling formulas of the losses for the off-axis shift  $r_d$  and for the tilt angle  $\theta$  are derived to be  $2.3(r_d/a)^2 - 2.2(r_d/a)^4$  and  $3.9(a\theta/\lambda)^2 - 5.6(a\theta/\lambda)^4$  for fixed  $w_0/a=0.643$ , respectively. To keep the mode-conversion loss  $\leq 1\%$  for the frequency of 168 GHz and  $2a=88.9$  mm, tilting angle and offset should be less than 0.1 degrees and 2.9 mm, respectively.

Keywords: gaussian beam, hybrid mode, HE<sub>11</sub> mode, corrugated wave guide, coupling, gyrotron, electron cyclotron heating

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# 1 Introduction

Recently, there has been an interest in electron cyclotron heating (ECH) of high temperature plasmas by high-power millimeter waves. To transmit efficiently the millimeter wave from a gyrotron, it is necessary to suppress a mode-conversion caused by offset and tilt in alignment of transmission system. Since the  $HE_{11}$  mode in the circular corrugated (CC) waveguide has advantages of low transmission loss [1], linear polarization and axisymmetry, the  $HE_{11}$  mode has been adopted as highly reliable transmission mode of millimeter waves [2, 3, 4]. The mode-conversion loss to the  $HE_{21}$  and  $TE_{01}$  modes for H-plane (or  $HE_{21}$  and  $TM_{02}$  modes for E-plane) [5] at the junction of the CC-waveguide with tilt  $\theta$  is given by  $2.2(a\theta/\lambda)^2$ , respectively, where  $2a$  is the waveguide diameter and  $\lambda$  is the wavelength in free space. Thus total loss of  $HE_{11}$  due to the higher mode-conversion is  $4.4(a\theta/\lambda)^2$ . For instance, in the  $HE_{11}$  mode transmission with the frequency of  $\omega/2\pi=170$  GHz by means of the CC-waveguide with  $2a = 88.9$  mm, waveguide tilting of 3 mrad gives the total mode-conversion loss of 2%. As for an axial offset between waveguides  $r_d$ , the mode-conversion loss for the  $HE_{21}$  and  $TE_{01}$  modes for H-plane (or  $HE_{21}$  and  $TM_{02}$  modes for E-plane) [5] is written by  $1.1(r_d/a)^2$ , respectively. The total loss  $2.2(r_d/a)^2$ , for  $r_d/a \sim 0.02$  is as small as 0.8%.

Experimental [3, 4] and theoretical [6, 7] studies on the coupling between a gaussian beam  $TEM_{00}$  from a gyrotron with a Vlasov converter and a hybrid mode  $HE_{11}$  in the CC-waveguide have been carried out. The results indicate that 98 % power of a incident beam can be coupled to the  $HE_{11}$  mode and that a part of remaining power propagates as an higher mode of  $HE_{12}$  in the waveguide and the other spills outside the waveguide. When a gaussian beam is injected to the CC-waveguide, offset and tilt as well as waist parameters affect strongly the coupling efficiency. By using the mode matching method, we analyze the effect of offset and tilt on the coupling efficiencies between a gaussian beam and hybrid modes.

This paper presents the evidence from the numerical calculation that  $TEM_{00}$ -to- $HE_{11}$  mode power coupling efficiency strongly decreases with the incident angle and offset. The scaling laws on frequency and diameter of waveguide are shown for designing the matching optics unit between the gyrotron and transmission line in ECH system. The paper consists of four sections. In section 2, the electromagnetic (EM) fields of gaussian beam and hybrid modes are presented. The mode matching at the entrance of the CC-waveguide is described. The components of electromagnetic fields of obliquely incident gaussian beam at the entrance of the CC-waveguide are calculated. In section 3, the numerical calculation of coupling in the ECH transmission system of the large helical device (LHD) is carried out. The values of off-axis shift and tilting angle are discussed to satisfy the specific coupling efficiency. For a general application, coupling

dependence on waveguide size and frequency for an obliquely or off-axis incident beam is shown and its characteristics are described. By changing the gaussian beam for the  $\text{HE}_{11}$  mode, effects of the tilting and offset of the CC-waveguide on coupling efficiency are given. Conclusion is given in section 4.

## 2 Theoretical consideration

### 2.1 Electromagnetic fields

In the cartesian coordinate system where the origin of  $(x, y, z)$  is located at the center of the CC-waveguide aperture, EM fields of  $\text{TEM}_{nm}$  mode with the waist point  $z_0 = 0$  are given by [8]

$$E_y = \frac{\epsilon_n}{w} \sqrt{\frac{2Z_0 m!}{\pi(m+n)!}} \left(\frac{\sqrt{2}r}{w}\right)^n L_m^n\left(\frac{2r^2}{w^2}\right) \exp\left[-r^2\left(\frac{1}{w^2} + \frac{jk}{2R}\right)\right] \\ \times \cos n\phi \exp\left[-jkz + j(2m+n+1)\tan^{-1}\left(\frac{\lambda z}{\pi w_0^2}\right)\right], \quad (1)$$

$$H_x = -E_y/Z_0, \quad (2)$$

where,  $Z_0 = \sqrt{\mu_0/\epsilon_0}$ ,  $R = z(1 + (\pi w_0^2/\lambda z)^2)$ ,  $w = w_0\sqrt{1 + (\lambda z/\pi w_0^2)^2}$  and  $\phi = \tan^{-1}(y/x)$  and  $L_m^n$  is the generalized Laguerre's polynomial and  $w_0$  is waist size at the waist point, and  $\epsilon_n = 1$  or  $1/\sqrt{2}$  for  $n = 0$  and  $1$ , respectively.

