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A Spectral Method in Spherical Coordinates with Coordinate Singularity at the Origin

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Abstract

A new spectral method in the spherical coordinate system with a coordinate singularity at the origin is proposed. An analytical condition of all spectral modes is satisfied exactly at the origin. Dependent functions are expanded in terms of Chebyshev polynomials of even order in radial direction. Unnecessarily increased resolution near the origin as well as the restriction of severe time step are avoided automatically. Numerical accuracy is confirmed by applying it to a free decay of magnetic field in spherical geometry This method is applicable to quadratic nonlinear problems.

Keywords: spectral method, coordinate singularity, spherical coordinates, pole condition

1 Introduction

In a spherical coordinate system (r, θ, ϕ), special cares should be taken not to degrade numerical accuracy and efficiency which might originate from coordinate singularities along the axis ($\theta = 0, \pi$) and at the origin $r = 0$. Although the coordinate singularity along the axis has been studied extensively so far, there are a very few literatures on that at the origin. The purpose of the present paper is to provide a new spectral method in spherical coordinates including the origin.

The difficulty in spectral methods with a coordinate singularity relates to an analytical property to be satisfied by infinitely differentiable solutions near the singularity. This is called the pole condition. In order to elucidate relations between the coordinate singularity and the pole condition, we review briefly a simple problem examined in Gottlieb & Orszag [1]. It is to find eigen-values and eigen-solutions of Bessel's equation,

$$y'' + \frac{1}{x}y' - \frac{n^2}{x^2}y = \lambda y, \qquad (1)$$

under the conditions that $y(x)$ be finite for $0 \le x \le 1$ and $y(1) = 1$. Here, n is a non-negative integer and a prime denotes differentiation. They showed that satisfactory numerical accuracy is not attained when y is expanded in terms of the Chebyshev polynomials unless a special care is taken to the existence of a coordinate singularity at $x = 0$. They also showed that the accuracy or the convergence rate is improved dramatically by imposing an additional boundary condition at the coordinate singularity such as $y'(0) = 0$ when $n > 1$. This improvement is closely related to a characteristic behavior of analytical solutions near the coordinate singularity. In the above example, eigen-solutions (Bessel functions) $J_n(\sqrt{\lambda}x)$ behave like x^n as $x \to 0$. This is the pole condition that is to be satisfied by analytical solutions of (1). It should be stressed here that the boundary condition $y'(0) = 0$ is only a part of the pole condition (x^n as $x \to 0$, $n \ge 1$), and it is valid for $n > 1$. See [2] for the case of $n = 1$.

Every coordinate singularity has its intrinsic form of the pole condition. In the spherical coordinate system, the coordinate singularity, the pole condition and the appropriate boundary condition along the axis are discussed in [3]. The coordinate singularity appearing in the cylindrical coordinate system in a pipe flow problem was investigated by Orszag and Patera [4]. They derived asymptotic forms, or the pole conditions, of velocity components near the axis ($r = 0$). The axial flow component u, for example, behaves like $r^{|m|}$ as $r \to 0$, where m is the wave number in the azimuthal direction. Velocity u is symmetric or anti-symmetric in the radial direction according as m is even or odd. They expanded dependent variables in radial direction in terms of the even or odd order Chebyshev polynomials for even or odd m. Therefore, the pole condition is satisfied exactly for $m = 0$ and 1 but not for $m \ge 2$ in which only parity is satisfied. In order to achieve high numerical accuracy and to avoid the severe time step restriction near the singularity, the pole condition must be satisfied exactly for all modes [3]. As for the spherical geometry, Bonazzola & Marck [5] presented a spectral method in spherical coordinates to solve gas dynamics equations. Although the pole condition at $\theta = 0$ and π is satisfied exactly in their method, another pole condition at $r = 0$ is only satisfied partially in the same sense as in the axial singularity in the above pipe flow problem.

