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Sparseness of Nonlinear Coupling

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Abstract

It is not weakness of nonlinearity but sparseness of nonlinear couplings that plays a key role in the direct-interaction approximation (DIA), which is an excellent moment closure theory for nonlinear dynamical systems. Homogeneous Navier-Stokes turbulence is an example of dynamical systems in which nonlinearity is strong in magnitude but sparse in coupling. In order to clarify the importance of sparseness of coupling, we formulate DIA for a model equation which has three parameters coupling density, strength of nonlinearity and the number of degrees of freedom. By the help of numerical simulations, it is shown that DIA is applicable when the coupling density is much smaller than the square root of the number of degrees of freedom, even if the strength of nonlinearity is infinitely large. This implies that the applicability of DIA has nothing to do with the Gaussianity of a dynamical variable, although DIA is often explained as a theory based on it.

Keywords: nonlinear dynamical system, direct-interaction approximation, turbulence, Gaussianity

1 Introduction

Nonlinearity of a dynamical system is characterized by two parameters: strength and coupling density. The former represents the ratio of magnitude of nonlinear and linear terms, which corresponds to the Reynolds number in the Navier-Stokes system. The latter is defined by the number of direct interactions between two modes. Let us consider the properties of these two parameters in homogeneous fluid turbulence described by the Navier-Stokes equation,

$$\frac{\partial}{\partial t} u_i(\boldsymbol{k},t) = \sum_{j=1}^{3}\sum_{m=1}^{3} M_{ijm}(\boldsymbol{k}) \times \sum_{\substack{\boldsymbol{p} \\ (\boldsymbol{k}+\boldsymbol{p}+\boldsymbol{q}=\boldsymbol{o})}}\sum_{\boldsymbol{q}} u_j(-\boldsymbol{p},t)\, u_m(-\boldsymbol{q},t) - \nu k^2\, u_i(\boldsymbol{k},t)\,, \quad (1)$$

where $u_i(\boldsymbol{k},t)$ is the Fourier component of velocity, $\boldsymbol{k}$ is the wavenumber, and

$$M_{ijm}(\boldsymbol{k}) = -\frac{\mathrm{i}}{2}\left(\frac{2\pi}{L}\right)^3 \times \left(k_m\,\delta_{ij} + k_j\,\delta_{im} - \frac{2k_i k_j k_m}{k^2}\right). \quad (2)$$

The flow is assumed to be periodic in three orthogonal directions with period L (see Chap.V of Ref. [1] for derivation). The Reynolds number is defined by the ratio of magnitude of nonlinear and linear dissipative terms on the right-hand side of (1), which is much larger than unity in fully developed turbulence. The coupling density is the number of direct interactions between two modes, say $u_i(\boldsymbol{k}_1)$ and $u_i(\boldsymbol{k}_2)$, and is exactly equal to unity. To observe this, direct interactions between $u_i(\boldsymbol{k}_1)$ and other modes are schematically depicted in Fig.1. There is only a single direct interaction between any pair of modes owning to the constraint $\boldsymbol{k}+\boldsymbol{p}+\boldsymbol{q}=\boldsymbol{o}$ in summations with respect to $\boldsymbol{p}$ and $\boldsymbol{q}$. In general, there are $O(N)$ direct interactions between a pair of modes in an N mode quadratic nonlinear dynamical system. Note that the number of active modes is extremely large in fully developed turbulence. Hence, homogeneous turbulence at large Reynolds numbers may be regarded as a dynamical system of strong nonlinearity with sparse coupling.

A nonlinear term causes an infinite hierarchy of moment equations. Closure theories of nonlinear dynamical systems are after all to truncate this hierarchy, and to obtain a closed set of equations for a finite number of

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moments. Traditional perturbation theories based on Reynolds-number expansions are not useful for a system at large Reynolds numbers. Fortunately, however, if nonlinear couplings are sparse as for the Navier-Stokes equation (1), there is a possibility to solve the closure problem. In the present paper, we discuss the direct-interaction approximation (DIA) [2, 3], which is based on sparseness of coupling. As will be shown later, the coupling density should be compared with the number of degrees of freedom. It is usual that any approximation may require at least one small parameter. A purpose of the present paper is to find such small parameters for validity condition of DIA which depends on the coupling density and the number of degrees of freedom and which can be small even in the large Reynolds number limit.

The rest of this paper is organized as follows. In the next section, we introduce a model equation, in which the strength of nonlinearity, the coupling density and the number of degrees of freedom are easily controlled. The DIA is formulated for this model in §3, and its accuracy and validity conditions are discussed in §4. In §5, we clarify relationship between DIA and Gaussianity. Concluding remarks are given in the last section.

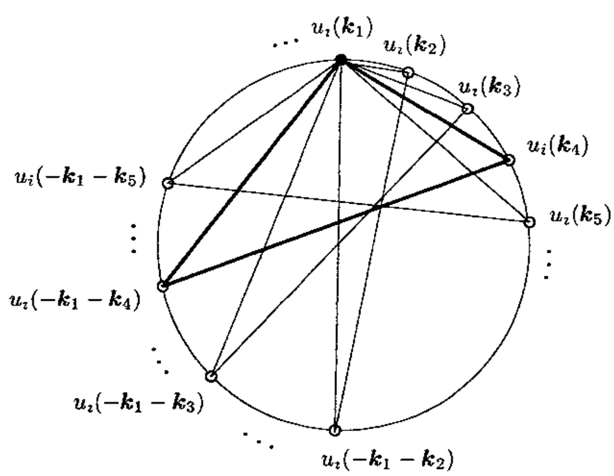

Figure.1 Schematic illustration of direct interactions between a Fourier mode $u_i(\boldsymbol{k}_1)$ and the others in the Navier-Stokes system. Each interaction is depicted by a triangle.

2 Model equation and coupling density

By introducing a model equation [4–6] for a set of real variables $\{X_i | i = 1, 2, \cdots, N\}$,

$$\frac{\mathrm{d}}{\mathrm{d}t} X_i(t) = \sum_j \sum_k C_{ijk}\, X_j(t)\, X_k(t) - \nu\, X_i(t) + f_i(t)\,, \qquad (3)$$

the coupling density of which is easily controlled, we shall discuss the essence of DIA in detail. Hereafter, $\sum_i$

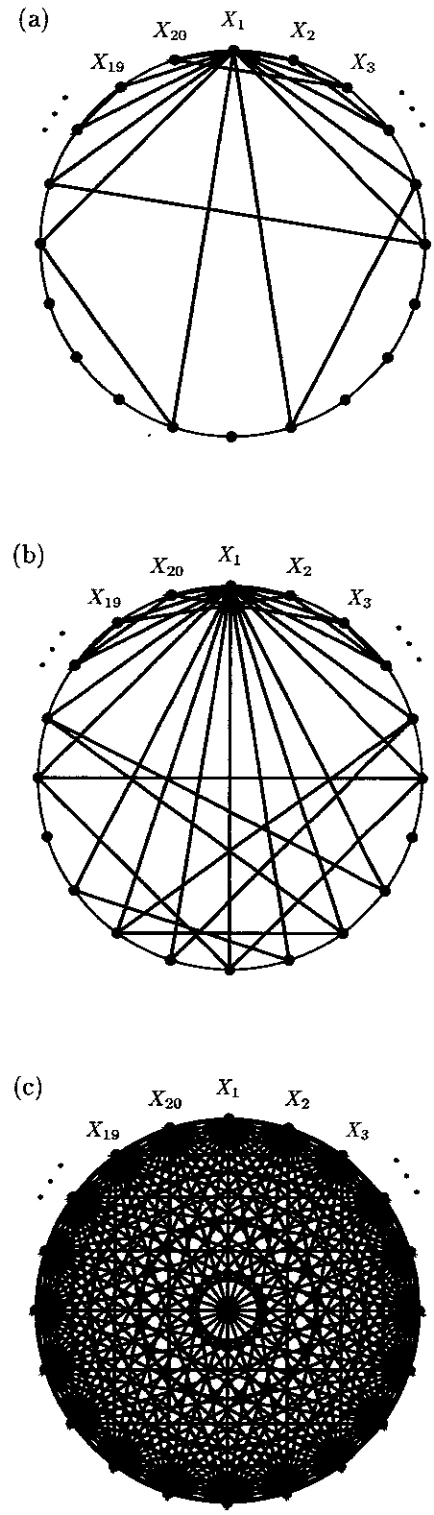

Figure.2 Direct interactions between X_1 and other modes in the model system (3). The coupling density is (a) $\rho \sim 1$, (b) 2 and (c) 18. $N = 20$.