For the hybrid mode with the mode number of  $(n, m)$  in the azimuthal and axial directions and the eigen value  $X_m$ , EM fields [9] can be written by

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = G_m \begin{pmatrix} \sin(n-1)\phi & -\sin(n+1)\phi & 0 \\ \cos(n-1)\phi & \cos(n+1)\phi & 0 \\ 0 & 0 & \sin n\phi \end{pmatrix} \\ \times \begin{pmatrix} (1-d_m)J_{n-1}\left(\frac{X_m r}{a}\right) \\ (1+d_m)J_{n+1}\left(\frac{X_m r}{a}\right) \\ \frac{j2X_m}{R_m Y} J_n\left(\frac{X_m r}{a}\right) \end{pmatrix} \exp(-j\beta_m z), \quad (3)$$

$$\begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = F_m \begin{pmatrix} -\cos(n-1)\phi & -\cos(n+1)\phi & 0 \\ \sin(n-1)\phi & -\sin(n+1)\phi & 0 \\ 0 & 0 & \cos n\phi \end{pmatrix}$$

$$\times \begin{pmatrix} (1 - d_m R_m^2) J_{n-1}(\frac{X_m r}{a}) \\ (1 + d_m R_m^2) J_{n+1}(\frac{X_m r}{a}) \\ \frac{j 2 X_m d_m R_m}{Y} J_n(\frac{X_m r}{a}) \end{pmatrix} \exp(-j \beta_m z). \quad (4)$$

The cross-polarized field is obtained by substituting the cosine function for the sine one and the  $-$ sine function for the cosine one. Here,  $d_m$ ,  $R_m$ ,  $G_m$  and  $F_m$  are defined as

$$d_m = \begin{cases} \frac{J_{n-1}(X_m) + J_{n+1}(X_m)}{J_{n-1}(X_m) - J_{n+1}(X_m)}, & n \neq 0 \\ 1, & \text{for TE}_{0m} \\ -\frac{J_0(X_m)}{X_m J_1(X_m)}, & \text{for TM}_{0m}, \end{cases} \quad (5)$$

$$R_m = \beta_m/k = \sqrt{[1 - (X_m/Y)^2]}; \quad Y = ka, \quad (6)$$

$$G_m = \frac{\sqrt{R_m Z_0}}{2a\sqrt{\pi D_m} J_n(X_m)}, \quad (7)$$

$$F_m = \frac{1}{2a\sqrt{\pi D_m} R_m Z_0 J_n(X_m)}, \quad (8)$$

where  $k = \omega/c = 2\pi/\lambda$  and

$$D_m = \begin{cases} \frac{1}{2} \left[ \left( 1 - \frac{n^2}{X_m^2} + \frac{2n}{d_m X_m^2} + \frac{n^2}{d_m^2 X_m^2} \right) \right. \\ \left. \times (1 + R_m^2 d_m^2) - 2(1 + R_m^2) \frac{n d_m}{X_m^2} \right], & n \neq 0 \\ 1 - \frac{2}{YZ} + \left( \frac{X_m}{YZ} \right)^2, & \text{for TM}_{0m} \\ R_m^2, & \text{for TE}_{0m}. \end{cases} \quad (9)$$

Here, the normalized reactance  $Z$  along  $z$ -direction [9] is given by

$$Z = \frac{j}{Z_0} \left[ \frac{E_z}{H_\phi} \right]_{r=a} = \begin{cases} \frac{X_m^2 d_m}{nY(d_m^2 R_m^2 - 1)}, & n \neq 0 \\ -\frac{X_m^2 d_m}{Y}, & \text{for TM}_{0m}. \end{cases} \quad (10)$$

For the  $\text{TM}_{0m}$  mode, equations describing the cross-polarized field should be used to calculate electromagnetic field. To calculate EM-field, it is necessary that  $X_m$  is determined. By using the Newton-Raphson method to solve the equation,  $X_m$  can be represented as a function of  $d$ . For the sake of simplicity, we use the ideal eigenvalue  $X_m$  satisfying  $J_{n-1}(X_m) = 0$  for  $\text{HE}_{nm}$ ,  $J_{n+1}(X_m) = 0$  for  $\text{EH}_{n,m+1}$  and  $J'_0(X_m) = -J_1(X_m) = 0$  for  $\text{TM}_{0m}$ , since both  $Z$  and  $2a$  have large values in this analysis. All the numerical coefficients of EM fields in gaussian beam and hybrid mode are normalized so as to satisfy flux  $P = \iint_S (\mathbf{E} \times \mathbf{H}^*) \cdot d\mathbf{S} = 1$  where  $\mathbf{S}$  is the whole plane of a cross section.

## 2.2 Evaluation of coupling coefficients

In the waveguide system, the following orthonormal relation between the  $i$ - and  $j$ -modes is valid:

$$\iint_S (\mathbf{E}_i \times \mathbf{H}_j^*) \cdot d\mathbf{S} = \delta_{ij}, \quad (11)$$

where  $S$  is the cross-section of the CC-waveguide and  $\delta_{ij}$  is 1 for  $i = j$  and 0 for  $i \neq j$ . The injected gaussian beam  $\mathbf{E}(x, y, 0)$  and  $\mathbf{H}(x, y, 0)$  at the plane of the waveguide aperture is expanded as

$$\mathbf{E} = \Sigma A_j \mathbf{E}_j + \mathbf{E}_{out}, \quad (12)$$

$$\mathbf{H} = \Sigma A_j \mathbf{H}_j + \mathbf{H}_{out}, \quad (13)$$

where  $A_j$  is the amplitude of the EM field for the  $j$ -mode, and  $\mathbf{E}_{out}(\mathbf{H}_{out})$  is the EM field outside the waveguide. Because the orthonormality in the waveguide is satisfied, the coefficient  $A_j$  is calculated from

$$A_j = \iint_S (\mathbf{E} \times \mathbf{H}_j^*) \cdot d\mathbf{S} = \iint_S (\mathbf{E}_j \times \mathbf{H}^*) \cdot d\mathbf{S}. \quad (14)$$

The power content of  $j$ -mode in the waveguide is equal to  $|A_j|^2$ . For the TEM<sub>00</sub> mode, the calculation of  $A_j$  is straightforward

$$A_j = - \iint_S E_{jy}^* H_x dx dy. \quad (15)$$

At the same time, the normalized flux of TEM<sub>00</sub> mode to waveguide is calculated by

$$P_{WG} = \frac{1}{Z_0} \iint_S |E_y|^2 \cos \alpha \cos \beta dx dy. \quad (16)$$

Here, if the center of waist point is located at  $z'_0 (\neq 0)$ ,  $z'$  variable should be substituted by  $z' - z'_0$ . To calculate  $A_j$ , the two dimensional data sampled with  $M \times M$  on the plane  $(x, y, 0)$  are provided for both injected gaussian beam and waveguide mode. For the numerical calculation in the following section, we use  $64 \times 64$  sampling points and the radius of 16 sampling points for the CC-waveguide. In order to examine the coupling efficiencies in the ECH system for the LHD, the numerical integration in two dimensions is carried out for waveguide modes of HE <sub>$nm$</sub> , EH <sub>$nm$</sub> , TE <sub>$0m$</sub>  and TM <sub>$0m$</sub> .