In this paper, we propose a spectral method in which the pole conditions are satisfied exactly both along the axis and at the origin. A key point is a direct use of asymptotic behavior of the pole condition in spectral expansions. Since the pole condition is such important in the present method, we briefly review it in §2 for the axial singularity both in the spherical and the cylindrical coordinate systems, and in §3 for that at the origin. Numerical algorithm is presented in §4. Validity and accuracy are examined in §5 where a diffusion problem of magnetic field in spherical geometry is solved and the results are compared with analytical solutions. Section 6 is devoted to summary.

2 Pole Condition on the Axis

In the spherical coordinate system, the pole condition of an analytical function $f(r,\theta,\phi)$ near the axis $\theta = 0, \pi$ is given by [3]

$$f_m(r,\theta) = O(\sin^{|m|}\theta) \quad (\theta \to 0, \pi), \tag{2}$$

where $f_m(r,\theta)$ is the Fourier coefficients of $f(r,\theta,\phi)$, i.e.,

$$f(r,\theta,\phi) = \sum_{m=-\infty}^{\infty} f_m(r,\theta)\, e^{im\phi}. \tag{3}$$

In the cylindrical polar coordinates (s,φ), where s is the radius ($0 \le s < \infty$) and φ is the azimuthal angle ($0 \le \varphi < 2\pi$), a similar pole condition is derived near the coordinate singularity $s = 0$ as

$$f_m(s) = O(s^{|m|}) \quad (s \to 0), \tag{4}$$

where

$$f(s,\varphi) = \sum_{m=-\infty}^{\infty} f_m(s)\, e^{im\varphi}. \tag{5}$$

There are two spectral approaches to satisfy the pole conditions exactly. The first one is to choose a set of complete orthogonal functions each of which satisfies the pole condition. An example of such functions for condition (2) in the spherical coordinates is the well-known spherical harmonics $Y_{lm}(\theta,\phi)$ [3]. (Actually, $Y_{lm}(\theta,\phi) \propto P_l^m(\cos\theta) = O(\sin^{|m|}\theta)$ for $\theta \to 0, \pi$, where $P_l^m(\cos\theta)$ are associated Legendre polynomials.) As for the cylindrical coordinate system, a family of polynomial sets which satisfies the pole condition (4) was derived by Matsushima and Marcus [6]. The second approach is to factorize the asymptotic form of the pole conditions and then to expand the residual part in terms of orthogonal functions. (This is the approach that we take in this paper at $r = 0$.) For example, in the case of condition (2), a variable $f(r,\theta,\phi)$ is expressed as $f(r,\theta,\phi) = \sin^{|m|}\theta\, F(r,\theta,\phi)$ and then $F(r,\theta,\phi)$ is expanded into Fourier series both in θ and ϕ directions on the basis of $\cos n\theta\, e^{im\phi}$. The combined expansion function $G_n^m(\theta,\phi) \equiv \sin^{|m|}\theta\, \cos n\theta\, e^{im\phi}$ is called the modified Robert functions [3, 7]. (The original Robert function [8] $\sin^{|m|}\theta\, \cos^n\theta\, e^{im\phi}$ is not suited for the fast Fourier transform.) There is, however, a serious problem in the Robert or the modified Robert expansions that inverse transforms are ill-conditioned since they invoke divisions by very small numbers ($\sin^{|m|}\theta$) for large m. We shall see that the same problem appears in radial expansion of our spectral method at the origin for nonlinear problems. A numerical technique to avoid it will be described in §4.