implies $\sum_{i=1}^{N}$. Real constant coefficients C_{ijk} are assumed to satisfy

$$C_{ijk} = C_{ikj} \quad \text{(symmetry)}, \tag{4}$$

$$C_{ijk} + C_{jki} + C_{kij} = 0 \quad \text{(detailed balance of energy)}, \tag{5}$$

$$C_{ijj} = 0 \quad \text{(absence of self-interaction)}, \tag{6}$$

$$C_{ijk} = C_{\mathrm{rem}\{i+m,N\}\mathrm{rem}\{j+m,N\}\mathrm{rem}\{k+m,N\}} \quad \text{for } m = 1, 2, \cdots, N-1 \quad \text{(homogeneity)}, \tag{7}$$

where rem denotes the remainder between 1 and N. Because of symmetric appearance of $X_j(t)$ and $X_k(t)$ in the nonlinear term, the anti-symmetric part of C_{ijk} is irrelevant so that we can always assume the symmetry condition (4). The detailed balance of energy (5) and the absence of self-interaction (6) are analogue of the Navier-Stokes equation (1). A stepwise random force $f_i(t)$, which is constant during a time interval Δt, is assumed to obey a normal distribution with zero mean and variance,

$$\sigma^2 = \frac{2\nu}{N\Delta t}\,. \tag{8}$$

Instantaneous values of forcing terms, $f_i(t)$ and $f_j(t')$, are independent of each other either if $i \neq j$ or $|t-t'| > \Delta t$. Because of the homogeneity (7) of C_{ijk} and $f_i(t)$, statistics of $X_i(t)$ can be independent of i.

We define the coupling density ρ quantitatively by the average number of direct interactions between pairs of modes. In an N mode system, ρ can take a value between 0 and $N-2$. Only two extreme cases were dealt with in Ref. [6], namely the cases either that there is a single, at the most, direct interaction between any pair of modes ($\rho \sim 1$), or that a pair of modes has $N-2$ direct interactions through all of the other modes ($\rho = N-2$). Here, we shall examine such systems that have intermediate coupling density. Incidentally, as stated in §1, ρ is exactly equal to unity in the Navier-Stokes system (1). Because of restrictions (4)—(7) of coefficients C_{ijk}, we cannot construct in general this model system (3) with ρ exactly equal to an integer except the densest coupling case. Direct interactions between $X_1(t)$ and the other modes in several cases of coupling density are schematically depicted in Fig.2 in the case of $N = 20$. We construct the coefficients C_{ijk} in such a way that the number of direct interactions between any pair of modes does not exceed $[\rho] + 1$ in order to make the nonlinear couplings as homogeneous as possible.

The model equation (3) has three parameters: the dissipation coefficient ν (or the Reynolds number $R = 1/\nu$), the number N of degrees of freedom and the coupling density ρ. We are particularly interested in the limit of strong nonlinearity, i.e.,

$$R \to \infty \quad \Longleftrightarrow \quad \nu \to 0\,, \tag{9}$$

in which both the dissipative and the forcing terms vanish (see (8)), and the system is governed by the nonlinear term only.

3 Direct-interaction approximation (DIA)

3.1 Correlation and response functions

To start with, we separate the variable $X_i(t)$ into two parts:

$$X_i(t) = x_i(t) + Y_i(t)\,, \tag{10}$$

where

$$x_i(t) = \overline{X_i(t)} \tag{11}$$

is an ensemble average (or a temporal average in statistically stationary cases), and $Y_i(t)$ is a fluctuation which satisfies

$$\overline{Y_i(t)} = 0\,. \tag{12}$$

It is easy to derive the evolution equation for $Y_i(t)$, from (3), (10)—(12), as

$$\frac{\mathrm{d}}{\mathrm{d}t} Y_i(t) = \sum_j \sum_k C_{ijk} \left\{ 2\, x_j(t)\, Y_k(t) - \overline{Y_j(t)\, Y_k(t)} + Y_j(t)\, Y_k(t) \right\} - \nu\, Y_i(t) + f_i(t)\,. \tag{13}$$

The purpose of second-order moment closure is to derive a closed set of equations for the correlation function of the fluctuation,

$$V(t,t') = \overline{Y_i(t)\, Y_i(t')}\,, \tag{14}$$

which is governed by

$$\left[\frac{\partial}{\partial t} + \nu \right] V(t,t') = \sum_j \sum_k C_{ijk} \left\{ 2\, x_j(t)\, \overline{Y_k(t)\, Y_i(t')} + \overline{Y_j(t)\, Y_k(t)\, Y_i(t')} \right\} \quad (t > t') \tag{15}$$

and

$$\left[\frac{\mathrm{d}}{\mathrm{d}t}+2\nu\right]V(t,t)=2\sum_j\sum_k C_{ijk}\left\{2\,x_j(t)\,\overline{Y_k(t)\,Y_i(t)}+\overline{Y_j(t)\,Y_k(t)\,Y_i(t)}\right\}+2\,\overline{f_i(t)\,Y_i(t)}\,, \tag{16}$$

and for a few other statistical quantities (if necessary). In DIA we introduce the response function,

$$G_{in}(t,t')=\frac{\delta Y_i(t)}{\delta Y_n(t')}\,, \tag{17}$$

where δ denotes a functional derivative and $G_{in}(t,t)=\delta_{in}$. The evolution equation of $G_{in}(t,t')$ is obtained by taking the functional derivative of (3) with respect to $Y_n(t')$ as

$$\frac{\partial}{\partial t}G_{in}(t,t')=2\sum_j\sum_k C_{ijk}\left\{x_j(t)+Y_j(t)\right\}G_{kn}(t,t')-\nu\,G_{in}(t,t')\qquad(t>t')\,, \tag{18}$$

an ensemble average of which leads to

$$\frac{\partial}{\partial t}\overline{G_{in}(t,t')}=2\sum_j\sum_k C_{ijk}\left\{x_j(t)\,\overline{G_{kn}(t,t')}+\overline{Y_j(t)\,G_{kn}(t,t')}\right\}-\nu\,\overline{G_{in}(t,t')}\qquad(t>t') \tag{19}$$

with initial condition $\overline{G_{in}(t,t)}=\delta_{in}$. The higher order statistical quantities appearing on the right-hand sides of (15), (16) and (19) must be expressed in terms of the quantities on the left-hand sides under some reasonable assumptions. In the following subsection, we solve this closure problem by DIA [2,3].