## 2.3 Oblique incidence of gaussian beam

As shown in Fig. 1, for an on-axis gaussian beam injected with a small angle, injected EM fields at the entrance of waveguide are calculated. The cartesian coordinate

system  $(x', y', z')$  rotated by  $\alpha$  and  $\beta$  around  $x$ - and  $y$ -axes of the  $(x, y, z)$  coordinate system is transformed as:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos \beta & 0 & 0 \\ \sin \alpha \sin \beta & \cos \alpha & 0 \\ \cos \alpha \sin \beta & \sin \alpha & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \quad (17)$$

When we consider the gaussian beam of TEM<sub>00</sub> with  $E'_y$  and  $H'_x$ , magnetic field  $H_x$  is given by

$$H_x = H'_x(x', y', z') \cos \beta. \quad (18)$$

### 3 Numerical calculation

In the LHD, the transmission lines which consists of the CC-waveguide with  $2a = 88.9\text{mm}$  for  $\omega/2\pi=84$  and  $168$  GHz are being prepared. In this section, the result from the coupling calculation based on those parameters is discussed. When the gaussian beam with waist point  $z_0$  located at the entrance of the CC-waveguide is injected normally, the contours of  $P_{\text{WG}}$  and  $|A_{\text{HE11}}|^2$  in the  $(z_0, w_0)$  plane for  $\omega/2\pi=168$  GHz are shown in Figs.2 (a)-(b).

The flux  $P_{\text{WG}}$  has small dependence on  $z_0$  in the range of  $|z_0| \leq 500$  mm. It is found from Fig. 2 (b) that the maximum of  $|A_{\text{HE11}}|^2$  is in  $w_0/a=0.643$  and  $z_0=0$ , and that those values agree well with the theoretical analysis by [6], [7]. It becomes evident that for large  $w_0$ , most of  $P_{\text{WG}}$  is coupled to HE<sub>11</sub> mode and that for small  $w_0$ , mode purity in the waveguide decreases. As shown in Fig. 2 (c),  $z_0$  and  $w_0$  at which the coupling is maximum of  $|A_{\text{HE11}}|^2$  for  $\omega/2\pi=84$  GHz are the same as that in Fig. 2 (b). In comparison with the result from  $\omega/2\pi=168$  GHz, the dependence of  $|A_{\text{HE11}}|^2$  on  $z_0$  increases due to the smaller  $R$  of wavefront at the entrance. As a result, the maximum coupling for larger  $z_0$  increases slightly from  $w_0/a=0.643$ .

In Fig. 3 (a), contours of  $|A_{\text{HE11}}|^2$  are plotted in the  $(w_0, \alpha)$  plane for  $\omega/2\pi=168$  GHz and  $2a=88.9$  mm. Here, the angle  $\alpha$  is equal to the deviation from the normal incidence. With increasing  $\alpha$ , the optimum  $w_0$  decreases from  $w_0/a=0.643$ . It is noted that  $|A_{\text{HE11}}|^2$  for  $\alpha = 1\text{deg}$  is as small as 0.45, even if  $P_{\text{WG}} \geq 0.98$ . For the gaussian beam with the smaller  $w_0$ , dependence of  $|A_{\text{HE12}}|^2$  on  $\alpha$  is weak since the main body of gaussian beam is located only near the axis and then the effect to the coupling due to the phase variation is weakened on an average. For reference,  $|A_{\text{XHE21}}|^2$ ,  $|A_{\text{TM02}}|^2$  and  $|A_{\text{HE12}}|^2$  are plotted in the  $(w_0, \alpha)$  plane in Figs. 3 (b) and (c). For tilting, the HE<sub>12</sub> mode is almost independent of  $\alpha$ ; however, XHE<sub>21</sub> and TM<sub>02</sub> modes are created as a result of non-uniformity of phase on the entrance.

In Figs. 4 (a)-(b), contours of  $|A_{\text{HE11}}|^2$ ,  $|A_{\text{HE21}}|^2$  and  $|A_{\text{TE01}}|^2$  are plotted in the  $(w_0, y_d)$  plane for the normal incidence. Here,  $y_d$  is the displacement value of gaussian beam along the  $y$ -axis. The optimum value of  $w_0$  increases with offset. Even for  $y_d$  equal to 5 mm,  $|A_{\text{HE11}}|^2$  is more than 0.9 because the phase distribution remains constant and only the amplitude becomes asymmetric. In Figs. 5 (a)-(c), contours of  $P_{\text{WG}}$ ,  $|A_{\text{HE11}}|^2$ ,  $|A_{\text{HE21}}|^2$  and  $|A_{\text{TE01}}|^2$ ,  $|A_{\text{XHE21}}|^2$  and  $|A_{\text{TM02}}|^2$  are plotted in the  $(x_d, y_d)$  plane for normal incidence, where  $x_d$  is the offset value of gaussian beam along the  $x$ -axis. It is noted that the contours of  $P_{\text{WG}}$  and  $|A_{\text{HE11}}|^2$  show axisymmetry. For  $r_d/a = \sqrt{x_d^2 + y_d^2}/a \leq 0.02$ ,  $|A_{\text{HE11}}|^2$  is nearly equal to 0.9 even if  $P_{\text{WG}} \simeq 0.98$ . The off-axis shift should be less than 2.9 mm for mode-conversion loss of  $\leq 1\%$ . The polynomial fitting of Fig. 5 (b) is

$$|A_{\text{HE11}}|^2 \simeq 0.98 - 1.2 \times 10^{-3} r_d^2 + 5.6 \times 10^{-7} r_d^4, \quad (19)$$

