

NATIONAL INSTITUTE FOR FUSION SCIENCE

Eigenfunctions for Vlasov Equation in Multi-species Plasmas

T. Yamagishi

(Received - Nov. 6, 1998)

NIFS-578

Nov. 1998

This report was prepared as a preprint of work performed as a collaboration research of the National Institute for Fusion Science (NIFS) of Japan. This document is intended for information only and for future publication in a journal after some rearrangements of its contents.

Inquiries about copyright and reproduction should be addressed to the Research Information Center, National Institute for Fusion Science, Oroshi-cho, Toki-shi, Gifu-ken 509-02 Japan.

RESEARCH REPORT
NIFS Series

Eigenfunctions for Vlasov Equation in Multi-species Plasmas

T. Yamagishi

Tokyo Metropolitan Institute of Technology

Abstract

The normal mode expansion method for the Vlasov equation has been developed to multi-species plasmas in an infinite system. The perturbed charge density for the Vlasov-Poisson system in multi-species plasmas is found to be the same form as the perturbed distribution function in a single species plasma. The eigenfunctions and adjoint eigenfunctions, and the orthogonality relations between them have been derived. The complete set of eigenfunctions for the charge density is constructed, which has been applied for the initial value problems. The continuum contribution consists of the ballistic mode and pure continuum contribution. Scalar potential and each species distribution function are also solved making use of the complete set of eigenfunctions.

Keywords: Vlasov equation, multi-species plasma, normal modes, adjoint equation, orthogonality, complete set of eigenfunctions, initial value problem, Van Kampen mode, ballistic mode.

1. Introduction

Van Kampen has developed the normal mode approach for the electron plasma oscillation described by the Vlasov equation. He proved the continuum contribution (Van Kampen mode), which is the contribution from the singular eigenfunction over the continuous eigenvalue spectrum induced from the wave-particle resonance condition, plays an essential role for the completeness of the Vlasov eigenfunctions.¹⁾ Case²⁾ has developed this method for the same electron oscillation problem, introduced the orthogonality relations, and constructed the complete set of eigenfunctions for the case where the discrete set of eigenvalues exists in addition to the continuous eigenvalue spectrum. This normal mode expansion method has also been developed in the field of neutron transport theory by Case and Zweifel³⁾, and applied to many problems.⁴⁾⁵⁾ The essence of the normal mode approach is that the solution of transport equation like

Vlasov equation consists of the discrete mode determined from the dispersion relation, and the continuum contribution. Depending on a parameter value, the discrete mode may vary and disappear, while the continuum contribution always exists, and never disappears.

In the linear plasma stability theory, the discrete eigenvalue evaluated from the dispersion relation is usually examined, because the discrete eigenvalue determines whether the system grows or decays in time, or the stability of the system. The continuum contribution which is independent of the dispersion relation does not grow in time. It represents the effect of wave-particle resonance, or the dynamics of particles trapped by potential wave in the linear theory.

The normal mode approach is applied, to the authors' knowledge, only to the electron plasma oscillation where cold ions are introduced only to keep the zero-th order charge neutrality. Since ions and electrons equally

contribute to the dielectric function and the dispersion relation if ion and electron temperatures are the same, it may be interesting and useful if the complete set of eigenfunctions is constructed for the cases including both electron and ion dynamics. The purpose of the present paper, therefore, is to construct the complete set of eigenfunctions for the Vlasov equation for multi-species plasmas in the electrostatic approximation.

2. Vlasov -Poisson system for multi-species plasmas

We consider the Vlasov equation for the j -th species charged particles perturbed distribution function f_j in the electrostatic approximation:

$$\frac{\partial f_j}{\partial t} + v \cdot \nabla f_j + \left(\frac{e}{m} \right)_j E \cdot \frac{\partial f_{0j}}{\partial v} = 0, \quad (1)$$

where $\mathbf{E} = -\nabla \phi$ is the perturbed electric field with ϕ being the perturbed scalar potential, e_j and m_j are charge and mass, and f_{0j} is the unperturbed distribution for the j -th species particle. We must solve eq.(1) combining with the Poisson equation:

$$\nabla^2 \phi = -4\pi \sum_j e_j \int d^3v f_j(r, v, t). \quad (2)$$

We assume for the perturbed quantities f_j and ϕ are proportional to the Fourier factor $\exp(i\mathbf{k} \cdot \mathbf{v})$. Combining with eq.(2), eq.(1) can be written in the form:

$$\frac{\partial f_j}{\partial t} + ik \cdot v f_j = ik \cdot \frac{\partial f_{0j}}{\partial v} \frac{4\pi}{k^2 m_j} \rho(v, t), \quad (3)$$

where the charge density ρ is defined by

$$\rho(v, t) = \sum_j e_j g_j(v), \quad (4)$$

and g_j is the integrated distribution function given by

$$g_j(v) = \int f_j(v, v_\perp) dv_\perp. \quad (5)$$

Integrating eq.(3) over v_\perp , we have

$$\frac{\partial g_j}{\partial t} + ik \cdot v g_j(v, t) = -ik \alpha_j(v) \int dv \rho(v, t), \quad (6)$$

where the quantity α_j is defined by

$$\alpha_j(v) = -\frac{4\pi e_j}{m_j k^2} \frac{\partial}{\partial v} \int f_{0j}(v, v_\perp) dv_\perp. \quad (7)$$

Multiplying e_j to both sides of eq.(6) and adding, we have an equation for ρ :

$$\frac{\partial \rho}{\partial t} + ikv\rho = -ik\alpha \int_{-\infty}^{\infty} dv \rho(v, t), \quad (8)$$

where

$$\alpha = \sum_j e_j \alpha_j. \quad (9)$$

When ρ is solved from eq.(8), the j -th distribution function g_j is obtained by solving eq.(6) with respect to t .

