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Reduction of Turbulence by Sheared Toroidal Flow
on a Flux Surface

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## RESEARCH REPORT
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# Reduction of Turbulence by Sheared Toroidal Flow on a Flux Surface

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## Abstract

When the tokamak plasma is heated by neutral beam injection, the pressure of the high energy ions is generally not uniform on a flux surface. The distribution depends on the injection geometry. The response of the electrons, described by Ohm's law results in the sheared toroidal rotation of the plasma on a flux surface. Because the turbulence has long correlation lengths parallel to the magnetic field, the rotational shear on a flux surface is more effective for suppressing the turbulences with short radial correlation length than the rotational shear across the flux surface.

# 1. Introduction

The improved confinement regimes of tokamak plasma with negative shear[1] are under intense theoretical study. The reduction of the turbulence[2] by shear of plasma flow across a flux surface have been proposed as the underlying cause of the improvement. The turbulence with a significant radial correlation length will be "washed away" by the flow shear. On the other hand, the turbulence with very short radial correlation length are difficult to suppress by this mechanism. Since the orbits of the electrons are much smaller than those of the ions, one might speculate that the electron heat transport is due to the turbulence of short radial correlation length and that is why only the ion heat transport is reduced in the experiments.

The recent experimental results[3] from JT-60U have shown that the electron heat transport is reduced as well as the ion heat transport. It suggests that there may be a different mechanism at work which reduces the electron heat transport.

Because the correlation along the magnetic field is much better than across the magnetic field, the turbulence is expected to have elongated shapes parallel to the magnetic field. If there is a shear in the toroidal flow of the plasma on a flux surface, the turbulence will quickly decorrelate independent of the radial correlation length. In plasma with the temperature and the density uniform on a flux surface, Ohm's law dictates there is no shear in the toroidal flow. However, when the plasma contains a significant amount of high energy ions from the neutral beam injection, the situation changes. The magnetic force must support the force exerted by the high energy ions for the equilibrium. The extra magnetic force in Ohm's law results in a sheared toroidal flow on a flux surface.

If the sheared flow reduces the electron heat transport, the electron temperature increases. As a result the beam slow down time lengthens and the high energy ion component increases. A positive feed back loop is established and the electron temperature reaches a high value until the beam slow down is mainly due to the collisions with the ions. It seems that this mechanism leads to a bifurcation, either the electron temperature stays low or reaches a high value.

In the following we discuss Ohm's law, high energy ion component, toroidal rotation and the effect on the turbulence.

## 2. Ohm's law

The electron heat conduction along the magnetic field lines is rapid and also the electron collision frequency is much higher than that of the ions. Therefore we assume that the electron temperature is uniform on a flux surface and the electron pressure is isotropic. The force balance equation of the electron fluid becomes

$$\vec{E} + \vec{v} \times \vec{B} = \frac{1}{en}(-\nabla p_e + \vec{j} \times \vec{B}) \tag{1}$$

Superficially the above equation looks like Ohm's law of the ordinary plasma. The effects of the high energy ion component are hidden in the magnetic force term $\vec{j} \times \vec{B}$ which must support the force exerted by the high energy ions as well as the thermal plasma pressure.

In the absence of the high energy ions, the equilibrium condition is given by

$$\vec{j} \times \vec{B} = \nabla p(\psi) \tag{2}$$

where p is the pressure and $\psi$ is the flux function. The combination of eq.(1) and eq.(2) results in the well known relationship given by

$$\Omega \equiv \frac{v_\phi}{R} = \Phi'(\psi) + \frac{1}{en}(-p_e'(\psi) + p'(\psi)) \tag{3}$$

where $\phi$ is the toroidal angle, R is the major radius, $\Phi$ is the electrostatic potential and the prime denotes the derivative. It shows that the toroidal rotation is uniform on a flux surface. In obtaining eq.(3) it has been assumed that the poloidal rotation is suppressed by the neo-classical parallel viscosity.

## 3. High energy ions

When a neutral beam is injected into the plasma the beam ions slow down by colliding with the electrons until the energy decreases to the critical energy $W_c$, below which the beam ions scatter and slow down by colliding with the plasma ions. The critical energy is give by

$$W_c = 2^{3/2}(m_i / m_e)^{1/3} k_B T_e \tag{4}$$

The velocity spectrum $f_b$ of the high energy ions[4] is approximately given by

$$f_b \simeq \frac{3n_b}{\ell n(1 + u_b^3 / u_c^3)} \frac{u^2}{u^3 + u_c^3} \tag{5}$$

where $n_b$ is the ion density above the critical energy, $u_b$ is the velocity at injection and $u_c$ is the velocity at the critical energy. The energy density of the high energy ions $p^*$ is given by

$$p^* \equiv \frac{m_i}{2} \int_{u_c}^{u_b} f_b u^2 du$$

$$= \frac{m_i}{4} \frac{3n_b}{\ell n(1 + u_b^3 / u_c^3)} \times \left[ u_b^2 - u_c^2 \left\{ 1 + \frac{1}{3} \ell n \left( \frac{4(u_b^2 - u_b u_c + u_c^2)}{(u_b + u_c)^3} \right) + \frac{2}{\sqrt{3}} \left( tan^{-1} \frac{2u_b - u_c}{\sqrt{3} u_c} - \frac{\pi}{6} \right) \right\} \right]$$

$$\simeq \frac{m_i}{4} \frac{u_b^2 n_b}{\ell n(u_b / u_c)} \qquad (u_b \gg u_c) \tag{6}$$