for  $r_d \leq 20$  mm. The normalized mode conversion loss can be written as

$$|A_{\text{HE11}}|^2 \simeq 0.98 - 2.3(r_d/a)^2 + 2.2(r_d/a)^4, \quad (20)$$

for  $r_d/a \leq 0.45$ . In Figs. 6 (a)-(c), the contours of  $|A_{\text{HE11}}|^2$ ,  $|A_{\text{HE21}}|^2$  and  $|A_{\text{TE01}}|^2$ ,  $|A_{\text{XHE21}}|^2$  and  $|A_{\text{TM02}}|^2$  are plotted in the  $(\alpha, \beta)$  plane with  $x_d = y_d = 0$ . Contours of  $|A_{\text{HE11}}|^2$ ,  $|A_{\text{HE12}}|^2$ , are axisymmetric. It is confirmed that power conservation  $P_{\text{WG}} \simeq \sum |A_{\text{MODE}}|^2$  is valid within  $n=3$  and  $m=3$ . At  $\alpha \simeq 0.5$  and  $\beta = 0$ , higher modes of 14% which mainly consist of  $|A_{\text{XHE21}}|^2$  and  $|A_{\text{TM02}}|^2$  are excited, however, a small content of the  $\text{HE}_{12}$  mode with axisymmetric contour exists as well. In Figs. 7 (a)-(b),  $P_{\text{WG}}$  and  $\sum |A_{\text{MODE}}|^2$  are plotted as a function of  $\alpha$  and  $\beta$ , respectively. In  $\alpha$  (or  $\beta$ )  $\simeq 0.16$  deg where  $|A_{\text{HE11}}|^2$  decreases to 0.96,  $|A_{\text{HE12}}|^2$  is equal to  $|A_{\text{XHE21}}|^2$  and  $|A_{\text{TM02}}|^2$  ( $|A_{\text{HE21}}|^2$  and  $|A_{\text{TE01}}|^2$ ). For  $\omega/2\pi=84$  GHz, the same figures as Figs. 7 (a) and (b) are obtained by exchanging the scale (-1 to 1) of  $\alpha$  and  $\beta$ -axes to scale (-2 to 2). To suppress the total higher mode content to a value less than 1%, the tilting angle should be less than 0.1 deg for  $\omega/2\pi=168$  GHz and  $2a=88.9$  mm. By comparing Fig. 7 (a) with Fig. 7 (b), the curves in (b) are easily obtained by exchanging the label of modes to the cross polarization mode.

As shown in Fig. 8, the contours  $|A_{\text{HE11}}|^2$  for  $w_0/a=0.643$  are plotted in the  $(1/2a, \alpha)$  plane as a parameter of  $|A_{\text{HE11}}|^2$ . All the contour curves are straight lines passing through the origin (0, 0) if each line is extrapolated. In the approximation of  $\tan \alpha \simeq \alpha$ ,  $\alpha$  is inversely proportional to  $a$  for constant  $|A_{\text{HE11}}|^2$ . Taking into account axisymmetry of  $|A_{\text{HE11}}|^2$ , it is found that  $|A_{\text{HE11}}|^2$  depends on the product of  $a\theta$ , where  $\theta = \sqrt{\alpha^2 + \beta^2}$ . The result shows that the careful adjustment of incident angle is required for effectual transmission in the large diameter of the CC-waveguide.



In Fig. 9 (a), contours of  $|A_{\text{HE}_{11}}|^2$  are plotted in the  $(\lambda, 2a)$  plane for the fixed angle of  $\alpha=0.1$  deg and  $w_0/a=0.643$ . In Fig. 9 (b), contours are also plotted as a parameter of the incident angle when  $|A_{\text{HE}_{11}}|^2=0.970$  and  $w_0/a=0.643$ . In both graphs, all the lines pass through the origin  $(0, 0)$ . For the ECH experimental ranges of  $2a=50 \sim 100$  mm and  $60 \sim 200$  GHz in the device such as LHD, Wendelstein-7 AS [4], Heliotron-E [3], and ITER [10] if the injection angle can be adjusted so as to satisfy  $|\alpha| \leq 0.1$  deg, the power coupling efficiency more than 97% is achieved. From both figures, it is found that  $|A_{\text{HE}_{11}}|^2$  depends on  $a/\lambda$ . By considering the discussion in Fig. 8, it becomes evident that  $|A_{\text{HE}_{11}}|^2$  is a function of  $a\theta/\lambda$ . In order to obtain the scaling formula, polynomial fitting of  $|A_{\text{HE}_{11}}|^2$  in Fig. 6 is carried out in the range of  $|\alpha| \leq 1$  deg:

$$|A_{\text{HE}_{11}}|^2 \simeq 0.98 - 0.73\alpha^2 + 0.20\alpha^4, \quad (21)$$

where the unit of  $\alpha$  is degree. By introducing the dependence of  $a\theta/\lambda$  on coupling coefficient for general injection angle  $\theta$ , the above equation is rewritten as

$$|A_{\text{HE}_{11}}|^2 \simeq 0.98 - 3.9(a\theta/\lambda)^2 + 5.6(a\theta/\lambda)^4, \quad (22)$$

for  $|a\theta/\lambda| \leq 0.43$ , where the unit of  $\theta$  is radian. When the tilting gaussian beam is injected with offset, the total mode conversion loss is approximately given by sum of the loss terms of Eqs. (20) and (22).

To confirm formulas of the mode-conversion loss at the junction of the CC-waveguide with tilt or offset [5], the EM fields of  $\text{HE}_{11}$  mode instead of the gaussian beam are used. The mode conversion loss at the junction of CC-waveguides obtained is written as

$$|A_{\text{HE}_{11}}|^2 \simeq 1 - 2.7(r_d/a)^2 + 3.2(r_d/a)^4, \quad (23)$$

for offset of the waveguide axis ( $r_d/a \leq .45$ ) and

$$|A_{\text{HE}_{11}}|^2 \simeq 1 - 4.2(a\theta/\lambda)^2 + 6.3(a\theta/\lambda)^4, \quad (24)$$

for tilting of the axis ( $|a\theta/\lambda| \leq 0.43$ ). The mode-conversion loss by the small offset and tilting are close to the results (Eqs. (20) and (22)) from gaussian beam injection and the calculations by coupled mode theory reported previously [5].

In analysis of the oversized miter bend, the input EM-field to the phase-corrected reflecting mirror in the bend is treated as the radiated fields from the CC-waveguide [11, 12]. The mode-conversion loss for a 90-degree plane miter bend is given by  $0.74(\lambda/a)^{3/2}$ . For instance, in the 90-degree miter bend, the setting accuracies on the angle and position of the reflecting mirror are closely related with the oblique and off-axis injection of gaussian beam, even though the near-field radiation from the CC-waveguide is not able to approximate as the only fundamental gaussian beam [13]. The setting tolerance on the angle and position of reflecting mirror is inferred to be similar to results from the present analysis.