3.2 Formulation of DIA

The DIA is based on the sparseness of nonlinear couplings, and consists of the following two assumptions, which have been justified numerically [6]. If we artificially remove a single direct interaction between $Y_{i_0}(t)$, $Y_{j_0}(t)$ and $Y_{k_0}(t)$, say, then (I) these three modes are statistically independent of each other, but (II) statistical properties of the entire system are hardly changed. In order to put these assumptions into practice, we introduce a decomposition (the direct-interaction decomposition) at $t=t_0$ that

$$Y_i(t)=Y^{(0)}_{i/i_0j_0k_0}(t|t_0)+Y^{(1)}_{i/i_0j_0k_0}(t|t_0)\,. \tag{20}$$

Here, an artificial field $Y^{(0)}_{i/i_0j_0k_0}(t|t_0)$, called the non-direct-interaction field associated with a triplet of modes $\{Y_{i_0},Y_{j_0},Y_{k_0}\}$, is governed by

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}Y^{(0)}_{i/i_0j_0k_0}(t|t_0)=&\sum_j\sum_k C_{ijk}\left\{2\,x_j(t)\,Y^{(0)}_{k/i_0j_0k_0}(t|t_0)-\overline{Y_j(t)\,Y_k(t)}\right\}\\&+\sum_{\substack{j\ \ k\\ \{i,j,k\}\neq\{i_0,j_0,k_0\}}}\sum C_{ijk}\,Y^{(0)}_{j/i_0j_0k_0}(t|t_0)\,Y^{(0)}_{k/i_0j_0k_0}(t|t_0)\;-\nu\,Y^{(0)}_{i/i_0j_0k_0}(t|t_0)+f_i(t)\,,\end{aligned} \tag{21}$$

and a deviation field $Y^{(1)}_{i/i_0j_0k_0}(t|t_0)$ by

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}Y^{(1)}_{i/i_0j_0k_0}(t|t_0)=&\sum_j\sum_k 2\,C_{ijk}\,x_j(t)\,Y^{(1)}_{k/i_0j_0k_0}(t|t_0)\\&+\sum_{\substack{j\ \ k\\ \{i,j,k\}\neq\{i_0,j_0,k_0\}}}\sum 2\,C_{ijk}\,Y_j(t\,Y^{(1)}_{k/i_0j_0k_0}(t|t_0)\;-\nu\,Y^{(1)}_{i/i_0j_0k_0}(t|t_0)\\&+\delta_{ii_0}\,2\,C_{i_0j_0k_0}\,Y^{(0)}_{j_0/i_0j_0k_0}(t|t_0)Y^{(0)}_{k_0/i_0j_0k_0}(t|t_0)\\&+\delta_{ij_0}\,2\,C_{j_0k_0i_0}\,Y^{(0)}_{k_0/i_0j_0k_0}(t|t_0)Y^{(0)}_{i_0/i_0j_0k_0}(t|t_0)\\&+\delta_{ii_0}\,2\,C_{k_0i_0j_0}\,Y^{(0)}_{i_0/i_0j_0k_0}(t|t_0)Y^{(0)}_{j_0/i_0j_0k_0}(t|t_0)\,.\end{aligned} \tag{22}$$

Observe that a difference between the governing equations for $Y_i(t)$ and $Y^{(0)}_{i/i_0j_0k_0}(t|t_0)$ is only in the absence of a single direct interaction between $\{Y_{i_0}, Y_{j_0}, Y_{k_0}\}$ in the latter. Then, the DIA assumptions are stated as follows:

DIA assumption 1

Three modes $\{Y^{(0)}_{i_0/i_0j_0k_0}, Y^{(0)}_{j_0/i_0j_0k_0}, Y^{(0)}_{k_0/i_0j_0k_0}\}$, between which the direct interaction is absent, are statistically independent of each other.

DIA assumption 2

The deviation field $Y^{(1)}_{i/i_0j_0k_0}(t|t_0)$ is much smaller in magnitude than the true field $Y_i(t)$ in a statistical sense as long as $t - t_0$ is within the time scale of the auto-correlation function $V(t, t')$.

Similar assumptions are imposed for the response function (see Ref [6] for detail). Hereafter, we omit the argument t_0 of $Y^{(0)}_{i/i_0j_0k_0}(t|t_0)$ for brevity of notations.

These two assumptions are satisfied better as the number N of degrees of freedom of the system increases. As for the first assumption, when N is large, many modes are coupled indirectly with those particular three modes, and such indirect interactions may be well randomized and tend to cancel out correlation between them in a statistical sense. As for the second assumption, an artificial removal of one direct interaction may have only tiny effects on the entire statistics of a large system. Furthermore, it should be stressed here that the first assumption requires also sparseness of nonlinear couplings. If couplings are denser, there exist more indirect interactions between a triplet of modes through a few modes (e.g., four-mode indirect interactions shown in Fig.3), which are not expected to be randomized enough to make correlation between the three modes negligible. Indeed, it is shown [6] that in the densest coupling system ($\rho = N - 2$) this assumption is violated, and the prediction of auto-correlation function by DIA is far from satisfaction. Density dependence of accuracy of prediction by DIA will be estimated quantitatively in §4.

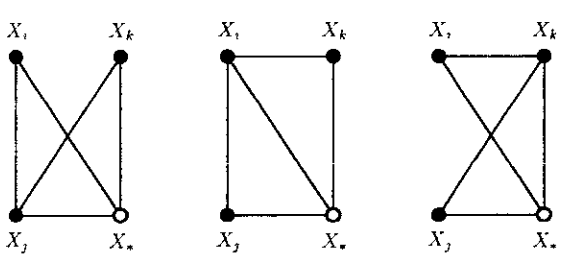

Figure.3 Direct interactions between X_1 and other modes in the model system (3). The coupling density is (a) $\rho \sim 1$, (b) 2 and (c) 18. $N = 20$.

The DIA assumption 1 will be employed as

$$\overline{Y^{(0)}_{i/ijk}\, Y^{(0)}_{j/ijk}\, Y^{(0)}_{k/ijk}} = 0 \qquad (23)$$

and

$$\overline{Y^{(0)}_{i/ijk}\, Y^{(0)}_{i/ijk}\, Y^{(0)}_{j/ijk}\, Y^{(0)}_{j/ijk}} = \overline{Y^{(0)}_{i/ijk}\, Y^{(0)}_{i/ijk}}\;\overline{Y^{(0)}_{j/ijk}\, Y^{(0)}_{j/ijk}} \qquad (24)$$

in the DIA formulation (see Appendix A). It is the guiding principle in moment closure theories to express higher-order moments in terms of lower-order ones. By a straightforward calculation, using (23) and (24), we can derive from (15), (16) and (19) a closed set of equations for the two-mode correlation and the response functions as

$$\left[\frac{\partial}{\partial t} + \nu\right] V(t,t') = - 2C \int_{t_0}^{t} \mathrm{d}t''\, G(t,t'')\, V(\max\{t', t''\}, \min\{t', t''\}) \times V(t,t'') + 2C \int_{t_0}^{t'} \mathrm{d}t''\, G(t',t'')\, V(t,t'')^2 \quad (t > t') , \qquad (25)$$

$$\left[\frac{\mathrm{d}}{\mathrm{d}t} + 2\nu\right] V(t,t) = \frac{2\nu}{N} , \qquad (26)$$

and

$$\left[\frac{\partial}{\partial t} + \nu\right] G(t,t') = -2C \int_{t'}^{t} \mathrm{d}t''\, V(t,t'')\, G(t,t'')\, G(t'',t') . \qquad (27)$$

Here, $G(t,t')$ and C are defined by

$$G(t,t') = \overline{G_{ii}(t,t')} \qquad (28)$$

and

$$C = \sum_j \sum_k C_{ijk}{}^2 , \qquad (29)$$

respectively. The linear equation (26) can be integrated as

$$V(t,t) = \frac{1}{N} + \left\{ V(0,0) - \frac{1}{N} \right\} e^{-2\nu t} . \qquad (30)$$

It is interesting to see that (25) and (27) permit a solution which satisfies

$$V(t,t') = V(t',t')\, G(t,t') , \qquad (31)$$

since each of them then leads to

$$\left[\frac{\partial}{\partial t} + \nu\right] G(t,t') = -2C \int_{t'}^{t} \mathrm{d}t''\, G(t,t'')\, G(t,t'')\, G(t'',t')\, V(t'',t'') . \qquad (32)$$