In addition to the pole condition, another numerical difficulty arises generally in radial expansion of the spherical coordinates. If a normalized radial coordinate r ($0 \le r \le 1$) is rescaled to r' so that dependent functions may be expanded in terms of, for example, Chebyshev polynomials $T_n(r')$, $-1 \le r' \le 1$, then spatial resolution is unnecessarily refined near $r = 0$ ($r' = -1$). In the method of Bonazzola & Marck [5], dependent functions are expanded in terms of Chebyshev polynomials of the first kind ($T_l(\theta) = \cos(l\theta)$) for the modified Robert functions $G_n^m(\theta,\phi)$ of even m, and in terms of those of the second kind ($T_l^\dagger(\theta) = \sin(l\theta)$) for odd m to resolve this too-high-resolution problem near $r = 0$. In our method proposed in this paper, only even-order Chebyshev polynomials of the first kind is employed. (A different method to resolve the resolution problem was proposed by Fornberg [9], in which functions are defined in $-1 \le r \le 1$ and $0 \le \theta < \pi/2$.)

3 Pole condition at the origin

The pole condition at the origin when functions are expanded in terms of the modified Robert functions in the (θ,ϕ) space is presented in [5]. We derive it here when functions are expanded in terms of the spherical harmonics.

Let an analytical function $f(x,y,z)$ be expanded into a Taylor series around the origin as

$$f(x,y,z) = \sum_{n=0}^{\infty} \Lambda_n(x,y,z), \tag{6}$$

where $\Lambda_n(x,y,z)$ is a homogeneous polynomial of degree n. Suppose that $V_n(x,y,z)$ is a spherical solid harmonics, i.e., $\nabla^2 V_n = 0$ and $V_n \propto r^n$, where $r^2 = x^2 + y^2 + z^2$. Then, it is shown from the following property of the spherical solid harmonics,

$$\nabla^2(r^j V_n) = j(j + 2n + 1) r^{j-2} V_n, \tag{7}$$

that $\Lambda_n(x,y,z)$ is expanded as [10]

$$\Lambda_n = \sum_{k=0}^{[n/2]} r^{2k}\, V_{n-2k}. \tag{8}$$

Since $V_n(x,y,z)$ is a spherical solid harmonics of degree n, it can be written as a linear combination of spherical harmonics $Y_{nm}(\theta,\phi)$.

$$V_n(x,y,z) = r^n \sum_{m=-n}^{n} v_{nm}\, Y_{nm}(\theta,\phi), \tag{9}$$

where

$$\int_0^{2\pi} \mathrm{d}\phi \int_0^{\pi} \sin\theta \; \mathrm{d}\theta\, Y_{nm}(\theta,\phi)\, Y^*_{n'm'}(\theta,\phi) = \delta_{nn'}\,\delta_{mm'}, \tag{10}$$

and Y^*_{nm} denotes the complex conjugate of Y_{nm}. A combination of (6), (8) and (9) leads to

$$f(x,y,z) = \sum_{n=0}^{\infty} r^n \sum_{l=0}^{n} \sum_{m=-l}^{l} f^m_{n,l}\, Y_{lm}(\theta,\phi), \tag{11}$$

where, from (8),

$$f^m_{n,l} = 0 \quad \text{for} \quad n+l = \text{odd}. \tag{12}$$

Exchanging the order of summations with respect to l and n in (11), we get

$$\begin{aligned} f(x,y,z) & \\ &= \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left(\sum_{n=l}^{\infty} f^m_{n,l}\, r^n \right) Y_{lm}(\theta,\phi) \\ &= \sum_{l=0}^{\infty} \sum_{m=-l}^{l} r^l \left(\sum_{j=0}^{\infty} f^m_{l+j,l}\, r^j \right) Y_{lm}(\theta,\phi) \\ &= \sum_{l=0}^{\infty} \sum_{m=-l}^{l} r^l\, F_{lm}(r)\, Y_{lm}(\theta,\phi). \end{aligned} \tag{13}$$

Here, a function

$$F_{lm}(r) = \sum_{j=0}^{\infty} f^m_{l+j,l}\, r^j \tag{14}$$

has been introduced, which is even in r (because of (12)), i.e.,

$$F_{lm}(r) = f^m_{l,l} + f^m_{l+2,l}\, r^2 + f^m_{l+4,l}\, r^4 + \cdots, \tag{15}$$

and finite at $r = 0$. This is the pole condition in the present expansion.