## 4 Conclusion

The mode-conversion loss increases and the optimum  $w_0$  changes when the gaussian beam is injected with offset or tilt into the circular corrugated waveguide. For the gaussian beam with waist size of  $w_0/a=0.643$ , the power losses depend on  $2.3(r_d/a)^2 - 2.2(r_d/a)^4$  and  $3.9(a\theta/\lambda)^2 - 5.6(a\theta/\lambda)^4$  for offset and tilt, respectively. In the recent high power ECH system, large diameter of waveguide and high frequency more than 100 GHz are required from the viewpoint of breakdown and penetration of millimeter waves to the core in the high density plasmas. It is pointed out that the careful alignment of injection beam to the CC-waveguide is necessary for such requirement and that deviation angle of injection from the axis and offset should be less than 0.1 degrees and 2.9 mm to achieve the loss of  $\leq 1\%$  for LHD.

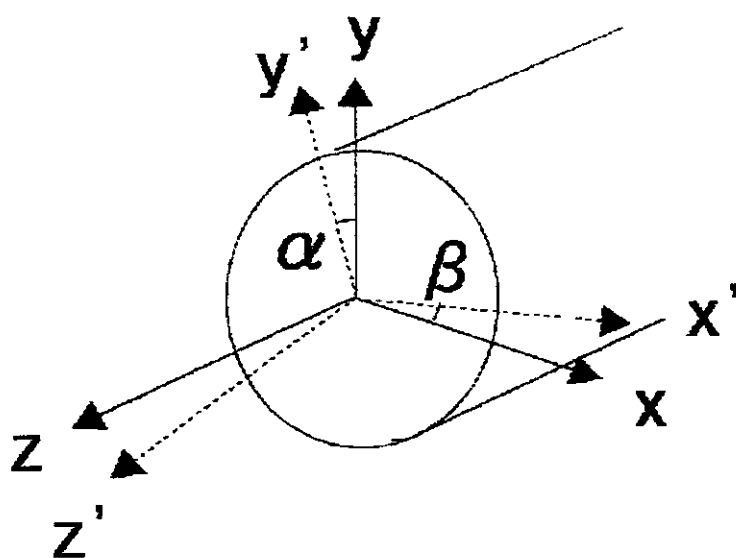
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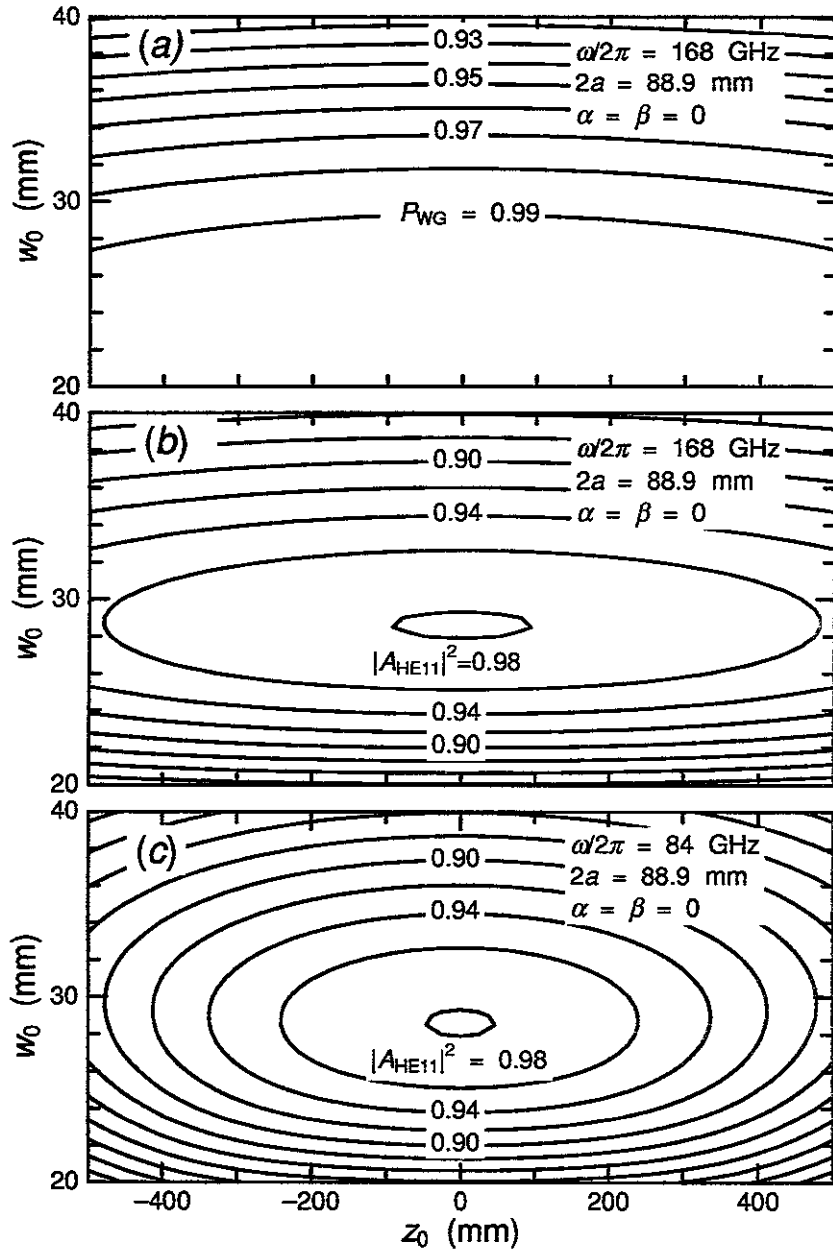
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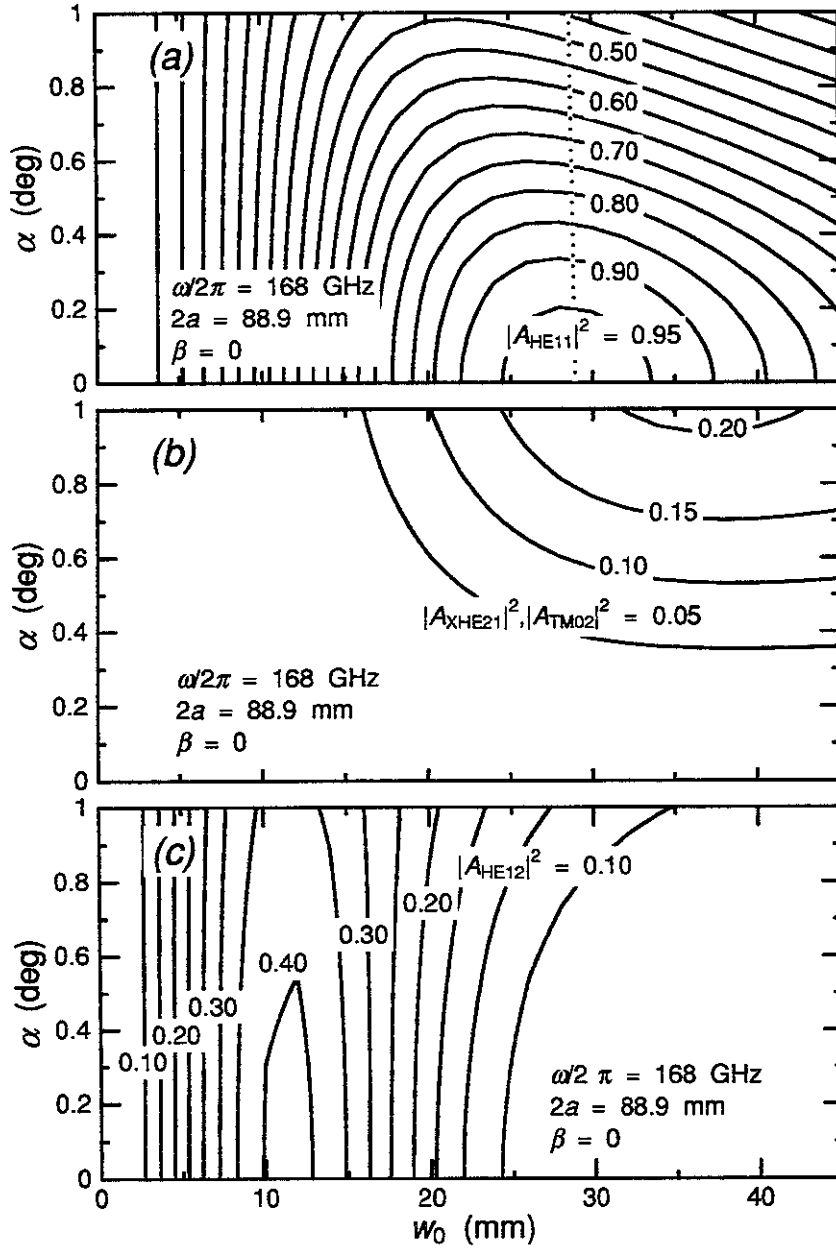
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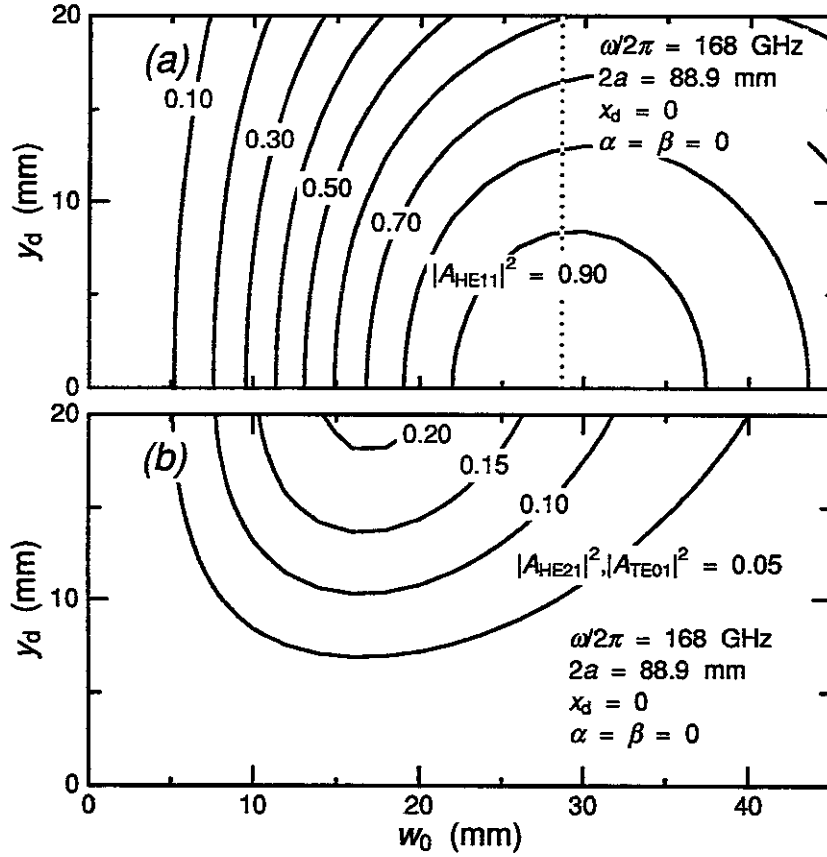
**Fig. 1:** Schematic drawing of the gaussian beam injected into the entrance of the waveguide. Here, the angle  $\alpha$  and  $\beta$  correspond to the rotation around the  $x$ - and  $y$ -axis, respectively.