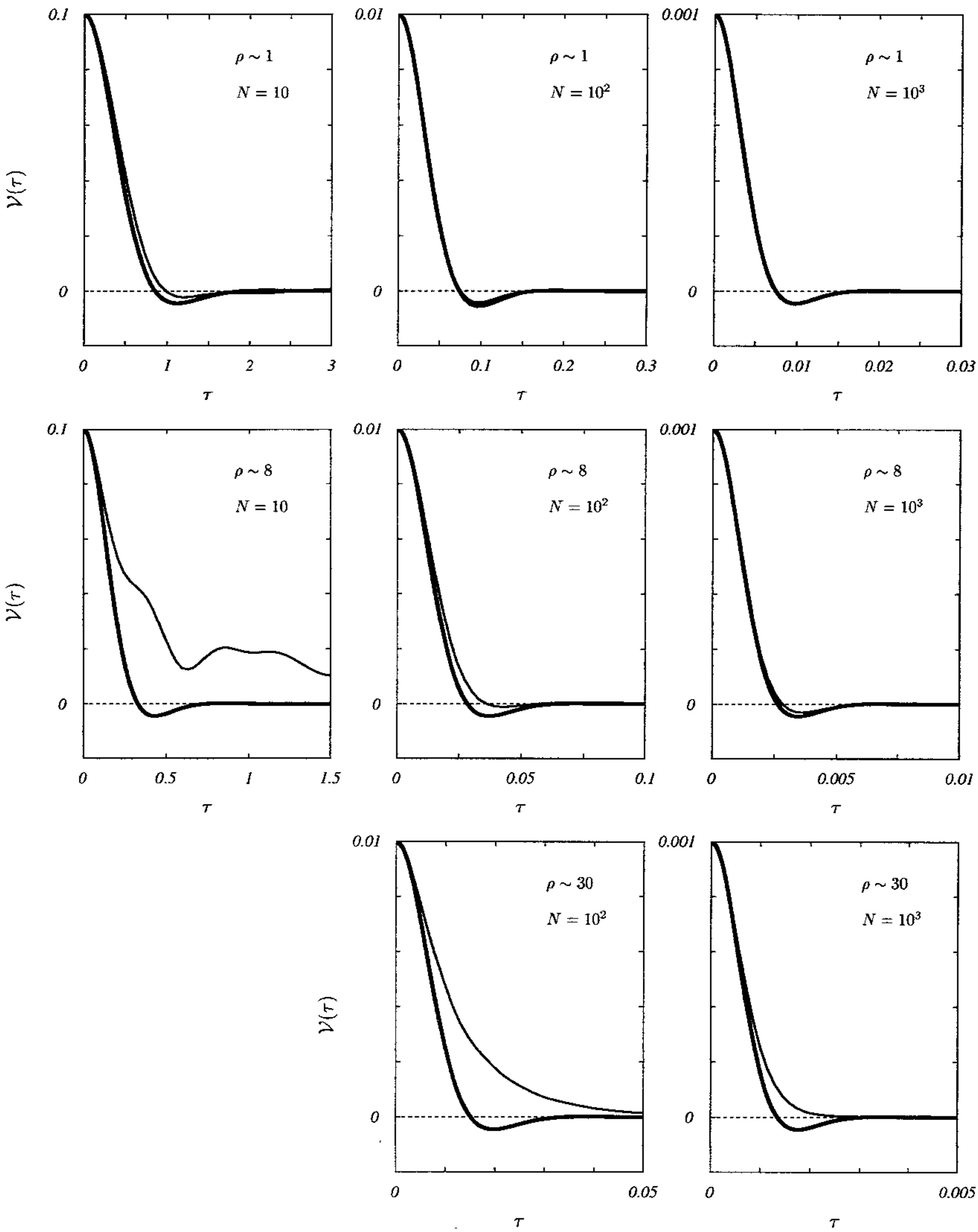

Figure.4 Auto-correlation function of the model equation (3) for various combinations of N and ρ in the case $R \to \infty$. Thick line, DIA; thin line, direct numerical simulation.

Equation (31) implies that the response function plays the role of a linear propagate operator on the correlation function. It should be noted that the artificial time t_0 in (25) disappears automatically when we assume (31). In the following, we call (30), (31) and (32) the DIA equations.

4 Accuracy and validity of DIA

Here, we show explicitly that the DIA equations give an extremely good prediction even in the limit of large Reynolds numbers ($R \to \infty$) by examining their solutions in the statistically stationary state, in which V and G may be expressed as

$$\begin{cases} \mathcal{V}(\tau) = V(t+\tau, t)\,, \\ \mathcal{G}(\tau) = G(t+\tau, t)\,, \end{cases} \tag{33}$$

where $\tau \geq 0$. Then the DIA equations (30), (31) and (32) are reduced to

$$\mathcal{V}(0) = \frac{1}{N}\,, \tag{34}$$

$$\mathcal{V}(\tau) = \mathcal{V}(0)\,\mathcal{G}(\tau) \tag{35}$$

and

$$\left[\frac{\mathrm{d}}{\mathrm{d}\tau} + \nu\right] \mathcal{V}(\tau) = -\frac{2\mathcal{C}}{\mathcal{V}(0)} \int_0^\tau \mathrm{d}\tau' \; \mathcal{V}(\tau')^2\, \mathcal{V}(\tau - \tau')\,. \tag{36}$$

For later use, we define here the auto-correlation time scale τ_c as

$$\tau_c(N) = \frac{1}{\sqrt{2\mathcal{C}\,\mathcal{V}(0)}}\,. \tag{37}$$

The DIA equation (36) does not depend on N in the limit $\nu \to 0$, if $\mathcal{V}$ and τ are respectively rescaled by $\mathcal{V}(0)$ and $\tau_c(N)$.

In Fig.4, we compare predictions $\mathcal{V}_{\text{DIA}}$ of the auto-correlation function by the DIA equations (36) and results $\mathcal{V}_{\text{DNS}}$ of direct numerical simulations of (3) for several combinations of (ρ, N) at the infinite Reynolds number (i.e., $\nu = 0$). The agreement is better for larger N or smaller ρ, and it is excellent in the case that $\rho \sim 1$ and $N \gg 1$. This is consistent with the intuitive argument made in §3.2 that DIA is valid for systems with sparse nonlinear couplings and a large number of degrees of freedom.

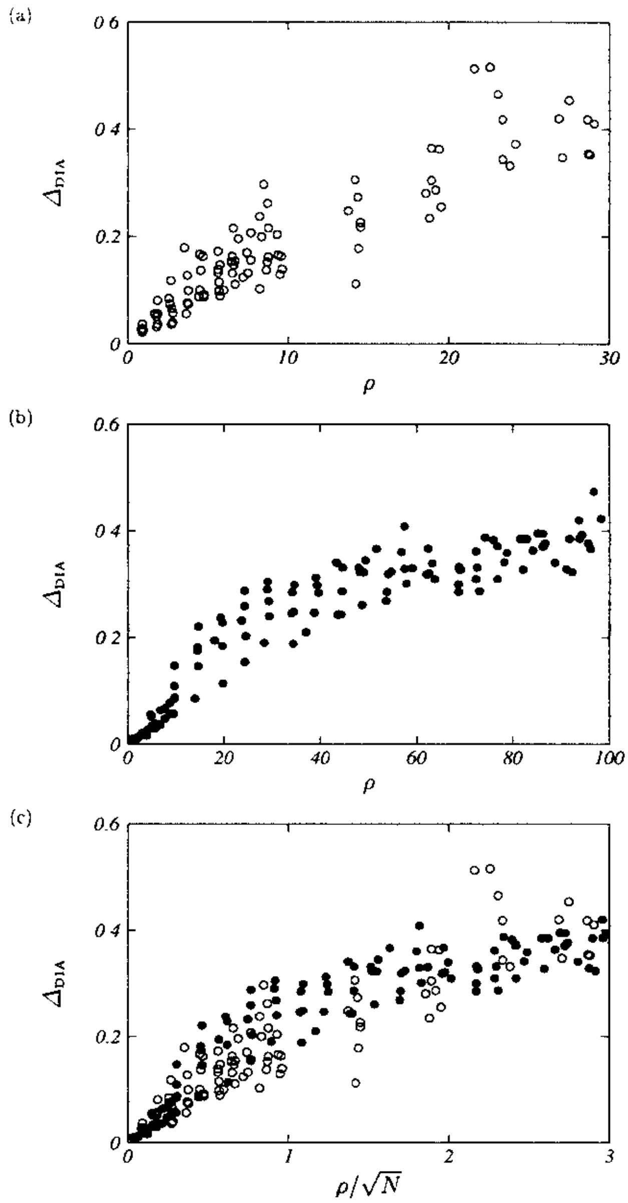

Figure.5 Coupling density dependence of the integrated difference Δ_{DIA}. $\nu = 0$. (a) $N = 10^2$, (b) 10^3 and (c) ∘, $N = 10^2$; •, $N = 10^3$.