3. Normal Modes

We here develop the normal modes given by Van Kampen and Case for multi-species plasmas. Since eq.(6) for the charge density ρ is the same form as the equation for the distribution function g treated by Case, we may apply the same method. We assume the solution of the form $\rho \sim \exp(-i\omega t)$, and introduce the parameter $\mu = \omega/k$. Then eq.(8) can be written by

$$(\mu - v)\rho = \alpha \int_{-\infty}^{\infty} dv \rho(v). \quad (10)$$

If we require the normalization condition to ρ :

$$\int_{-\infty}^{\infty} dv \rho(v) = 1, \quad (11)$$

then from eq.(10), ρ is given in the form

$$\rho(v) = \frac{\alpha(v)}{\mu - v}. \quad (12)$$

Since v varies between $-\infty$ to ∞ , the solution (12) becomes singular when μ is on the real axis, i.e. $\mu \in \Sigma$. When μ is not real, introducing eq.(12) into eq.(11), we have the discrete eigenvalue condition or dispersion relation:

$$\varepsilon(\mu) \equiv 1 - \int_{-\infty}^{\infty} \frac{\alpha(v)}{\mu - v} dv = 0. \quad (13)$$

The corresponding eigenfunction for i -th eigenvalue becomes

$$\rho_{\mu_i}(v) \equiv \rho_i(v) = \frac{\alpha(v)}{\mu_i - v}. \quad (14)$$

For $\mu \in \Sigma$, we assume the singular solution in the form

$$\rho_\mu(v) = P \frac{\alpha(v)}{\mu - v} + \lambda(\mu) \delta(\mu - v), \quad (15)$$

where P in eq.(15) means the Cauchy principal value integration. The coefficient λ is obtained by introducing eq.(15) into eq.(11):

$$\lambda(\mu) \equiv 1 - P \int_{-\infty}^{\infty} \frac{\alpha(v)}{\mu - v} dv. \quad (16)$$

The second δ -function term in eq.(15) is the characteristics of the Van Kampen mode, which represents the trapped particle resonance with the potential wave ϕ in the linear theory. It is also a source of incoherent fluctuations.

Let us here introduce the adjoint equation

$$(\mu - v) \rho^* = \int_{-\infty}^{\infty} dv \alpha \rho^*(v). \quad (17)$$

For the adjoint function ρ^* , we also require the normalization condition:

$$\int_{-\infty}^{\infty} dv \alpha \rho^*(v) = 1. \quad (18)$$

From eqs.(17) and (18), we have the same dispersion relation as given by eq.(13). Then for the discrete eigenvalue μ_i , the adjoint eigenfunction becomes

$$\rho_i^*(v) = \frac{1}{\mu_i - v}. \quad (19)$$

For $\mu \in \Sigma$, the singular adjoint eigenfunction can be written as

$$\rho^*(v) = P \frac{1}{\mu - v} + \lambda^*(\mu) \delta(\mu - v). \quad (20)$$

Introducing eq.(20) into eq.(18), the coefficient λ^* is obtained in the form

$$\lambda^*(\mu) = \frac{\lambda(\mu)}{\alpha(\mu)} \quad (21)$$

Multiplying eq.(1) by $\rho_\mu(v)$, eq.(17) by $\rho_\mu(v)$, subtracting one from the other, and integrating over all v yields for $\mu \neq \mu'$

$$(\mu - \mu') \int_{-\infty}^{\infty} dv \rho_\mu(v) \rho_{\mu'}^*(v) = 0. \quad (22)$$

If we introduce the scalar product by

$$\langle \rho_\mu, \rho_{\mu'}^* \rangle \equiv \int_{-\infty}^{\infty} dv \rho_\mu(v) \rho_{\mu'}^*(v) \quad (23)$$

then eq.(22) means the orthogonality relation:

$$\langle \rho_\mu, \rho_{\mu'}^* \rangle = 0 \quad \text{for} \quad \mu \neq \mu' \quad (24)$$

For the discrete eigenvalues μ_i and μ_j , the scalar product of eigenfunctions ρ_i, ρ_j^* , becomes

$$\langle \rho_i, \rho_j^* \rangle = C_i \delta_{ij}, \quad (25)$$

where the coefficient C_i is given by

$$C_i = \int_{-\infty}^{\infty} \frac{\alpha(v)}{(\mu_i - v)^2} dv. \quad (26)$$

From the definition of the dielectric function given by eq.(11), C_i is also written as

$$C_i = \left. -\frac{d\mathcal{E}(\mu)}{d\mu} \right|_{\mu=\mu_i}. \quad (27)$$

For μ and $\mu' \in \Sigma$, the product of the singular eigenfunctions $\rho_\mu(v)$ and $\rho_{\mu'}^*(v)$ can be written by^{4,15)}

$$\rho_\mu(v) \rho_{\mu'}^*(v) = \frac{1}{\mu' - \mu} \left\{ \rho_\mu(v) - \alpha \rho_{\mu'}^*(v) \right\} + \frac{\lambda^2 + \pi^2 \alpha^2}{\alpha} \delta(\mu - v) \delta(\mu' - v).$$

Integration of this equation over v yields the orthogonality relation for these singular eigenfunctions

$$\langle \rho_\mu, \rho_{\mu'}^* \rangle = C_\mu \delta(\mu - \mu'), \quad (28)$$

where the coefficient C_μ is given by

$$C_\mu = \frac{\lambda^2(\mu) + \pi^2 \alpha^2(\mu)}{\alpha(\mu)}. \quad (29)$$

This coefficient can also be written in terms of the dielectric function making use of the formula

$$\lim_{x \rightarrow 0} \mathcal{E}(\mu \pm ix) \equiv \mathcal{E}^\pm(\mu) = \lambda(\mu) \mp i\pi\alpha(\mu), \quad (30)$$

in the form:

$$C_\mu = \frac{\mathcal{E}^+(\mu) \mathcal{E}^-(\mu)}{\alpha(\mu)}. \quad (31)$$

Applying the orthogonality relations (25) and (28), the set of eigenfunctions $\{\rho_i(v), \rho_\mu(v)\}$ is complete will be proved in the Appendix.