The density $n_b$ is determined by the injection current $I_b$ into the volume $V$. We obtain

$$n_b = \frac{I_b}{Ve\nu_{ie}}$$

$$\simeq 0.81 \times 10^{30} \frac{I_b}{Vn} \left( \frac{k_B T_e}{e} \right)^{3/2} \tag{7}$$

where $\nu_{ie}$ is the reciprocal of the slow down time.

Since the collisions with electrons do not result in the pitch angle scattering, the spatial and velocity space distribution of the high energy ions are determined by the injection geometry. We consider two extremes, the perpendicular and the tangential injection.

The perpendicular injection puts the beam ions on the trapped orbits. The spatial distribution of the ions is weighted on the outboard side. To the first order of the inverse aspect ratio $\varepsilon$, the distribution of the perpendicular pressure $p_\perp$ can be approximated by

$$p_{\perp} \simeq 2p^* \frac{R - R_I}{R_2 - R_I}$$ (8)

where $R_2$ and $R_I$ are the maximum and the minimum radius of the flux surface .

When the injection is tangential; co, counter or balanced, the centrifugal force $F_C$ must be supported. It is given by

$$F_c = \frac{2p^*}{R}$$ (9)

In term of the equivalent pressure $p_C$ representing the centrifugal force, we obtain

$$p_c = 4p^* \frac{R - R_I}{R_2 - R_I}$$ (10)

Another way of looking at it is that the beam equilibrium is shifted radially outward relative to the flux surface. By comparing eq.(8) and eq.(10) one can see the effect of the centrifugal force is smaller by $\varepsilon$.

Although the generalized Grad-Shafranov equation[5] with anisotropic pressure and rotation is available, here we parameterize the contribution from the high energy ions as an additional pressure to be supported by the magnetic force. We assume that the contribution $p_I$ of the high energy ions to the total pressure is given by

$$p_I = 2\alpha p^*(\psi) \frac{R - R_I}{R_2 - R_I}$$ (11)

where $\alpha$ is a parameter. For the perpendicular injection $\alpha$ is closed to unity and for the tangential injection it is of the order of $\varepsilon$. If the injection is into the barely trapped orbits the distribution of the energetic ions is similar to the "sloshing ions" in the mirror device and $\alpha$ can be a negative value.

We write the $\psi$-component of the equilibrium equation

$$\left(\vec{j} \times \vec{B}\right)_\psi = \nabla_\psi \bar{p}(\psi) + 2\alpha \frac{R - R_1}{R_2 - R_1} \nabla_\psi p^*(\psi) \tag{12}$$

where a low $\beta$-value is assumed and $B^{-1}\nabla_\psi B$ term is neglected. The second term represents the force needed to support the high energy ions.

## 4. Toroidal rotation

We calculate the toroidal rotation from Ohm's law using the pressure of the high energy ions estimated in the previous section. The component parallel to the magnetic field of Ohm's law yields

$$\Phi = \bar{\Phi}(\psi) + \frac{k_B T_e(\psi)}{e} \ell n\left(\frac{n}{\bar{n}(\psi)}\right) \tag{13}$$

where the bar indicates the values averaged over the flux surface. By using eq.(12), we obtain

$$\Omega = \bar{\Phi}' + \frac{k_B T_e'}{e}\left\{\ell n\left(\frac{n}{\bar{n}}\right) - 1\right\} - \frac{k_B T_e}{e}\frac{\bar{n}'}{\bar{n}} + \frac{1}{en}\left(\bar{p}' + 2\alpha \frac{R - R_1}{R_2 - R_1} p^{*\prime}\right) \tag{14}$$

The high energy ions may contribute to the pressure significantly because of their energy but contribute very little to the density. Consequently we may neglect the density nonuniformity on the flux surface and obtain

$$\Omega = \bar{\Phi}' + \frac{1}{e\bar{n}}\left(\bar{p}' - p_e' + 2\alpha \frac{R - R_1}{R_2 - R_1} p^{*\prime}\right) \tag{15}$$

The modulated part of the rotation frequency $\Omega^*$ is given by

$$\Omega^* = \frac{2\alpha}{e\bar{n}}\frac{R - R_1}{R_2 - R_1} p^{*\prime} \tag{16}$$

## 5. Reduction of turbulence

The turbulent electrostatic potential $\tilde{\Phi}$ may be expanded in Fourier components

$$\Phi(r, \theta, \phi) = \sum_{M, N} \Phi_{N,M}(r) \, exp(iN\phi - iM\theta - i\omega t) \tag{17}$$

where r is the minor radius, $\theta$ is the poloidal angle, N and M are the toroidal and the poloidal mode numbers and $\omega$ is the frequency. The plasma is assumed to have a circular cross section and the cylindrical coordinates are used.