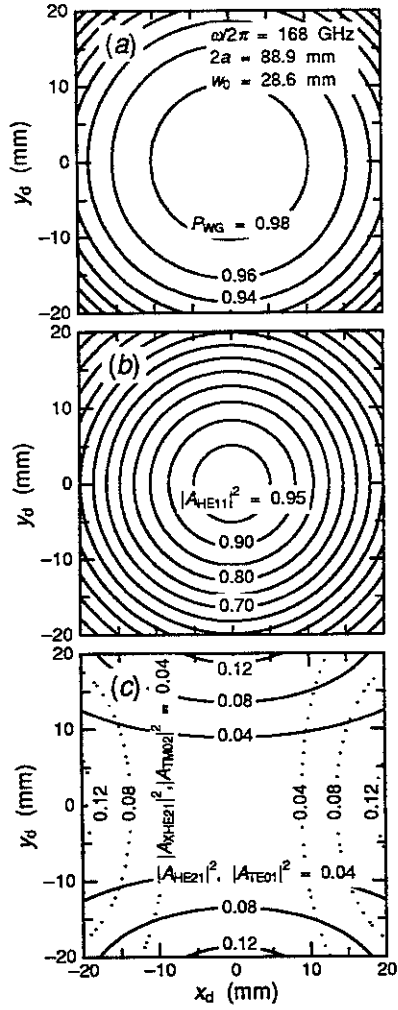
**Fig. 2:** Contour plots of (a)  $P_{WG}$ , (b)  $|A_{HE11}|^2$  for  $\omega/2\pi=168$  GHz, and (c)  $|A_{HE11}|^2$  for  $\omega/2\pi=84$  GHz for normally injected beam at the entrance of the waveguide with  $2a=88.9$  mm.