Another proof of the even parity of $F_{lm}(r)$ is given by writing $f(x,y,z)$ in the spherical coordinate system as

$$f(x,y,z) = f(r\sin\theta\cos\phi,\ r\sin\theta\sin\phi,\ r\cos\theta). \tag{16}$$

Suppose the identical transformation,

$$(r,\theta,\phi) \to (r',\theta',\phi') = (-r, \pi-\theta, \pi+\phi), \tag{17}$$

under which the function $f(x,y,z)$ should be invariant, i.e.,

$$\begin{aligned} f \quad \to \quad & f' \\ &= f(r'\sin\theta'\cos\phi',\ r'\sin\theta'\sin\phi',\ r'\cos\theta') \\ &= f(-r\sin(\pi-\theta)\cos(\pi+\phi), \\ &\quad -r\sin(\pi-\theta)\sin(\pi+\phi), \\ &\quad -r\cos(\pi-\theta)) \\ &= f(r\sin\theta\cos\phi,\ r\sin\theta\sin\phi,\ r\cos\theta) \\ &= f, \end{aligned} \tag{18}$$

while the right-hand side of (13) is transformed as

$$\begin{aligned} r^l\, F_{lm}(r)\, Y_{lm}(\theta,\phi) \quad \to \quad & (r')^l\, F_{lm}(r')\, Y_{lm}(\theta',\phi') \\ &= (-1)^l\, r^l F_{lm}(-r) \\ &\quad \times Y_{lm}(\pi-\theta, \pi+\phi) \\ &= r^l\, F_{lm}(-r)\, Y_{lm}(\theta,\phi). \end{aligned} \tag{19}$$

Therefore, $F_{lm}(r) = F_{lm}(-r)$ must be satisfied.

To summarize, any analytical function $f(r,\theta,\phi)$ is expanded around the origin as

$$f(r,\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} r^l F_{lm}(r) Y_{lm}(\theta,\phi), \tag{20}$$

where $F_{lm}(r)$ is an even function of r,

$$F_{lm}(-r) = F_{lm}(r), \tag{21}$$

and

$$|\, F_{lm}(0)\, | < \infty. \tag{22}$$

Equations (20)–(22) are the pole condition at the origin in the spherical coordinate system when it is expanded in terms of the spherical harmonics in the (θ,ϕ) space. The pole condition for the modified Robert expansion is found in [5].

4 Spectral Method without Singularity Problem

Our spectral method is constructed on the basis of the pole condition described in the preceding section. Suppose that a function $f(r,\theta,\phi)$ is governed by a differential equation defined in a sphere $0 \le r \le 1$, $0 \le \theta \le \pi$, and $0 \le \phi < 2\pi$. We expand $f(r,\theta,\phi)$ as (20) and the coefficient $F_{lm}(r)$ in terms of Chebyshev polynomials of even order as,

$$F_{lm}(r) = \sum_{n=0}^{N} F_{lmn}\, T_{2n}(r), \tag{23}$$

which is justified by the property of (21) and $T_{2n}(-r) = T_{2n}(r)$. Here, N is the truncation mode number. Note that the problem of unnecessarily refined resolution near the origin has been avoided automatically in this expansion.

By choosing such radial node points as

$$r_j = \cos\left(\frac{j\pi}{2N}\right), \quad j = 0, 1, 2, \ldots, N, \tag{24}$$

we can invoke the fast Fourier cosine transformation to calculate the summation in (23) as

$$\begin{aligned} F_{lm}(r_j) &= \sum_{n=0}^{N} F_{lmn} T_{2n}(r_j) \\ &= \sum_{n=0}^{N} F_{lmn} \cos(2n \cos^{-1} r_j) \\ &= \sum_{n=0}^{N} F_{lmn} \cos(nj\pi/N). \end{aligned} \tag{25}$$