Here we briefly examine the discrete eigenvalue for a Maxwellian plasma. If we assume the Maxwell distribution for the j -species unperturbed distribution function:

$$f_{0j} = n_0 \left(\frac{\pi}{v_j^2} \right)^{\frac{3}{2}} \exp\left(-\frac{v^2}{v_j^2} \right), \quad (32)$$

where v_j is j -th thermal velocity. In this case, α is reduced

to

$$\alpha = -\sum_j \frac{1}{\sqrt{\pi}} \left(\frac{k_{Dj}}{k} \right)^2 \frac{v}{v_j} \exp \left(-\left(\frac{v}{v_j} \right)^2 \right), \quad (33)$$

where $k_{Dj} = (4\pi n_j e^2 / T_j)^{1/2}$ with $T_j = m_j v_j^2 / 2$ and m_j being j -th species mass, is the Debye wave number. The dispersion function defined in eq.(13) becomes

$$\varepsilon(\mu) = 1 + \sum_j \left(\frac{k_{Dj}}{k} \right)^2 \left\{ 1 + \frac{\mu}{v_j} Z \left(\frac{\mu}{v_j} \right) \right\}, \quad (34)$$

where Z is the plasma dispersion function.

If we assume the cold ion limit with $\alpha_j = 0$, and the large argument approximation for electrons, $Z(x) = -(1 + 1/2x^2)/x$ for $x \gg 1$ to the argument of the Z in eq.(34), we find the discrete eigenvalue determined by eq.(13), as $\mu_0 = v_e k_D / k \sqrt{2}$, which can be rewritten as $\omega_0 = \omega_{pe}$ with $\omega_{pe} = (4\pi n_e e^2 / m_e)^{1/2}$ being the electron plasma frequency. In the cold ion limit, we recover the electron plasma oscillation. By the same manner, for cold electron limit with $\alpha_e = 0$, we can derive the ion plasma oscillation. In general the dispersion relation (13) yields a hybrid oscillation frequency.

4. Initial Value Problem

We here apply the orthogonality relations obtained in the previous section to the initial value problem. Let us expand the charge density at $t=0$ in terms of the eigenfunctions:

$$\rho(v, 0) = \sum_j a_j \rho_j(v) + \int A(\mu) \rho_\mu(v) d\mu. \quad (35)$$

Applying the relation (25) to the discrete terms in eq.(35), the coefficients are determined by

$$a_j = \frac{1}{C_j} \int \frac{\rho(v, 0)}{\mu_j - v} dv. \quad (36)$$

By the same manner for $\mu \in \mathbb{C}$, applying eq.(28), the coefficient A is determined in the form

$$A(\mu) = \frac{1}{\varepsilon'(\mu)\varepsilon'(\mu)} \left\{ \alpha(\mu) P \int \frac{\rho(v, 0)}{\mu - v} dv + \lambda(\mu) \rho(\mu, 0) \right\}. \quad (37)$$

Making use of these coefficients, the time dependent solution is given in the form

$$\rho(v, t) = \sum_j a_j e^{-ik\mu_j t} \rho_j(v) + \int A(\mu) e^{-ik\mu t} \rho_\mu(v) d\mu. \quad (38)$$

Depending on the parameter values, the discrete eigenvalues μ_j may vary in the complex μ -plane. If $\text{Im}(\mu_j) < 0$, we must seek the discrete eigenvalue in the lower half μ -plane by analytical continuation of the dispersion function ε like Landau did. In this case, the discrete mode damps as time goes on. When the discrete eigenvalue does not exist in the whole μ -plane, the first term disappears and the system is described by the continuum contribution alone.

If we take into account the space dependence, eq.(38) may be written for a fixed k in the form:

$$\rho(x, v, t) = \sum_j a_j e^{ik(x - \mu_j t)} \rho_j(v) + \int A(\mu) e^{ik(x - \mu t)} \rho_\mu(v) d\mu. \quad (39)$$

The velocity dependence of the charge density can be seen by substituting the eigenfunctions:

$$\rho(x, v, t) = \sum_j a_j e^{ik(x - \mu_j t)} \frac{\alpha(v)}{\mu_j - v} + \lambda(v) A(v) e^{ik(x - vt)} + P \int \frac{\alpha}{\mu - v} A(\mu) e^{ik(x - \mu t)} d\mu. \quad (40)$$

Equation (40) indicates that the discrete mode term may show singularity when the discrete eigenvalue is real and $\mu_j = v$. The second term is the ballistic mode which becomes steady on the orbit $x=vt$. The third singular integral term is the pure continuum contribution. As a function of velocity, each term varies in complicated manner depending on temperatures and forms of distribution functions particularly for multi-species plasmas.

When $\rho(x, v, t)$ is obtained, from eq.(2), the scalar potential is determined by

$$\phi(x, t) = \frac{4\pi}{k^2} \int \rho(x, v, t) dv. \quad (41)$$

Since the eigenfunctions are normalized by eq.(11), eq.(41) can be rewritten as

$$\phi(x, t) = \frac{4\pi}{k^2} \left\{ \sum_j a_j e^{ik(x - \mu_j t)} + \int A(\mu) e^{ik(x - \mu t)} d\mu \right\}. \quad (42)$$

Substitution of coefficients given by eqs.(36) and (37), and the initial charge density $\rho(v, 0) = \rho_0 \delta(v - v_0)$ for

the sake of simplicity, into eq.(42), we have

$$\phi(x, t) = \frac{4\pi}{k^2} \rho_0 \left\{ \sum_j \frac{e^{ik(x-\mu_j t)}}{\mathcal{E}'(\mu_j)} \frac{\alpha(v_0)}{\mu_j - v_0} + \frac{\lambda(v_0)e^{ik(x-v_0 t)}}{\lambda^2(v_0) + \pi^2 \alpha^2(v_0)} + P \int \frac{e^{ik(x-\mu t)}}{\mu - v_0} \frac{\alpha(\mu) d\mu}{\lambda^2(\mu) + \pi^2 \alpha^2(\mu)} \right\}. \quad (43)$$

Equation (43) also indicates that the first discrete mode becomes singular when the initial beam velocity v_0 coincides with the eigenvalue μ_0 .