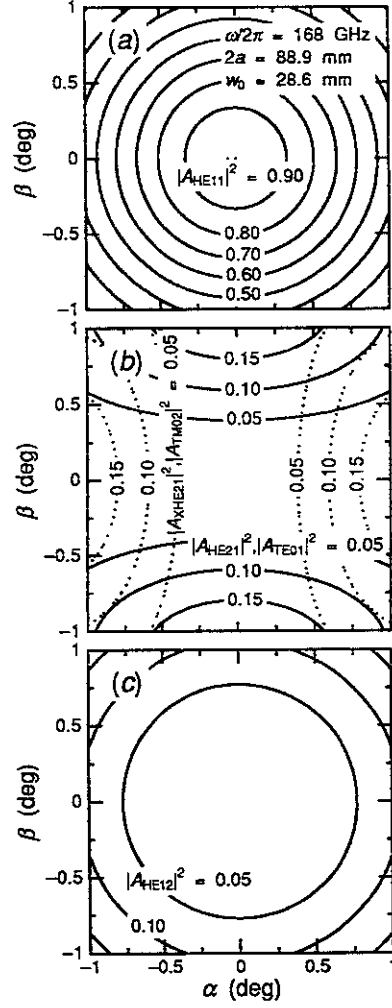
**Fig. 3:** Contour plots of (a)  $|A_{HE11}|^2$ , (b)  $|A_{XHE21}|^2$  and  $|A_{TM02}|^2$ , (c)  $|A_{HE12}|^2$  in the  $(w_0, \alpha)$  plane for  $\omega/2\pi=168$  GHz, and  $2a=88.9$  mm when a gaussian beam  $TEM_{00}$  is injected with  $x_d=y_d=0$ .



**Fig. 4:** Contour plots of (a)  $|A_{HE11}|^2$ , (b)  $|A_{HE21}|^2$  and  $|A_{TE01}|^2$ , in the  $(w_0, y_d)$  plane for  $\omega/2\pi=168$  GHz, and  $2a=88.9$  mm when a gaussian beam  $TEM_{00}$  is injected with  $x_d=0$  and  $\alpha=\beta=0$ .

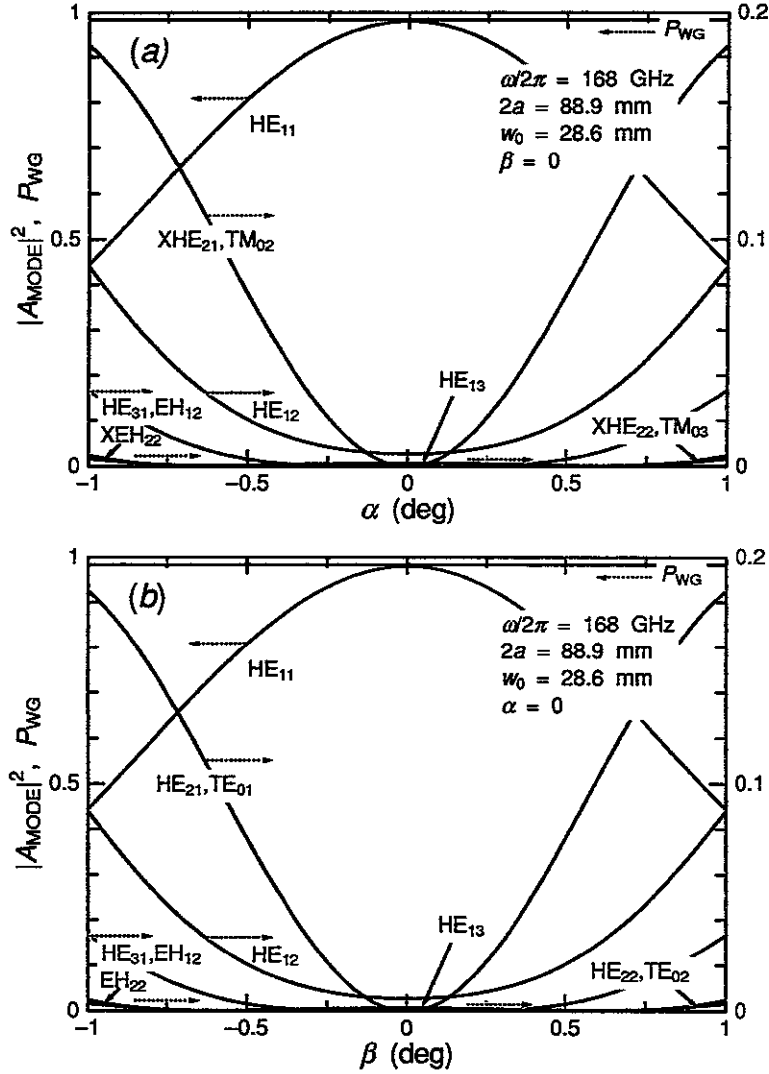


**Fig. 5:** Contour plots of (a)  $P_{WG}$ , (b)  $|A_{HE11}|^2$ , (c)  $|A_{HE21}|^2$  and  $|A_{TE01}|^2$ ,  $|A_{XHE21}|^2$  and  $|A_{TM02}|^2$  in the  $(x_d, y_d)$  plane for  $\omega/2\pi = 168$  GHz when a gaussian beam  $TEM_{00}$  with  $w_0/a = 0.643$  is injected normally to the entrance of the waveguide with  $2a = 88.9$  mm.