When the problem to be solved is quadratically nonlinear, a care should be taken in dealing with the nonlinear terms. Suppose a nonlinear term of

$$h(r,\theta,\phi) = f(r,\theta,\phi)\ g(r,\theta,\phi). \tag{26}$$

According to our algorithm, we expand f, g, and h as

$$\begin{aligned} f(r,\theta,\phi) &= \sum_{l=0}^{\infty} \sum_{m=-l}^{l} r^l\ F_{lm}(r)\ Y_{lm}(\theta,\phi) \\ &= \sum_{l=0}^{\infty} r^l\ \widetilde{F}_l(r,\theta,\phi), \end{aligned} \tag{27}$$

$$\begin{aligned} g(r,\theta,\phi) &= \sum_{l=0}^{\infty} \sum_{m=-l}^{l} r^l\ G_{lm}(r)\ Y_{lm}(\theta,\phi) \\ &= \sum_{l=0}^{\infty} r^l\ \widetilde{G}_l(r,\theta,\phi), \end{aligned} \tag{28}$$

$$\begin{aligned} h(r,\theta,\phi) &= \sum_{l=0}^{\infty} \sum_{m=-l}^{l} r^l\ H_{lm}(r)\ Y_{lm}(\theta,\phi) \\ &= \sum_{l=0}^{\infty} r^l\ \widetilde{H}_l(r,\theta,\phi), \end{aligned} \tag{29}$$

where

$$\widetilde{F}_l(r,\theta,\phi) = \sum_{m=-l}^{l} F_{lm}(r)\ Y_{lm}(\theta,\phi), \tag{30}$$

$$\widetilde{G}_l(r,\theta,\phi) = \sum_{m=-l}^{l} G_{lm}(r)\ Y_{lm}(\theta,\phi), \tag{31}$$

$$\widetilde{H}_l(r,\theta,\phi) = \sum_{m=-l}^{l} H_{lm}(r)\ Y_{lm}(\theta,\phi). \tag{32}$$

Then, what is required in our method is to calculate $H_{lm}(r)$ from $F_{lm}(r)$ and $G_{lm}(r)$. Notice that numerical errors would be amplified if a spherical expansion of $h(r,\theta,\phi)$ were divided by r^l to obtain $H_{lm}(r)$, since r^l could be too small when l is large and r is small. (A similar problem appears near $\theta = 0$ and π in the modified Robert expansion [3, 7].) This problem may be resolved if $H_{lm}(r)$ is calculated from $\widetilde{F}_l(r,\theta,\phi)$ and $\widetilde{G}_l(r,\theta,\phi)$ [11] as follows.

By substituting (27) and (28) in (26) and rearranging the terms, we get

$$h(r,\theta,\phi) = \sum_{i=0}^{\infty} r^i\ \overline{H}_i(r,\theta,\phi), \tag{33}$$

where $\overline{H}_i(r,\theta,\phi)$ are even functions of r, defined by

$$\overline{H}_i(r,\theta,\phi) = \sum_{j=0}^{i} \widetilde{F}_j(r,\theta,\phi)\, \widetilde{G}_{i-j}(r,\theta,\phi). \tag{34}$$

It should be noticed that although the expansions in (29) and (33) are same in appearance, $\widetilde{H}_l(r,\theta,\phi)$ is different from $\overline{H}_l(r,\theta,\phi)$ in general.