In order to make a quantitative comparison, we introduce an integrated difference,

$$\Delta_{\text{DIA}} = \frac{\displaystyle\int_0^\infty \mathrm{d}\tau \left|\mathcal{V}_{\text{DIA}}(\tau) - \mathcal{V}_{\text{DNS}}(\tau)\right|}{\displaystyle\int_0^\infty \mathrm{d}\tau \left|\mathcal{V}_{\text{DNS}}(\tau)\right|}\,, \tag{38}$$

which represents a discrepancy between a direct numerical simulation result and a prediction by the DIA equations, that is, small Δ_{DIA} implies validness of the DIA equations. Numerical results of Δ_{DIA} evaluated for various ρ ($\ll N$) are plotted in Figs.5(a) for $N = 10^2$ and

(b) 10^3 in the limit of $R \to \infty$. They are qualitatively consistent with the above results on the auto-correlation function. As seen in Fig.5(c), $\Delta_{\rm DIA}$ seems to be a function of $\rho/\sqrt{N}$. Thus, it is numerically suggested that the DIA equations may be valid as long as

$$\rho \ll \sqrt{N}\,, \tag{39}$$

even if $R \to \infty$.

This validity condition (39) of the DIA equations is consistent with that of DIA assumption 1, which is grounded on sparseness of nonlinear couplings and on largeness of the number of degrees of freedom. This may be understood as follows. The simplest indirect interactions between a triplet of modes $\{X_i, X_j, X_k\}$ are four-mode interactions which involves one more mode X_* (Fig.3). Since such a small-number-mode interaction may bring non-negligible correlation between the three modes, the independency assumption (DIA assumption 1) deteriorates. (It requires that indirect interactions should be randomized enough so that their effects to the correlation can be neglected in a statistical sense.) The probability q of the existence of a four-mode interaction is evaluated as

$$q = \begin{cases} 0 & (\rho \le 1)\,, \\ \dfrac{3\rho(\rho-1)}{N} & (\rho \ge 1) \end{cases} \tag{40}$$

for $N \gg 1$. It is expected that $\Delta_{\rm DIA}$ is small for small q. This may be a reason why $\Delta_{\rm DIA}$ is a function of $\rho/\sqrt{N}$; note that $q \approx 3(\rho/\sqrt{N})^2$ for $\rho \gg 1$ and $N \gg 1$. It is concluded therefore that DIA is valid for systems which satisfy the condition (39) irrespective of the Reynolds number.

5 Irrelativeness of DIA to Gaussianity

It is known (see e.g., §19.6 of Ref. [7]) that the DIA equations are derived by a variety of methods based on different idea and procedures. In contrast with the method described in §4, many of the authors (e.g., Refs. [4,8,9]) assumed the Gaussianity of variables, and employed the well-known mathematical properties of a set of probability variables, $\{Y_i^{(G)} | i = 1, 2, 3, \cdots\}$ which obeys a joint Gaussian distribution with zero mean: An odd-order moment vanishes, e.g.,

$$\overline{Y_i^{(G)} Y_j^{(G)} Y_k^{(G)}} = 0\,, \tag{41}$$

and an even-order moment is expressed in terms of second-order ones, e.g.,

$$\overline{Y_i^{(G)} Y_j^{(G)} Y_k^{(G)} Y_l^{(G)}} = \overline{Y_i^{(G)} Y_j^{(G)}}\;\overline{Y_k^{(G)} Y_l^{(G)}} + \overline{Y_i^{(G)} Y_l^{(G)}}\;\overline{Y_j^{(G)} Y_k^{(G)}} + \overline{Y_i^{(G)} Y_k^{(G)}}\;\overline{Y_j^{(G)} Y_l^{(G)}}\,. \tag{42}$$

It should be emphasized that (41) and (42) are superficially similar to, but essentially different from (23) and (24). In the following we shall give two examples which show that DIA is never based on Gaussianity. In one case DIA does not work though the pdf of $X_i(t)$ is nearly Gaussian. In the other case the pdf is far from Gaussian but the prediction by DIA is excellent.

The m-th order moment of $X_i(t)$ is defined by

$$M_m(t) = \overline{X_i(t)^{\,m}}\,. \tag{43}$$

Here, we deal with a non-stationary homogeneous system, in which an average is defined by

$$\overline{z_i(t;j)} = \frac{1}{NJ}\sum_{i=1}^{N}\sum_{j=1}^{J} z_i(t;j) \tag{44}$$

instead of the temporal average adopted in a stationary case. Index j denotes a realization of $J(\gg 1)$ initial conditions. Equation (44) implies, therefore, that we take an average over both N modes and J initial conditions. For the purpose of showing the irrelativeness of DIA to Gaussianity, the following four initial pdfs $P(X_i, t=0)$, which satisfy

$$M_2(0) = \frac{1}{N}\,, \tag{45}$$

are examined:

$$\text{(a)}: \;\; P(X_i, 0) = \frac{1}{\sqrt{2\pi{\sigma_1}^2}} \exp\left[-\frac{{X_i}^2}{2{\sigma_1}^2}\right], \tag{46}$$

$$\text{(b)}: \;\; P(X_i, 0) = \tfrac{1}{2}\Big\{\, \delta(X_i - \tfrac{1}{2}) + \delta(X_i + \tfrac{1}{2}) \,\Big\}, \tag{47}$$

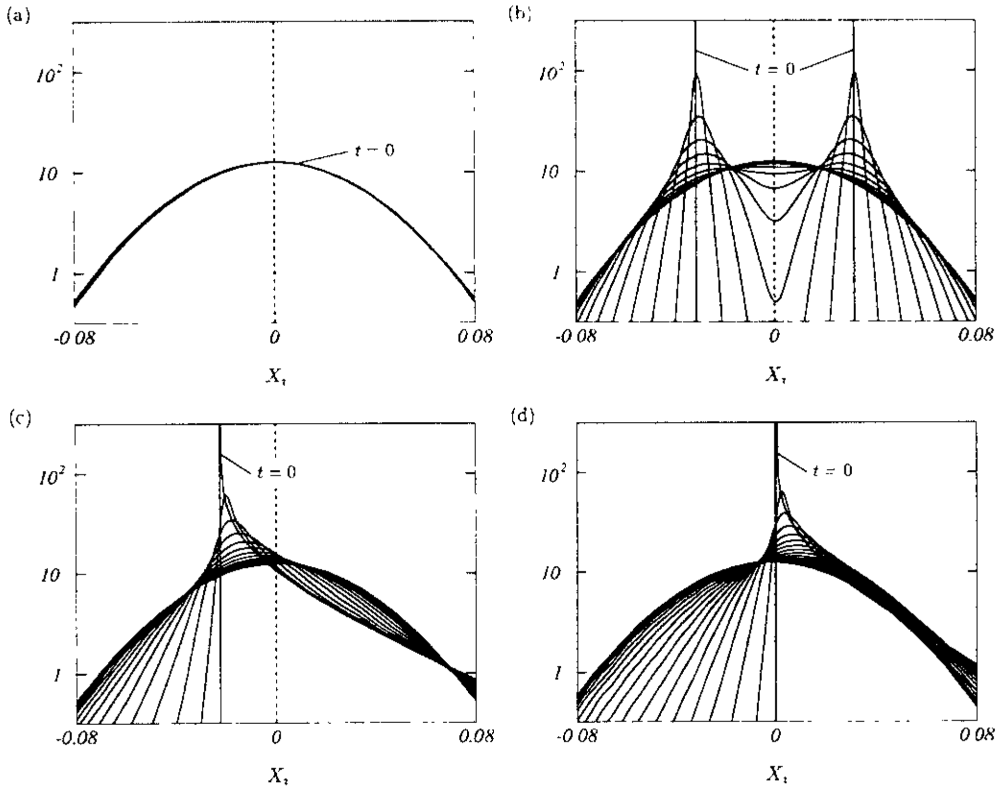

Figure.6 Temporal evolution of the pdf $P(X_i, t)$. Each line denotes $P(X_i, t)$; $t = 0, \Delta t, 2\Delta t, 3\Delta t, \cdots$ ($\Delta t \approx 3.8 \times 10^{-4}$). Figures (a)—(d) correspond to the initial pdfs (a)—(d), respectively. $N = 10^3$, $\nu = 0$, $\rho \sim 1$.