Let us apply the normal mode expansion method to solve each distribution function. Since arbitrary function can be expanded by the complete set of the eigenfunction $\{\rho_j(v), \rho_\mu(v)\}$, we may apply the complete set of eigenfunctions for the charge density ρ to solve the distribution function $g_j(v, t)$. If we expand $g_j(v, 0)$ in the form

$$g_j(v, 0) = \sum_i a'_i \rho_i(v) + \int A^i(\mu) \rho_\mu(v) d\mu, \quad (44)$$

the expansion coefficients can be obtained making use of the orthogonality relations: multiplying both sides of eq.(44) by ρ_i^* , and integrating over v , we have from eq.(25)

$$a'_i = \frac{1}{C_i} \int \frac{g_j(v, 0)}{\mu_i - v} dv. \quad (45)$$

By the same manner, making use of eq.(28), we obtain

$$A^i(\mu) = \frac{1}{\mathcal{E}'(\mu)\mathcal{E}(\mu)} \left\{ \alpha(\mu) P \int \frac{g_j(v, 0)}{\mu - v} dv + \lambda(\mu) g(\mu, 0) \right\}. \quad (46)$$

Applying these coefficients, the time dependent distribution function is then given by

$$g_j(v, t) = \sum_i a'_i e^{-i\mu_i t} \rho_i(v) + \int A^i(\mu) e^{-i\mu t} \rho_\mu(v) d\mu. \quad (47)$$

The solution (47) can recover eq.(38) if we multiply by e_i both sides of eq.(47) and add with respect to j . The distribution function of each species is also obtained by solving eq.(6) with respect to time t .

5. Summary

The normal mode expansion method for the Vlasov equation in an electron plasma has been developed to the case of multi-species plasmas in an infinite system. The

equation of charge density for the Vlasov-Poisson system in the multi-species plasmas is found to be the same form as that of electron plasma treated by Van Kampen and Case. The eigenfunctions have been constructed, and the orthogonality relations have been derived by the same manner as Case. The completeness has also been proved by applying the orthogonality. The orthogonality relations of eigenfunctions has been applied for the initial value problem of the charge density. Once the charge density is solved, the scalar potential, and each species distribution function is easily derived. When the discrete eigenvalue is real (purely oscillatory), the discrete mode is found to show a singularity in the velocity space at the phase velocity of eigenfrequency. The continuum contribution is separated into the ballistic mode and pure continuum contribution (singular principal value integral over the continuum), which may be a source of incoherent fluctuations.

Appendix Completeness of Eigenfunction

The completeness of eigenfunctions has already been proved by Case. Here we show the completeness directly making use of the orthogonality relation,^{4,5)} and compare with Case's result. First we show briefly Case's proof: if arbitrary function $g(v)$ expanded as

$$g(v) = \sum_i a_i \rho_i(v) + \int A(\mu) \rho_\mu(v) d\mu, \quad (A.1)$$

then it is sufficient to show that it is always possible to solve eq.(A.1) with respect to $A(v)$. If we introduce g' by

$$g'(v) = g(v) - \sum_i a_i \rho_i(v), \quad (A.2)$$

then, eq.(A.1) is rewritten as follow

$$g'(v) = \lambda(v) A(v) + \alpha(v) P \int \frac{A(\mu)}{\mu - v} d\mu. \quad (A.3)$$

In order to solve $A(v)$, Case introduce three complex functions:

$$N(z) = \frac{1}{2\pi i} \int \frac{A(\mu)}{\mu - z} d\mu, \quad (A.4)$$

$$Q(z) = \frac{1}{2\pi i} \int \frac{\alpha(v)}{v-z} dv, \quad (\text{A.5})$$

$$M(z) = \frac{1}{2\pi i} \int \frac{g'(v)}{v-z} dv, \quad (\text{A.6})$$

which are analytic with cut along the real axis, and tend to zero at infinity. Each quantity in eq.(A.3) can be expressed by the limiting values of these functions for $v \in \Sigma$:

$$N^+(v) - N^-(v) = A(v), \quad (\text{A.7})$$

$$\pi i \{N^+(v) + N^-(v)\} = P \int \frac{A(\mu)}{\mu-v} d\mu, \quad (\text{A.8})$$

$$Q^+(v) - Q^-(v) = \alpha(v), \quad (\text{A.9})$$

$$\lambda(v) = 1 + \pi i \{Q^+(v) + Q^-(v)\}, \quad (\text{A.10})$$

$$M^+(v) - M^-(v) = g'(v). \quad (\text{A.11})$$

Substitution of these expressions into eq.(3) yields

$$N^+(v)\epsilon^+(v) - M^+(v) = N^-(v)\epsilon^-(v) - M^-(v)$$

where $\epsilon(z)=1+2\pi iQ(z)$ is the dielectric function. This equation means that the complex function

$$J(z)=N(z)\epsilon(z)-M(z)$$

is continuous across the real axis, i.e., analytic in the whole complex z -plane, and tends to zero at infinity. From the Louville theorem $J(z)=0$ in the whole complex plane, that is, the function N is determined by $N(z)=M(z)/\epsilon(z)$. Since $M(\mu_i)=0$ at the discrete eigenvalue condition $\epsilon(\mu_i)=0$, N is analytic at this point, and the gap condition of N from eq.(A.7) gives A , we have

$$A(v) = \frac{M^+(v)}{\epsilon^+(v)} - \frac{M^-(v)}{\epsilon^-(v)}. \quad (\text{A.12})$$