If there is toroidal rotation $\Omega[r, \theta]$, the Doppler shifted phase difference $\Delta\phi$ between the points $(r_2, \theta_2)$ and $(r_1, \theta_1)$ increases with time as

$$\Delta\phi = (\Omega(r_2, \theta_2) - \Omega(r_1, \theta_1))Nt \tag{18}$$

The rotation has the uniform part $\overline{\Omega}$ and the modulated part $\Omega_1$

$$\Omega = \overline{\Omega}(r) + \Omega_1 cos \, \theta \tag{19}$$

The phase shift between $r_2$ and $r_1$ is mainly caused by the derivative of the first term whereas the second term causes the phase shift between different poloidal positions.

The turbulence is expected to have very small parallel wave numbers compared to the perpendicular wave numbers. For very small parallel wave number, the mode numbers satisfy

$$N \sim Mq^{-1} \tag{20}$$

where q is the safety factor. For these modes the correlation length extends along the magnetic field line from $\theta = 0$ to $\theta = \pi$. The phase shift across the correlation lengths is given by

$$\Delta\phi \simeq \left(\frac{\overline{\Omega}\Delta r}{\lambda} + 2\Omega_1\right)Nt \tag{21}$$

where $\Delta r$ is the radial correlation length and $\lambda$ is the gradient length of the rotation.

The radial correlation length can be very small. For example, if $\Delta r$ is of the order of the ion gyroradius or London length [collisionless skin depth], $\Delta r / \lambda$ may be $\sim 10^{-2}$. Hence $\Omega_l$ can be much smaller than $\bar{\Omega}$ and yet effective in decorrelating the turbulence with small radial correlation length. In the limit of $\Delta r \rightarrow 0$, the phase shift becomes

$$\Delta \varphi \simeq \frac{4\alpha p^* N t}{enRB_\theta \lambda}$$
$$\simeq \left(\frac{p^*}{p_e}\right) 2\alpha \omega^* t$$

(22)

where $\omega^*$ is the drift wave frequency without the gyroradius correction. The turbulence may be suppressed if the phase shift become large, $\Delta \varphi \sim 1$, during the correlation time or the reciprocal of the growth rate $\gamma$. The condition is given by

$$\gamma < \left(\frac{p^*}{p_e}\right) 2\alpha \omega^*$$

(23)

It shows that the turbulence may be suppressed when the pressures of the high energy ions becomes a significant fraction of the plasma pressure.

When the beam is injected into the plasma at a low electron temperature, the pressure of the high energy ions may not be sufficient to suppress the turbulence because the pressure is proportional to 3/2 power of the electron temperature for a given plasma density. The turbulent transport of the electron heat continues and the electron temperature remains low. By either a separate electron heating or by increasing the beam power, the pressure of the high energy ions may reach the value sufficient to suppress the turbulence, The electron temperature rises because of the reduced transport. Consequently the pressure of the high energy ions rises further closing a positive feed back loop. Finally the critical energy becomes higher than injection energy and the pressure of the high energy ions becomes insensitive to the electron temperature. Also the beam ions cease to heat the electrons directly. There seem to exist two states, one at a low electron temperature with a poor electron heat confinement and the other at a high electron temperature with a good confinement.

## 6. Discussion

The recent experimental data of JT-60U is tantalizingly suggestive. At the final electron temperature at 9 keV and the beam energy of 90 keV the critical energy is above the injection energy. The beam ions scatter and slow down by colliding with the plasma ions. At the density of $3 \times 10^{19}$ m$^{-3}$, the pressure of the high energy ions is estimated to be several percent of the electron pressure. To estimate the value of $\alpha$ more detailed informations on the injection geometry is needed. The most direct test for the model is to measure the toroidal rotation velocity at both the inboard side and the outboard side. If the difference in the rotational frequency can be correlated with the improved electron heat confinement, the experimental support for the model is strong. The measured rotational velocity is in the range of $10^5$m/s. If the model is correct, the velocity difference between the outboard side and the inboard side of roughly $10^4$m/s is expected. Also there may be methods to enhance the effect such as a higher beam energy [500 keV] or a separate electron heating to test the model indirectly.

A question may arise why the negative shear is necessary for the improvement of the confinement. If the improvement is due to the effects described here, the high energy ion component is required. In the positive shear regimes these ions in the trapped orbits drift in the "bad" direction and are likely to cause "fish bone" type of the mhd activity. With the negative shear, the drift velocity of the shallowly trapped ions becomes in the "good" direction and the mhd activity may not occur. Thus the negative shear allows the presence of the energetic trapped ions without the deleterious mhd activities.

In this note we discussed the beam heated plasma. However, non-uniform ion pressure can be produced by r-f heating. The ICRF heating may create and maintain high energy trapped ion populations which resemble the high energy ion component produced by a neutral beam injection. If the model is confirmed by the experimental measurement, the design of future devices would be able to incorporate the injection geometry or an ion heating method to enhance the effect.

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