Now, we expand $\overline{H}_i(r,\theta,\phi)$ in terms of the spherical harmonics as

$$\overline{H}_i(r,\theta,\phi) = \sum_{l=0}^{i} \sum_{m=-l}^{l} \overline{H}_{i,lm}(r)\, Y_{lm}(\theta,\phi), \tag{35}$$

where

$$\overline{H}_{i,lm}(r) = \int_0^{2\pi} \mathrm{d}\phi \int_0^{\pi} \sin\theta\, \mathrm{d}\theta\ \overline{H}_i(r,\theta,\phi)\, Y^*_{lm}(\theta,\phi). \tag{36}$$

Since a product of two spherical solid harmonics of degree j and $i-j$ is written by a linear combination of spherical solid harmonics of degree l with $l \le i$, the summation for l in (35) is taken up to i. Therefore, it follows from (33) and (35) that

$$\begin{aligned} & h(r,\theta,\phi) \\ &= \sum_{i=0}^{\infty} r^i \sum_{l=0}^{i} \sum_{m=-l}^{l} \overline{H}_{i,lm}(r) Y_{lm}(\theta,\phi) \\ &= \sum_{l=0}^{\infty} r^l \sum_{m=-l}^{l} \sum_{i=l}^{\infty} r^{i-l}\, \overline{H}_{i,lm}(r) Y_{lm}(\theta,\phi). \end{aligned} \tag{37}$$

Comparing (29) and (37), we obtain

$$H_{lm}(r) = \sum_{i=0}^{\infty} r^i\, \overline{H}_{i+l,lm}(r). \tag{38}$$

Thus, we can calculate $H_{lm}(r)$ from $F_{lm}(r)$ and $G_{lm}(r)$ as follows. First calculate $\widetilde{F}_l(r,\theta,\phi)$ and $\widetilde{G}_l(r,\theta,\phi)$ from $F_{lm}(r)$ and $G_{lm}(r)$ by (30) and (31). Then, we get $\overline{H}_i(r,\theta,\phi)$ by the convolution sum (34), $\overline{H}_{i,lm}(r)$ by (36), and finally $H_{lm}(r)$ by (38).

The convolution sum (34) would be inefficient when it were performed in its straightforward form. Here we provide a fast algorithm by the use of the fast Fourier transform to resolve this problem. Suppose $\{f_j\}$ and $\{g_j\}$, $(j = 0, 1, 2, \cdots, L)$, and their convolution sum,

$$h_l = \sum_{j=0}^{l} f_j\, g_{l-j}, \qquad l = 0, 1, 2, \cdots, L. \tag{39}$$

Note that h_0 is simply given by $f_0\, g_0$. Let $\{\widetilde{f}_k\}$ and $\{\widetilde{g}_k\}$ be Fourier transforms of $\{f_j\}$ and $\{g_j\}$ as

$$\widetilde{f}_k = \sum_{j=0}^{2L-1} f_j\, e^{i\pi jk/L}, \tag{40}$$

$$\widetilde{g}_k = \sum_{j=0}^{2L-1} g_j\, e^{i\pi jk/L}, \tag{41}$$

where $f_j = g_j = 0$ for $j = L+1, L+2, \cdots, 2L-1$. Then, h_l $(1 \le l \le L)$ is directly given by the inverse Fourier transform,

$$h_l = \frac{1}{2L} \sum_{k=0}^{2L-1} \widetilde{h}_k\, e^{-i\pi kl/L} \tag{42}$$

of the product $\widetilde{h}_k \equiv \widetilde{f}_k \widetilde{g}_k$, since

$$\begin{aligned} & \frac{1}{2L} \sum_{k=0}^{2L-1} \widetilde{h}_k\, e^{-i\pi lk/L} \\ = \; & \frac{1}{2L} \sum_{k=0}^{2L-1} \sum_{j=0}^{2L-1} \sum_{m=0}^{2L-1} f_j\, g_m\, e^{i\pi(j+m-l)k/L} \\ = \; & \sum_{j=0}^{2L-1} \sum_{m=0}^{2L-1} f_j\, g_m \delta_{j+m,l} \\ = \; & \sum_{j=0}^{L} f_j\, g_{l-j}, \end{aligned} \tag{43}$$

where use has been made of $|j + m - l| < 2L$ for non-zero terms on the right-hand side of the equality of (43). Therefore, our spectral method is amenable to the fast Fourier transforms in quadratic nonlinear problems with keeping the pole condition.