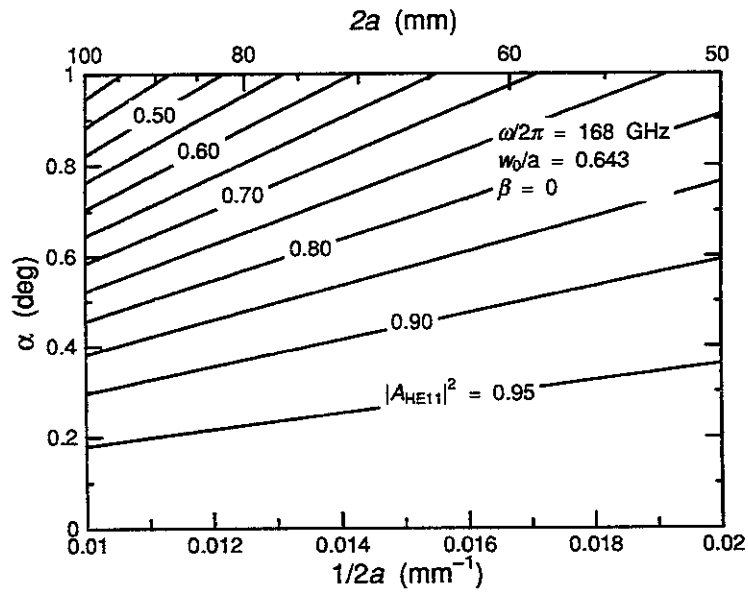


**Fig. 6:** Contour plots of (a)  $|A_{HE11}|^2$ , (b)  $|A_{HE21}|^2$  and  $|A_{TE01}|^2$ ,  $|A_{XHE21}|^2$  and  $|A_{TM02}|^2$ , (c)  $|A_{HE12}|^2$  in the  $(\alpha, \beta)$  plane for  $\omega/2\pi = 168$  GHz when a gaussian beam  $TEM_{00}$  with  $w_0/a = 0.643$  is injected obliquely into the center ( $x_d = y_d = 0$ ) of the CC-waveguide with  $2a=88.9$  mm.

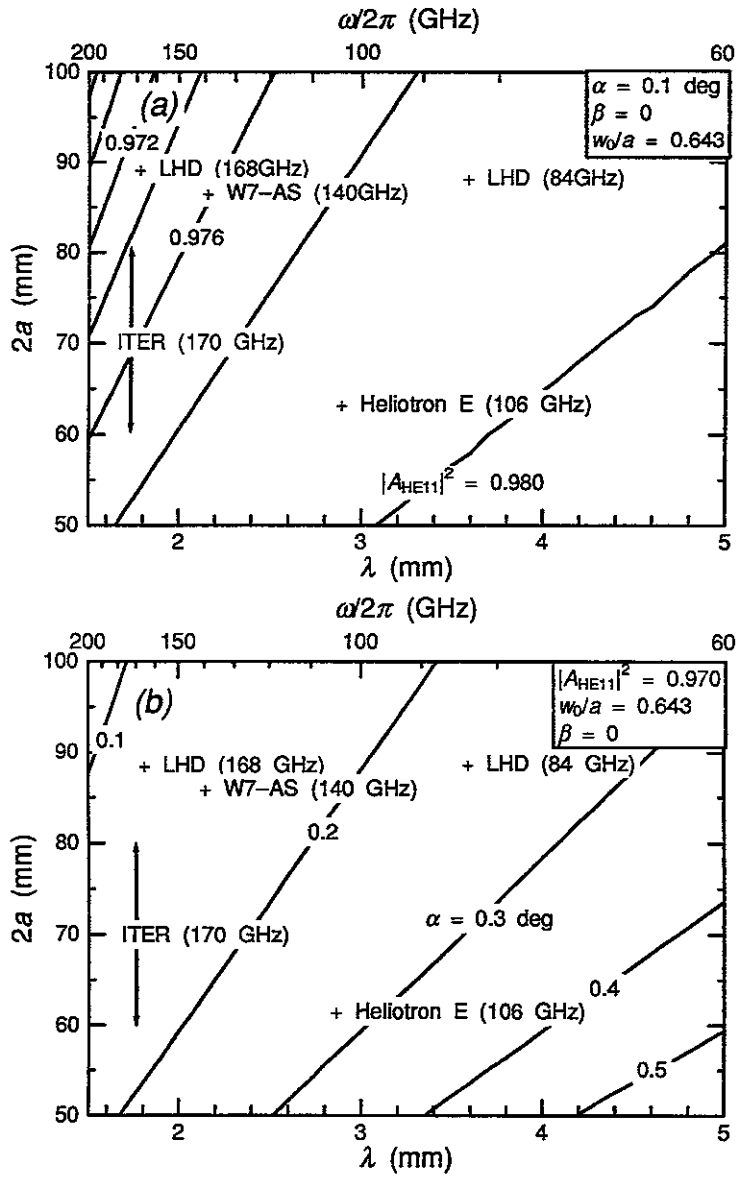




**Fig. 7:** (a)  $P_{\text{WG}}$  and  $|A_{\text{MODE}}|^2$  are plotted as a function of  $\alpha$ . Here, MODE denotes  $\text{HE}_{11}$ ,  $\text{XHE}_{21}$  and  $\text{TM}_{02}$ ,  $\text{HE}_{12}$ ,  $\text{HE}_{31}$  and  $\text{EH}_{12}$ ,  $\text{XEH}_{22}$ ,  $\text{XHE}_{22}$  and  $\text{TM}_{03}$ ,  $\text{HE}_{13}$ , respectively. (b)  $P_{\text{WG}}$  and  $|A_{\text{MODE}}|^2$  are plotted as a function of  $\beta$ . Here, MODE denotes  $\text{HE}_{11}$ ,  $\text{HE}_{21}$  and  $\text{TE}_{01}$ ,  $\text{HE}_{12}$ ,  $\text{HE}_{31}$  and  $\text{EH}_{12}$ ,  $\text{EH}_{22}$ ,  $\text{HE}_{22}$  and  $\text{TE}_{02}$ ,  $\text{HE}_{13}$ , respectively.



**Fig. 8:** Contour plots of  $|A_{HE11}|^2$  in the  $(1/2a, \alpha)$  plane for  $\omega/2\pi=168$  GHz and  $\beta=0$  when a gaussian beam  $TEM_{00}$  with  $w_0/a=0.643$  is injected obliquely into the center of the CC-waveguide. All the lines pass through the origin  $(0, 0)$ .



**Fig. 9:** (a) Contour plots of  $|A_{HE11}|^2$  in the  $(\lambda, 2a)$  plane for  $\alpha=0.1$  deg and  $\beta=0$  when a gaussian beam  $TEM_{00}$  with  $w_0/a=0.643$  is injected. All the lines pass through the origin  $(0, 0)$ . (b) Contour plots of  $\alpha$  in the  $(\lambda, 2a)$  plane for satisfying  $|A_{HE11}|^2=0.970$  when a gaussian beam  $TEM_{00}$  with  $w_0/a=0.643$  is injected with  $\beta=0$ . All the lines pass through the origin  $(0, 0)$ .

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