$$
\text{(c)}: \quad P(X_i, 0) = \begin{cases} 0 & (X_i \leq -{\sigma_2}^2)\,, \\ \dfrac{1}{\sqrt{2\pi{\sigma_2}^2(X_i + {\sigma_2}^2)}} \exp\left[-\dfrac{X_i + {\sigma_2}^2}{2{\sigma_2}^2}\right] & (X_i > -{\sigma_2}^2)\,, \end{cases} \tag{48}
$$

$$
\text{(d)}: \quad P(X_i, 0) = \begin{cases} 0 & (X_i \leq 0)\,, \\ \dfrac{1}{\sqrt{2\pi{\sigma_3}^2 X_i}} \exp\left[-\dfrac{X_i}{2{\sigma_3}^2}\right] & (X_i > 0)\,. \end{cases} \tag{49}
$$

where

$$
{\sigma_1}^2 = \frac{1}{N}\,, \quad {\sigma_2}^2 = \left(\frac{1}{2N}\right)^{\frac{1}{2}} \text{ and } {\sigma_3}^2 = \left(\frac{1}{3N}\right)^{\frac{1}{2}}\,. \tag{50}
$$

Initial pdf (a) is a Gaussian, the results of which are compared with those of other three initial pdfs far from a Gaussian. Initial pdf (b) has two peaks; (c) and (d) have non-vanishing skewness factors; (d) has non-vanishing mean $M_1(0) = {\sigma_3}^2$ as well.

Temporal evolutions of the pdfs obtained numerically are plotted in Fig.6. It is observed that each system relaxes to a statistically stationary state. This stationary state may be Ergodic, if $\nu = 0$, in which a state point migrates densely and homogeneously on an equi-energy surface in the N-dimensional phase space. The pdf in the statistically stationary state is then estimated as

$$
P(X_i, \infty) = \frac{1}{\sqrt{\pi}} \frac{\Gamma(N/2)}{\Gamma((N-1)/2)} \left(1 - {X_i}^2\right)^{\frac{N-3}{2}}\,. \tag{51}
$$

The numerically obtained pdf actually tends to (51) at $t \gg \tau_c$ (Fig.7). Here, we should note that expression (51) is valid irrespective of coupling density ρ as long as a conservative system is concerned, and that (51) tends to a Gaussian in the limit of $N \gg 1$. These facts imply that applicability of DIA, which strongly depends on coupling density, is not necessarily stuck with Gaussianity. Note that in the statistically stationary ($t \gg \tau_c$) system with *dense* couplings and large degrees of freedom, the pdf is nearly Gaussian but DIA never works. This is the first example which shows that DIA has no relation with Gaussianity.

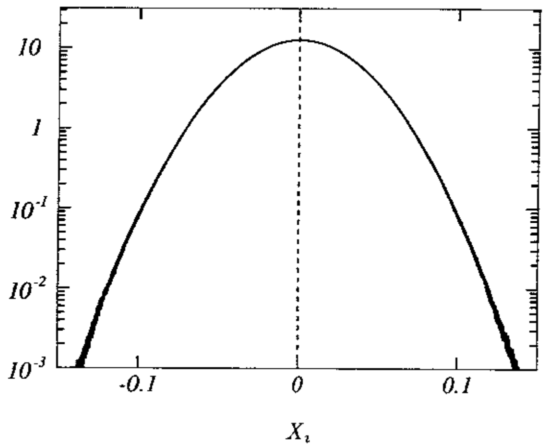

Figure.7 Pdf of $X_i(t)$ in the statistically stationary state ($t \gg \tau_c$) obtained by direct numerical simulations. Coupling density $\rho \approx$ 1, 10, 20, 30, $\cdots$, 100; these curves overlap not only with each other but also with the theoretical prediction (51). $N = 10^3$, $\nu = 0$.

By estimating the skewness factor,

$$s(t) = M_3(t) \Big/ M_2(t)^{\frac{3}{2}} , \tag{52}$$

and the flatness factor,

$$f(t) = M_4(t) \Big/ M_2(t)^2 , \tag{53}$$

we may determine the relaxation time scale from the initial non-Gaussian pdf to the one expressed by (51). Simulation results are plotted in Fig.8, in which time t is normalized by the auto-correlation time scale τ_c defined by (37). It is seen in this figure that the deviation of pdf $P(X_i, t)$ from a Gaussian is significant in the early stage of evolution ($t \lesssim \tau_c$) even if N is large when the initial pdf is not Gaussian.

The above results enable us to investigate the auto-correlation function,

$$\widetilde{V}(t) = V(t, 0) \tag{54}$$

as a statistical quantity the pdf of which is non-Gaussian when the pdf $P(X_i, 0)$ is taken as one of (b)—(d). It is easy to show from (30), (31) and (32) that the DIA equation for $\widetilde{V}(t)$ is the same as (36) in the limit of $\nu \to 0$. In Fig.9, direct numerical simulation results of $\widetilde{V}(t)$ are compared with the prediction by the DIA equation (36) for each of the four initial pdfs. We can see that DIA predictions are excellent irrespective of the initial pdfs. This is the second example which explicitly exhibits irrelativeness of DIA and Gaussianity.

6 Concluding remarks

The moment closure problem for a quadratic nonlinear dynamical system at very large Reynolds number is solved at the second order level by DIA. This approximation is based on the assumption (DIA assumption 1 in §3.2) that the three-mode correlation is induced mainly by direct interactions between them. Sparseness of nonlinear couplings and largeness of the number of degrees of freedom play key roles in this assumption. We applied DIA to the model equation (3), and obtained the DIA equations for the two-mode correlation and the average response functions, which give excellent predictions of the auto-correlation function both in the statistically stationary and non-stationary states (Figs.4 and 9), when the coupling density ρ (i.e., the number of direct interactions between two modes) is much smaller than the square root of the number N of degrees of freedom (Fig.5). It is emphasized again that validity of DIA is related neither with the nonlinearity (the Reynolds number) nor with the Gaussianity of pdf of variable. Indeed, the Reynolds number in Fig.4 is infinitely large, and the pdf of X_i in the cases of Fig.9 is far from Gaussian. The present success of DIA may be a little surprising because it is well-known that a naive application of DIA to the Navier-Stokes turbulence (i.e., the Eulerian DIA [3]) does not necessarily work well, and that an introduction of Lagrangian quantities is necessary [10,11]. Note that the model equation (3) is so general that the Navier-Stokes equation (1) can be included. In this paper, however, we simplify the system by homogeneous viscous constant $\nu_i = \nu$, and by homogeneity condition (7) of the coupling coefficient C_{ijk}. On the other hand, in the Navier-Stokes system, both of the coefficients are larger for Fourier modes of larger wavenumbers. It seems to be significant, therefore, to study an inhomogeneous system, i.e., model equation (3) without the condition (7) and with inhomogeneous viscosity ν_i. We are studying such a system, hoping to report interesting conclusions elsewhere in the near future.