5 Free Decay of Magnetic Field

5.1 Numerical Scheme

In this section, our spectral method is applied to a free decay of magnetic field in a sphere to check its validity and accuracy. Consider an electrically conducting solid sphere of radius a with a finite electrical resistivity. A magnetic field is given at an initial time $t = 0$ with an arbitrary distribution. Since the type of boundary condition is not important in the present algorithm, the outer region of the sphere $(r > a)$ is supposed to be a perfect insulator, or a vacuum for simplicity. It is physically evident that the magnetic field decays with time due to the finite resistivity. An analytical expression of decaying magnetic field is provided in [12].

We take the spherical coordinate system with its origin $r = 0$ at the center of the sphere, where r is the radial coordinate normalized by the sphere radius $(0 \le r \le 1)$. Magnetic field $\mathbf{b}$ is governed by the diffusion equation,

$$\frac{\partial \mathbf{b}}{\partial t} = \frac{\eta}{a^2} \nabla^2 \mathbf{b}, \tag{44}$$

where η denotes electrical resistivity.

Since $\nabla \cdot \mathbf{b} = 0$, the magnetic field may be written as [12]

$$\mathbf{b} = \nabla \times \nabla \times (B\mathbf{r}) + \nabla \times (J\mathbf{r}), \tag{45}$$

where J and B are the toroidal and poloidal potential functions respectively, and $\mathbf{r} = r\hat{\mathbf{r}}$, $\hat{\mathbf{r}}$ being a unit vector in radial direction. Just as (20), we expand the potentials as

$$B(\mathbf{r}, t) = \sum_{l=1}^{L} \sum_{m=-l}^{l} r^l\, \widetilde{B}_{lm}^t(r)\, Y_{lm}(\theta, \phi), \tag{46}$$

$$J(\mathbf{r}, t) = \sum_{l=1}^{L} \sum_{m=-l}^{l} r^l\, \widetilde{J}_{lm}^t(r)\, Y_{lm}(\theta, \phi). \tag{47}$$

Substituting (45)–(47) in (44), we obtain

$$\frac{\partial}{\partial t} \widetilde{B}_{lm}^t(r) = \frac{\eta}{a^2} D_l \widetilde{B}_{lm}^t(r), \tag{48}$$

$$\frac{\partial}{\partial t} \widetilde{J}_{lm}^t(r) = \frac{\eta}{a^2} D_l \widetilde{J}_{lm}^t(r), \tag{49}$$

where

$$D_l = \frac{1}{r^2} \left\{ r\frac{\partial}{\partial r} \left(r\frac{\partial}{\partial r} \right) + kr\frac{\partial}{\partial r} \right\} \tag{50}$$

with

$$k \equiv 2l + 1. \tag{51}$$

The magnetic field inside the sphere is connected continuously with the outer vacuum field on the surface $(r = 1)$ [12] as

$$\frac{\mathrm{d}\widetilde{B}_{lm}^t}{\mathrm{d}r}(1) + k\widetilde{B}_{lm}^t(1) = 0, \tag{52}$$

$$\widetilde{J}_{lm}^t(1) = 0. \tag{53}$$

Equations (48)–(53) constitute the fundamental equations to be solved in our spectral method.

The time derivatives in (48) and (49) are discretized by the Crank-Nicolson scheme as

$$\begin{aligned} & \left\{ r\frac{\mathrm{d}}{\mathrm{d}r} \left(r\frac{\mathrm{d}}{\mathrm{d}r} \right) + k\,r\frac{\mathrm{d}}{\mathrm{d}r} - c\,r^2 \right\} \widetilde{B}_{lm}^{t+\Delta t}(r) \\ & = - \left\{ r\frac{\mathrm{d}}{\mathrm{d}r} \left(r\frac{\mathrm{d}}{\mathrm{d}r} \right) + k\,r\frac{\mathrm{d}}{\mathrm{d}r} + c\,r^2 \right\} \widetilde{B}_{lm}^t(r), \end{aligned} \tag{54}$$