Let us apply the orthogonality relations to solve

eq.(A.3). Multiplying both sides of eq.(A.3) by ρ^* , integrating over v , and applying eq.(28), we have

$$A(v) = \frac{1}{\epsilon^+(v)\epsilon^-(v)} \left\{ \lambda(v)g'(v) - \alpha(v)P \int \frac{g'(\mu)}{\mu-v} d\mu \right\}. \quad (\text{A.13})$$

Introducing the relations $\epsilon^+ + \epsilon^- = 2\lambda(v)$, $\epsilon^+ - \epsilon^- = 2\pi i\alpha(v)$,

$$M^+(v) + M^-(v) = \frac{1}{\pi i} P \int \frac{g'(\mu)}{\mu-v} d\mu, \quad (\text{A.14})$$

and eq.(A.11) into eq.(A.13), we find

$$A(v) = \frac{1}{\epsilon^+(v)\epsilon^-(v)} \{M^+(v)\epsilon^-(v) - M^-(v)\epsilon^+(v)\}. \quad (\text{A.15})$$

which reduces to eq.(A.12), and the completeness is proved by applying the orthogonality relations.

Acknowledgment

This study is a joint research effort with the National Institute for Fusion Science.

References

- 1) N. Van Kampen, *Physica*, **21**(1955)945.
- 2) K.M.Case, *Annals of Physics*, **7**(1959)349.
- 3) K.M.Case and P.F.Zweifel, *Linear Transport Theory*, Addison-Wesley, New York, 1967.
- 4) T.Yamagishi, *J. Nucl. Science and Technol.*, **9**(1971)420.
- 5) T.Yamagishi, *Transport Theory and Statistical Physics*, **2**(1973)107.

Recent Issues of NIFS Series

- NIFS-514 T Shjmozuma, S Monimoto, M Sato, Y Takita, S Ito, S Kubo, H Idei, K Ohkubo and T Watan,
A Forced Gas-Cooled Single-Disk Window Using Silicon Nitride Composite for High Power CW Millimeter Waves, Oct 1997
- NIFS-515 K Akaishi,
On the Solution of the Outgassing Equation for the Pump-down of an Unbaked Vacuum System, Oct 1997
- NIFS-516 *Papers Presented at the 6th H-mode Workshop (Sceon, Germany)*; Oct. 1997
- NIFS-517 John L. Johnson,
The Quest for Fusion Energy, Oct. 1997
- NIFS-518 J. Chen, N Nakajima and M. Okamoto,
Shift-and-Inverse Lanczos Algorithm for Ideal MHD Stability Analysis, Nov. 1997
- NIFS-519 M Yokoyama, N. Nakajima and M. Okamoto,
Nonlinear Incompressible Poloidal Viscosity in $L=2$ Heliotron and Quasi-Symmetric Stellarators, Nov 1997
- NIFS-520 S Kida and H. Miura,
Identificaiton and Analysis of Vortical Structures, Nov 1997
- NIFS-521 K. Ida, S Nishimura, T Minami, K Tanaka, S. Okamura, M Osakabe, H Idei, S Kubo, C Takahashi and K Matsuoka,
High Ion Temperature Mode in CHS Heliotron/torsatron Plasmas, Nov 1997
- NIFS-522 M Yokoyama, N Nakajima and M. Okamoto,
Realization and Classification of Symmetric Stellarator Configurations through Plasma Boundary Modulations; Dec. 1997
- NIFS-523 H. Kitauchi,
Topological Structure of Magnetic Flux Lines Generated by Thermal Convection in a Rotating Spherical Shell; Dec. 1997
- NIFS-524 T Ohkawa,
Tunneling Electron Trap; Dec. 1997
- NIFS-525 K Itoh, S.-I Itoh, M. Yagi, A Fukuyama,
Solitary Radial Electric Field Structure in Tokamak Plasmas, Dec 1997
- NIFS-526 Andrey N. Lyakhov,
Alfven Instabilities in FRC Plasma; Dec. 1997
- NIFS-527 J Uramoto,
Net Current Increment of negative Muonlike Particle Produced by the Electron and Positive Ion Bunch-method; Dec. 1997
- NIFS-528 Andrey N Lyakhov,
Comments on Electrostatic Drift Instabilities in Field Reversed Configuraiton; Dec 1997
- NIFS-529 J. Uramoto,
Pair Creation of Negative and Positive Pionlike (Muonlike) Particle by Interaction between an Electron Bunch and a Positive Ion Bunch; Dec 1997
- NIFS-530 J Uramoto,
Measuring Method of Decay Time of Negative Muonlike Particle by Beam Collector Applied RF Bias Voltage; Dec 1997
- NIFS-531 J. Uramoto,
Confirmation Method for Metal Plate Penetration of Low Energy Negative Pionlike or Muonlike Particle Beam under Positive Ions; Dec 1997
- NIFS-532 J. Uramoto,
Pair Creations of Negative and Positive Pionlike (Muonlike) Particle or K Mesonlike (Muonlike) Particle in