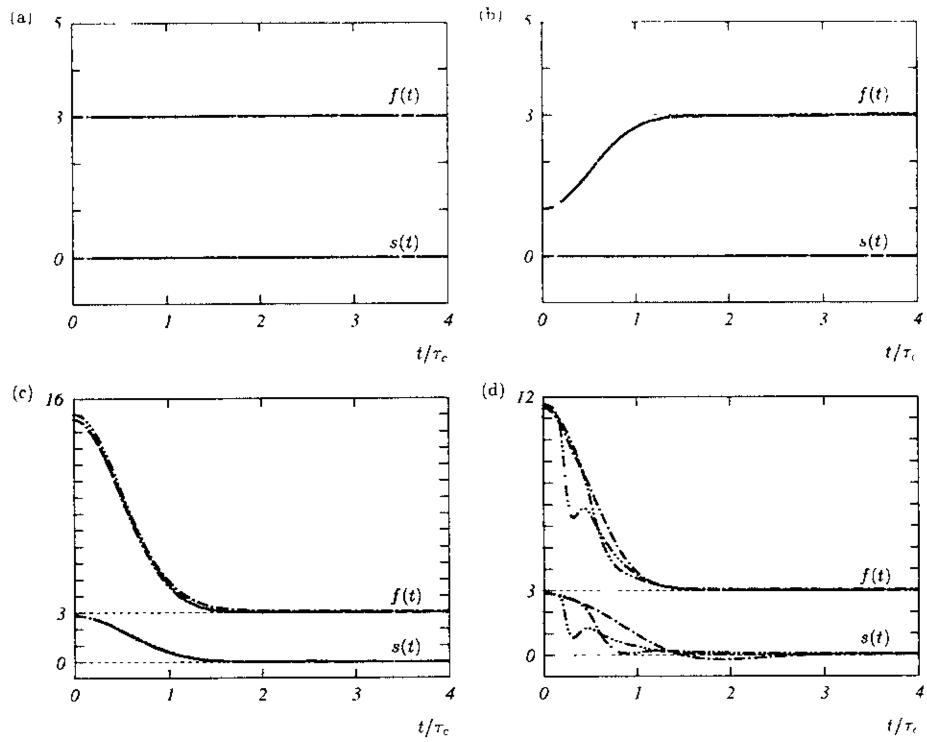

Figure.8 Temporal evolutions of the skewness $s(t)$ and the flatness factors $f(t)$ of $X_i(t)$. ----, $N = 10^3$; ---- , 10^4; ----- , 10^5. Figures (a)—(d) correspond to initial pdfs (a) --(d). $\nu = 0$, $\rho \sim 1$.

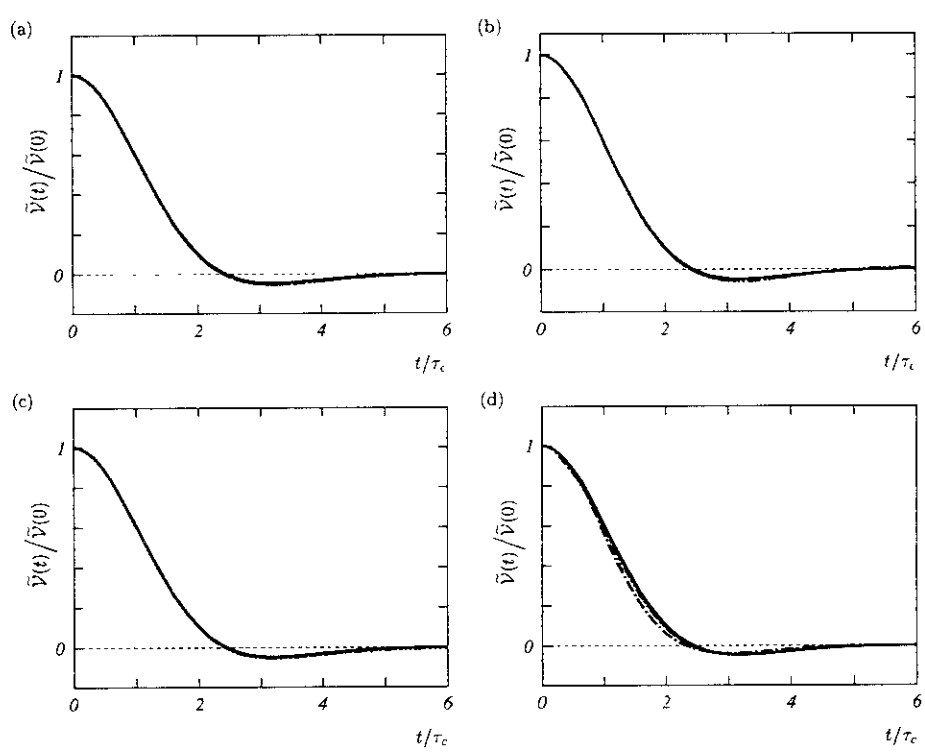

Figure.9 Auto-correlation functions $\widetilde{V}(t)$. In each figure, direct numerical simulation results for $N = 10^3$ (---), 10^4 (----) and 10^5 (-----) are plotted together with the solution (—) to the DIA equation. Figures (a)—(d) correspond to initial pdfs (a)—(d). $\nu = 0$, $\rho \sim 1$.

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Appendix A DIA formulation for a system with mean

Similarly to (20), the response function $G_{in}(t,t')$ is also decomposed as

$$G_{in}(t,t') = G^{(0)}_{in/i_0j_0k_0}(t,t') + G^{(1)}_{in/i_0j_0k_0}(t,t')\,, \tag{A.1}$$

where the two quantities on the right-hand side are respectively governed by

$$\begin{aligned}\frac{\partial}{\partial t} G^{(0)}_{in/i_0j_0k_0}(t,t') =& \sum_j \sum_k 2\,C_{ijk}\, x_j(t)\, G^{(0)}_{kn/i_0j_0k_0}(t,t') - \nu\, G^{(0)}_{in/i_0j_0k_0}(t,t') \\ &+ \sum_{\substack{j \\ \{i,j,k\}\neq\{i_0,j_0,k_0\}}} \sum_k 2\,C_{ijk}\, Y_j(t)\, G^{(0)}_{kn/i_0j_0k_0}(t,t')\end{aligned} \tag{A.2}$$

and

$$\begin{aligned}\frac{\partial}{\partial t} G^{(1)}_{in/i_0j_0k_0}(t,t') =& \sum_j \sum_k 2\,C_{ijk}\, x_j(t)\, G^{(1)}_{kn/i_0j_0k_0}(t,t') - \nu\, G^{(1)}_{in/i_0j_0k_0}(t,t') \\ &+ \sum_{\substack{j \\ \{i,j,k\}\neq\{i_0,j_0,k_0\}}} \sum_k 2\,C_{ijk}\, Y_j(t)\, G^{(1)}_{kn/i_0j_0k_0}(t,t') \\ &+ \delta_{ii_0}\, 2\,C_{i_0j_0k_0} \left\{ Y_{j_0}(t)\, G^{(0)}_{k_0n/i_0j_0k_0}(t,t') + Y_{k_0}(t)\, G^{(0)}_{j_0n/i_0j_0k_0}(t,t') \right\} \\ &+ \delta_{ij_0}\, 2\,C_{j_0k_0i_0} \left\{ Y_{k_0}(t)\, G^{(0)}_{i_0n/i_0j_0k_0}(t,t') + Y_{i_0}(t)\, G^{(0)}_{k_0n/i_0j_0k_0}(t,t') \right\} \\ &+ \delta_{ik_0}\, 2\,C_{k_0i_0j_0} \left\{ Y_{i_0}(t)\, G^{(0)}_{j_0n/i_0j_0k_0}(t,t') + Y_{j_0}(t)\, G^{(0)}_{i_0n/i_0j_0k_0}(t,t') \right\}\end{aligned} \tag{A.3}$$

with $G^{(0)}_{in/i_0j_0k_0}(t,t) = \delta_{in}$ and $G^{(1)}_{in/i_0j_0k_0}(t,t) = 0$. Then, by employing $G^{(0)}_{in}(t,t')$ as Green's function, we can express formal solutions of (22) and (A.3) as