H2 or D2 Gas Discharge in Magnetic Field; Dec. 1997

- NIFS-533 S. Kawata, C. Boonmee, T. Teramoto, L. Drska, J. Limpouch, R. Liska, M. Sinor,
Computer-Assisted Particle-in-Cell Code Development; Dec. 1997
- NIFS-534 Y. Matsukawa, T. Suda, S. Ohnuki and C. Namba,
Microstructure and Mechanical Property of Neutron Irradiated TiNi Shape Memory Alloy; Jan. 1998
- NIFS-535 A. Fujisawa, H. Iguchi, H. Idei, S. Kubo, K. Matsuoka, S. Okamura, K. Tanaka, T. Minami, S. Ohdachi, S. Morita, H. Zushi, S. Lee, M. Osakabe, R. Akiyama, Y. Yoshimura, K. Toi, H. Sanuki, K. Itoh, A. Shimizu, S. Takagi, A. Ejiri, C. Takahashi, M. Kojima, S. Hidekuma, K. Ida, S. Nishimura, N. Inoue, R. Sakamoto, S.-I. Itoh, Y. Hamada, M. Fujiwara,
Discovery of Electric Pulsation in a Toroidal Helical Plasma; Jan. 1998
- NIFS-536 Lj.R. Hadzievski, M.M. Skoric, M. Kono and T. Sato,
Simulation of Weak and Strong Langmuir Collapse Regimes; Jan. 1998
- NIFS-537 H. Sugama, W. Horton,
Nonlinear Electromagnetic Gyrokinetic Equation for Plasmas with Large Mean Flows; Feb. 1998
- NIFS-538 H. Iguchi, T.P. Crowley, A. Fujisawa, S. Lee, K. Tanaka, T. Minami, S. Nishimura, K. Ida, R. Akiyama, Y. Hamada, H., Idei, M. Isobe, M. Kojima, S. Kubo, S. Morita, S. Ohdachi, S. Okamura, M. Osakabe, K. Matsuoka, C. Takahashi and K. Toi,
Space Potential Fluctuations during MHD Activities in the Compact Helical System (CHS); Feb. 1998
- NIFS-539 Takashi Yabe and Yan Zhang,
Effect of Ambient Gas on Three-Dimensional Breakup in Coronet Formation Process; Feb. 1998
- NIFS-540 H. Nakamura, K. Ikeda and S. Yamaguchi,
Transport Coefficients of InSb in a Strong Magnetic Field; Feb. 1998
- NIFS-541 J. Uramoto,
Development of v_{μ} Beam Detector and Large Area v_{μ} Beam Source by H_2 Gas Discharge (I); Mar. 1998
- NIFS-542 J. Uramoto,
Development of \bar{v}_{μ} Beam Detector and Large Area \bar{v}_{μ} Beam Source by H_2 Gas Discharge (II),
Mar. 1998
- NIFS-543 J. Uramoto,
Some Problems inside a Mass Analyzer for Pions Extracted from a H_2 Gas Discharge; Mar. 1998
- NIFS-544 J. Uramoto,
Simplified v_{μ} \bar{v}_{μ} Beam Detector and v_{μ} \bar{v}_{μ} Beam Source by Interaction between an Electron Bunch and a Positive Ion Bunch; Mar. 1998
- NIFS-545 J. Uramoto,
Various Neutrino Beams Generated by D_2 Gas Discharge; Mar. 1998
- NIFS-546 R. Kanno, N. Nakajima, T. Hayashi and M. Okamoto,
Computational Study of Three Dimensional Equilibria with the Bootstrap Current, Mar. 1998
- NIFS-547 R. Kanno, N. Nakajima and M. Okamoto,
Electron Heat Transport in a Self-Similar Structure of Magnetic Islands; Apr. 1998
- NIFS-548 J.E. Rice,
Simulated Impurity Transport in LHD from MIST; May 1998
- NIFS-549 M.M. Skoric, T. Sato, A.M. Maluckov and M.S. Jovanovic,
On Kinetic Complexity in a Three-Wave Interaction; June 1998
- NIFS-550 S. Goto and S. Kida,
Passive Saclar Spectrum in Isotropic Turbulence: Prediction by the Lagrangian Direct-interaction Approximation; June 1998

- NIFS-551 T Kuroda, H Sugama, R Kanno, M Okamoto and W Horton,
Initial Value Problem of the Toroidal Ion Temperature Gradient Mode ; June 1998
- NIFS-552 T Mutoh, R Kumazawa, T Seki, F Simpo, G Nomura, T Ido and T Watari,
Steady State Tests of High Voltage Ceramic Feedthroughs and Co-Axial Transmission Line of ICRF Heating System for the Large Helical Device ; June 1998
- NIFS-553 N. Noda, K Tsuzuki, A. Sagara, N Inoue, T Muroga,
oronazation in Future Devices -Protecting Layer against Tritium and Energetic Neutrals-; July 1998
- NIFS-554 S. Murakami and H Saleem,
Electromagnetic Effects on Rippling Instability and Tokamak Edge Fluctuations; July 1998
- NIFS-555 H Nakamura , K Ikeda and S. Yamaguchi .
Physical Model of Nernst Element, Aug 1998
- NIFS-556 H Okumura, S. Yamaguchi, H. Nakamura, K Ikeda and K. Sawada,
Numerical Computation of Thermoelectric and Thermomagnetic Effects; Aug 1998
- NIFS-557 Y Takeiri, M Osakabe, K. Tsumon, Y. Oka, O Kaneko, E Asano, T. Kawamoto, R. Akiyama and M Tanaka,
Development of a High-Current Hydrogen-Negative Ion Source for LHD-NBI System; Aug.1998
- NIFS-558 M. Tanaka, A Yu Grosberg and T Tanaka.
Molecular Dynamics of Structure Organization of Polyampholytes, Sep. 1998
- NIFS-559 R. Honuchi, K. Nishimura and T. Watanabe,
Kinetic Stabilization of Tilt Disruption in Field-Reversed Configurations, Sep. 1998
(IAEA-CN-69/THP1/11)
- NIFS-560 S Sudo, K. Kholopenkov, K. Matsuoka, S Okamura, C. Takahashi, R. Akiyama, A Fujisawa, K Ida, H Idei, H. Iguchi, M Isobe, S. Kado, K. Kondo, S. Kubo, H. Kuramoto, T Minami, S. Morita, S. Nishimura, M. Osakabe, M. Sasao, B. Peterson, K. Tanaka, K. Toi and Y. Yoshimura,
Particle Transport Study with Tracer-Encapsulated Solid Pellet Injection; Oct. 1998
(IAEA-CN-69/EXP1/18)
- NIFS-561 A. Fujisawa, H. Iguchi, S. Lee, K. Tanaka, T. Minami, Y. Yoshimura, M. Osakabe, K. Matsuoka, S. Okamura, H. Idei, S. Kubo, S. Ohdachi, S. Morita, R. Akiyama, K. Toi, H. Sanuki, K. Itoh, K. Ida, A. Shimizu, S. Takagi, C. Takahashi, M. Kojima, S. Hidekuma, S. Nishimura, M. Isobe, A. Ejiri, N. Inoue, R. Sakamoto, Y. Hamada and M. Fujiwara,
Dynamic Behavior Associated with Electric Field Transitions in CHS Heliotron/Torsatron, Oct 1998
(IAEA-CN-69/EX5/1)
- NIFS-562 S. Yoshikawa ,
Next Generation Toroidal Devices , Oct 1998
- NIFS-563 Y Todo and T. Sato,
Kineuc-Magnetohydrodynamic Simulation Study of Fast Ions and Toroidal Alfvén Eigenmodes; Oct 1998
(IAEA-CN-69/THP2/22)
- NIFS-564 T Watari, T. Shimozuma, Y. Takeiri, R. Kumazawa, T Mutoh, M. Sato, O. Kaneko, K. Ohkubo, S. Kubo, H. Idei, Y. Oka, M. Osakabe, T. Seki, K. Tsumon, Y. Yoshimura, R. Akiyama, T. Kawamoto, S. Kobayashi, F. Shimpō, Y. Takita, E. Asano, S. Itoh, G. Nomura, T. Ido, M. Hamabe, M. Fujiwara, A. Iiyoshi, S. Morimoto, T. Bigelow and Y.P. Zhao,
Steady State Heating Technology Development for LHD; Oct 1998
(IAEA-CN-69/FTP/21)
- NIFS-565 A. Sagara, K.Y. Watanabe, K. Yamazaki, O. Motojima, M. Fujiwara, O. Mitarai, S. Imagawa, H. Yamanishi, H. Chikaraishi, A. Kohyama, H. Matsui, T. Muroga, T. Noda, N. Ohyabu, T. Satow, A.A. Shishkin, S. Tanaka, T. Terai and T. Uda,
LHD-Type Compact Helical Reactors, Oct 1998
(IAEA-CN-69/FTP/03(R))
- NIFS-566 N. Nakajima, J. Chen, K. Ichiguchi and M. Okamoto,
Global Mode Analysis of Ideal MHD Modes in L=2 Heliotron/Torsatron Systems, Oct 1998
(IAEA-CN-69/THP1/08)
- NIFS-567 K. Ida, M. Osakabe, K. Tanaka, T. Minami, S. Nishimura, S. Okamura, A. Fujisawa, Y. Yoshimura, S. Kubo, R. Akiyama, D.S. Darrow, H. Idei, H. Iguchi, M. Isobe, S. Kado, T. Kondo, S. Lee, K. Matsuoka, S. Morita, I. Nomura, S. Ohdachi, M. Sasao, A. Shimizu, K. Tsumon, S. Takayama, M. Takechi, S. Takagi, C. Takahashi, K. Toi and T. Watari,
Transition from L Mode to High Ion Temperature Mode in CHS Heliotron/Torsatron Plasmas, Oct 1998
(IAEA-CN-69/EX2/2)