$$Y^{(1)}_{i/i_0j_0k_0}(t|t_0) = \int_{t_0}^{t} \mathrm{d}t' \left\{ 2\,G^{(0)}_{ii_0/i_0j_0k_0}(t,t')\,C_{i_0j_0k_0}\,Y^{(0)}_{j_0/i_0j_0k_0}(t')\,Y^{(0)}_{k_0/i_0j_0k_0}(t') + 2\,G^{(0)}_{ij_0/i_0j_0k_0}(t,t')\,C_{j_0k_0i_0}\,Y^{(0)}_{k_0/i_0j_0k_0}(t')\,Y^{(0)}_{i_0/i_0j_0k_0}(t') + 2\,G^{(0)}_{ik_0/i_0j_0k_0}(t,t')\,C_{k_0i_0j_0}\,Y^{(0)}_{i_0/i_0j_0k_0}(t')\,Y^{(0)}_{j_0/i_0j_0k_0}(t') \right\}. \tag{A.4}$$

and

$$\begin{aligned} G^{(1)}_{in}(t,t') = &\int_{t'}^{t} \mathrm{d}t'' \Big[2\,G^{(0)}_{ii_0/i_0j_0k_0}(t,t'')\,C_{i_0j_0k_0} \left\{ Y_{j_0}(t'')\,G^{(0)}_{k_0n/i_0j_0k_0}(t'',t') + Y_{k_0}(t'')\,G^{(0)}_{j_0n/i_0j_0k_0}(t'',t') \right\} \\ &+2\,G^{(0)}_{ij_0/i_0j_0k_0}(t,t'')\,C_{j_0k_0i_0} \left\{ Y_{k_0}(t'')\,G^{(0)}_{i_0n/i_0j_0k_0}(t'',t') + Y_{i_0}(t'')\,G^{(0)}_{k_0n/i_0j_0k_0}(t'',t') \right\} \\ &+2\,G^{(0)}_{ik_0/i_0j_0k_0}(t,t'')\,C_{k_0i_0j_0} \left\{ Y_{i_0}(t'')\,G^{(0)}_{j_0n/i_0j_0k_0}(t'',t') + Y_{j_0}(t'')\,G^{(0)}_{i_0n/i_0j_0k_0}(t'',t') \right\} \Big]. \end{aligned} \tag{A.5}$$

Decompositions (20) and (A.1) are naive extensions to a system with non-zero mean of that of zero mean $\overline{x_i} = 0$ (see [6]). The resultant DIA equations (25)–(27) are the same between the two cases as shown below.

The first term of the evolution equation (15) for the auto-correlation function is written as

$$\begin{aligned} \text{(First term on rhs of (15))} = &\sum_j\sum_k 2\,C_{ijk}\,x_j(t)\,\overline{Y^{(0)}_{k/ijk}(t)\,Y^{(0)}_{i/ijk}(t')} \\ &+\sum_j\sum_k 2\,C_{ijk}\,x_j(t)\,\overline{Y^{(1)}_{k/ijk}(t)\,Y^{(0)}_{i/ijk}(t')} \\ &+\sum_j\sum_k 2\,C_{ijk}\,x_j(t)\,\overline{Y^{(0)}_{k/ijk}(t)\,Y^{(1)}_{i/ijk}(t')} \\ &+\sum_j\sum_k 2\,C_{ijk}\,x_j(t)\,\overline{Y^{(1)}_{k/ijk}(t)\,Y^{(1)}_{i/ijk}(t')}\,. \end{aligned} \tag{A.6}$$

The first term of (A.6) vanishes because $Y^{(0)}_{i/ijk}$ and $Y^{(0)}_{k/ijk}$ are statistically independent of each other. and because $\overline{Y^{(0)}_{i/ijk}(t)} = 0$. Furthermore, by substituting the formal solution (A.4) of $Y^{(1)}_{i/i_0j_0k_0}(t)$, the other three terms are also shown to be zero. For example, the second term of (A.6) is rewritten as

$$\begin{aligned} &\text{(Second term of (A.6))} \\ &= \sum_j\sum_k 4\,C_{ijk}\,x_j(t) \int_{t_0}^{t} \mathrm{d}t'' \Big\{ C_{ijk}\,\overline{G^{(0)}_{ki/ijk}(t,t'')}\;\overline{Y^{(0)}_{j/ijk}(t'')\,Y^{(0)}_{k/ijk}(t'')\,Y^{(0)}_{i/ijk}(t')} \\ &\qquad +C_{jki}\,\overline{G^{(0)}_{kj/ijk}(t,t'')}\;\overline{Y^{(0)}_{k/ijk}(t'')\,Y^{(0)}_{i/ijk}(t'')\,Y^{(0)}_{i/ijk}(t')} \\ &\qquad +C_{kij}\,\overline{G^{(0)}_{kk/ijk}(t,t'')}\;\overline{Y^{(0)}_{i/ijk}(t'')\,Y^{(0)}_{j/ijk}(t'')\,Y^{(0)}_{i/ijk}(t')} \Big\}. \end{aligned} \tag{A.7}$$

where we have assumed that $G^{(0)}_{in/ijk}$ and $Y^{(0)}_{j/ijk}$ are statistically independent of each other because they have no direct interaction. (This assumption has been checked numerically [6].) Then, all the terms in (A.7) vanish because of independency between $X^{(0)}_{i/ijk}$, $X^{(0)}_{j/ijk}$ and $X^{(0)}_{k/ijk}$. Thus, it is shown that

$$\text{(First term on rhs of (15))} = 0 \tag{A.8}$$

Next, the second term of (15) is written as

$$\begin{aligned}(\text{Second term on rhs of (15)}) &= \sum_j \sum_k 2\, C_{ijk}\, \overline{Y^{(0)}_{j/ijk}(t)\, Y^{(0)}_{k/ijk}(t)\, Y^{(0)}_{i/ijk}(t')} \\ &+ \sum_j \sum_k 4\, C_{ijk}\, \overline{Y^{(1)}_{j/ijk}(t)\, Y^{(0)}_{k/ijk}(t)\, Y^{(0)}_{i/ijk}(t')} \\ &+ \sum_j \sum_k 2\, C_{ijk}\, \overline{Y^{(0)}_{j/ijk}(t)\, Y^{(0)}_{k/ijk}(t)\, Y^{(1)}_{i/ijk}(t')}\,. \end{aligned} \tag{A.9}$$

where higher-order terms have been neglected. The first term vanishes. A straightforward calculation after substitution of the formal solution (A.4) into the second and the third terms leads to the right-hand side of (25). The DIA equation (26) for one-time correlation function is derived from (16) by the same procedures.

The DIA formulation for the response function equation (19) is also straightforward. Since only auto-response functions $G(t,t') = \overline{G_{ii}(t,t')}$ appear in the DIA equations (25) and (26), we put $i = n$ in (19). Then, the first term on right-hand side of (19) is written as

$$(\text{First term on rhs of (19)}) = \sum_j \sum_k 2\, C_{ijk}\, x_j(t) \left\{ \overline{G^{(0)}_{ki/ijk}(t,t')} + \overline{G^{(1)}_{ki/ijk}(t,t')} \right\}. \tag{A.10}$$

It can be shown, by substituting (A.5) in the second term of (A.10), that both terms of (A.10) vanish because $\overline{G^{(0)}_{ki/ijk}(t,t')} = 0$ and $\overline{Y_i(t)} = 0$. Next, the second term on right-hand side of (19) is rewritten by substituting the direct-interaction decompositions (A.1) as

$$(\text{Second term on rhs of (19)}) = \sum_j \sum_k 2\, C_{ijk} \left\{ \overline{Y_j(t)\, G^{(0)}_{ki/ijk}(t,t')} + \overline{Y_j(t)\, G^{(1)}_{ki/ijk}(t,t')} \right\}. \tag{A.11}$$

The first term of (A.11) vanishes, since Y_j and $G^{(0)}_{ki/ijk}$ are statistically independent of each other, and since $\overline{Y_j(t)} = 0$. Using (A.5), the second term of (A.11) is shown to lead to the right-hand side of (27). Thus, we arrive at (27).

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