- NIFS-568 S. Okamura, K. Matsuoka, R. Akiyama, D.S. Darrow, A. Ejiri, A. Fujisawa, M. Fujiwara, M. Goto, K. Ida, H. Idei, H. Iguchi, N. Inoue, M. Isobe, K. Itoh, S. Kado, K. Khlopenkov, T. Kondo, S. Kubo, A. Lazaros, S. Lee, G. Matsunaga, T. Minami, S. Morita, S. Murakami, N. Nakajima, N. Nikai, S. Nishimura, I. Nomura, S. Ohdachi, K. Ohkuni, M. Osakabe, R. Pavlichenko, B. Peterson, R. Sakamoto, H. Sanuki, M. Sasao, A. Shimizu, Y. Shirai, S. Sudo, S. Takagi, C. Takahashi, S. Takayama, M. Takechi, K. Tanaka, K. Toi, K. Yamazaki, Y. Yoshimura and T. Watari,
Confinement Physics Study in a Small Low-Aspect-Ratio Helical Device CHS; Oct 1998
(IAEA-CN-69/OV4/5)
- NIFS-569 M.M. Skoric, T. Sato, A. Maluckov, M.S. Jovanovic.
Micro- and Macro-scale Self-organization in a Dissipative Plasma; Oct. 1998
- NIFS-570 T. Hayashi, N. Mizuguchi, T.-H. Watanabe, T. Sato and the Complexity Simulation Group,
Nonlinear Simulations of Internal Reconnection Event in Spherical Tokamak; Oct 1998
(IAEA-CN-69/TH3/3)
- NIFS-571 A. Iiyoshi, A. Komori, A. Ejiri, M. Emoto, H. Funaba, M. Goto, K. Ida, H. Idei, S. Inagaki, S. Kado, O. Kaneko, K. Kawahata, S. Kubo, R. Kumazawa, S. Masuzaki, T. Minami, J. Miyazawa, T. Morisaki, S. Morita, S. Murakami, S. Muto, T. Muto, Y. Nagayama, Y. Nakamura, H. Nakanishi, K. Narihara, K. Nishimura, N. Noda, T. Kobuchi, S. Ohdachi, N. Ohyabu, Y. Oka, M. Osakabe, T. Ozaki, B.J. Peterson, A. Sagara, S. Sakakibara, R. Sakamoto, H. Sasao, M. Sasao, K. Sato, M. Sato, T. Seki, T. Shimozumura, M. Shoji, H. Suzuki, Y. Takeiri, K. Tanaka, K. Toi, T. Tokuzawa, K. Tsumon, I. Yamada, H. Yamada, S. Yamaguchi, M. Yokoyama, K.Y. Watanabe, T. Watari, R. Akiyama, H. Chikaraishi, K. Haba, S. Hamaguchi, S. Iima, S. Imagawa, N. Inoue, K. Iwamoto, S. Kitagawa, Y. Kubota, J. Kodaira, R. Maekawa, T. Mito, T. Nagasaka, A. Nishimura, Y. Takita, C. Takahashi, K. Takahata, K. Yamauchi, H. Tamura, T. Tsuzuki, S. Yamada, N. Yanagi, H. Yonezu, Y. Hamada, K. Matsuoka, K. Murai, K. Ohkubo, I. Ohtake, M. Okamoto, S. Sato, T. Satow, S. Sudo, S. Tanahashi, K. Yamazaki, M. Fujiwara and O. Motojima,
An Overview of the Large Helical Device Project; Oct. 1998
(IAEA-CN-69/OV1/4)
- NIFS-572 M. Fujiwara, H. Yamada, A. Ejiri, M. Emoto, H. Funaba, M. Goto, K. Ida, H. Idei, S. Inagaki, S. Kado, O. Kaneko, K. Kawahata, A. Komori, S. Kubo, R. Kumazawa, S. Masuzaki, T. Minami, J. Miyazawa, T. Morisaki, S. Morita, S. Murakami, S. Muto, T. Muto, Y. Nagayama, Y. Nakamura, H. Nakanishi, K. Narihara, K. Nishimura, N. Noda, T. Kobuchi, S. Ohdachi, N. Ohyabu, Y. Oka, M. Osakabe, T. Ozaki, B. J. Peterson, A. Sagara, S. Sakakibara, R. Sakamoto, H. Sasao, M. Sasao, K. Sato, M. Sato, T. Seki, T. Shimozumura, M. Shoji, H. Suzuki, Y. Takeiri, K. Tanaka, K. Toi, T. Tokuzawa, K. Tsumon, I. Yamada, S. Yamaguchi, M. Yokoyama, K.Y. Watanabe, T. Watari, R. Akiyama, H. Chikaraishi, K. Haba, S. Hamaguchi, M. Iima, S. Imagawa, N. Inoue, K. Iwamoto, S. Kitagawa, Y. Kubota, J. Kodaira, R. Maekawa, T. Mito, T. Nagasaka, A. Nishimura, Y. Takita, C. Takahashi, K. Takahata, K. Yamauchi, H. Tamura, T. Tsuzuki, S. Yamada, N. Yanagi, H. Yonezu, Y. Hamada, K. Matsuoka, K. Murai, K. Ohkubo, I. Ohtake, M. Okamoto, S. Sato, T. Satow, S. Sudo, S. Tanahashi, K. Yamazaki, O. Motojima and A. Iiyoshi,
Plasma Confinement Studies in LHD; Oct. 1998
(IAEA-CN-69/EX2/3)
- NIFS-573 O. Motojima, K. Akaishi, H. Chikaraishi, H. Funaba, S. Hamaguchi, S. Imagawa, S. Inagaki, N. Inoue, A. Iwamoto, S. Kitagawa, A. Komori, Y. Kubota, R. Maekawa, S. Masuzaki, T. Mito, J. Miyazawa, T. Morisaki, T. Muroga, T. Nagasaka, Y. Nakamura, A. Nishimura, K. Nishimura, N. Noda, N. Ohyabu, S. Sagara, S. Sakakibara, R. Sakamoto, S. Satoh, T. Satow, M. Shoji, H. Suzuki, K. Takahata, H. Tamura, K. Watanabe, H. Yamada, S. Yamada, S. Yamaguchi, K. Yamazaki, N. Yanagi, T. Baba, H. Hayashi, M. Iima, T. Inoue, S. Kato, T. Kato, T. Kondo, S. Moriuchi, H. Ogawa, I. Ohtake, K. Ooba, H. Sekiguchi, N. Suzuki, S. Takami, Y. Taniguchi, T. Tsuzuki, N. Yamamoto, K. Yasui, H. Yonezu, M. Fujiwara and A. Iiyoshi,
Progress Summary of LHD Engineering Design and Construction; Oct. 1998
(IAEA-CN-69/FT2/1)
- NIFS-574 K. Toi, M. Takechi, S. Takagi, G. Matsunaga, M. Isobe, T. Kondo, M. Sasao, D.S. Darrow, K. Ohkuni, S. Ohdachi, R. Akiyama, A. Fujisawa, M. Goto, H. Idei, K. Ida, H. Iguchi, S. Kado, M. Kojima, S. Kubo, S. Lee, K. Matsuoka, T. Minami, S. Morita, N. Nikai, S. Nishimura, S. Okamura, M. Osakabe, A. Shimizu, Y. Shirai, C. Takahashi, K. Tanaka, T. Watari and Y. Yoshimura,
Global MHD Modes Excited by Energetic Ions in Heliotron/Torsatron Plasmas; Oct. 1998
(IAEA-CN-69/EXP1/19)
- NIFS-575 Y. Hamada, A. Nishizawa, Y. Kawasumi, A. Fujisawa, M. Kojima, K. Narihara, K. Ida, A. Ejiri, S. Ohdachi, K. Kawahata, K. Toi, K. Sato, T. Seki, H. Iguchi, K. Adachi, S. Hidekuma, S. Hirokura, K. Iwasaki, T. Ido, R. Kumazawa, H. Kuramoto, T. Minami, I. Nomura, M. Sasao, K.N. Sato, T. Tsuzuki, I. Yamada and T. Watari,
Potential Turbulence in Tokamak Plasmas; Oct 1998
(IAEA-CN-69/EXP2/14)
- NIFS-576 S. Murakami, U. Gasparino, H. Idei, S. Kubo, H. Maassberg, N. Marushchenko, N. Nakajima, M. Romé and M. Okamoto.
5D Simulation Study of Suprathermal Electron Transport in Non-Axisymmetric Plasmas; Oct. 1998
(IAEA-CN-69/THP1/01)
- NIFS-577 S. Fujiwara and T. Sato,
Molecular Dynamics Simulation of Structure Formation of Short Chain Molecules; Nov. 1998
- NIFS-578 T. Yamagishi,
Eigenfunctions for Vlasov Equation in Multi-species Plasmas Nov. 1998