$$\begin{aligned} & \left\{ r\frac{\mathrm{d}}{\mathrm{d}r} \left(r\frac{\mathrm{d}}{\mathrm{d}r} \right) + k\,r\frac{\mathrm{d}}{\mathrm{d}r} - c\,r^2 \right\} \widetilde{J}_{lm}^{t+\Delta t}(r) \\ & = - \left\{ r\frac{\mathrm{d}}{\mathrm{d}r} \left(r\frac{\mathrm{d}}{\mathrm{d}r} \right) + k\,r\frac{\mathrm{d}}{\mathrm{d}r} + c\,r^2 \right\} \widetilde{J}_{lm}^t(r), \end{aligned} \tag{55}$$

where $c = 2a^2/\eta\Delta t$. Functions $\widetilde{B}^t_{lm}(r)$ and $\widetilde{J}^t_{lm}(r)$ are expanded in terms of Chebyshev polynomials of even order as

$$\widetilde{B}^t_{lm}(r) = \sum_{i=0}^{N} \widetilde{B}^t_{lmi} T_{2i}(r), \quad (56)$$

$$\widetilde{J}^t_{lm}(r) = \sum_{i=0}^{N} \widetilde{J}^t_{lmi} T_{2i}(r), \quad (57)$$

where N is the truncation mode number. Similar expansions are performed for $\widetilde{B}^{t+\Delta t}_{lm}(r)$ and $\widetilde{J}^{t+\Delta t}_{lm}(r)$. Equations (54) and (55) are then written in matrix form as

$$L_{ij}\widetilde{B}^{t+\Delta t}_{lmj} = R_{ij}\widetilde{B}^t_{lmj} \quad (0 \le i \le N,\ 1 \le l \le L,\ -l \le m \le l), \quad (58)$$

$$L_{ij}\widetilde{J}^{t+\Delta t}_{lmj} = R_{ij}\widetilde{J}^t_{lmj} \quad (0 \le i \le N,\ 1 \le l \le L,\ -l \le m \le l). \quad (59)$$

Here, summation is implicit for repeated subscripts. The $(N+1, N+1)$ matrices $\underline{\mathbf{L}}$ and $\underline{\mathbf{R}}$ are defined by

$$\underline{\mathbf{L}} = \underline{\mathbf{X}}^2 + k\underline{\mathbf{X}} - c\underline{\mathbf{Y}}, \quad (60)$$

$$\underline{\mathbf{R}} = -\left(\underline{\mathbf{X}}^2 + k\underline{\mathbf{X}} + c\underline{\mathbf{Y}}\right), \quad (61)$$

with $\underline{\mathbf{X}}$ and $\underline{\mathbf{Y}}$ are operator matrices for $r\mathrm{d}/\mathrm{d}r$ and r^2 in this Chebyshev expansion of even order. The derivation and explicit forms of $\underline{\mathbf{X}}$ and $\underline{\mathbf{Y}}$ are described in Appendix A.

Since $T_n(1) = 1$ and $T_n'(1) = n^2$, where a prime denotes differentiation, the boundary conditions (52) and (53) are respectively written as

$$\sum_{j=0}^{N} \left\{(2j)^2 + k\right\} \widetilde{B}^{t+\Delta t}_{lmj} = 0, \quad (62)$$

and

$$\sum_{j=0}^{N} \widetilde{J}^{t+\Delta t}_{lmj} = 0 \quad (63)$$

at time $t + \Delta t$. Replacing the last (Nth) rows of (58) and (59) by (62) and (63) respectively, we obtain

$$L^B_{ij}\widetilde{B}^{t+\Delta t}_{lmj} = R^\dagger_{ij}\widetilde{B}^t_{lmj}, \quad (0 \le i \le N,\ 0 \le l \le L,\ -l \le m \le l) \quad (64)$$

and

$$L^J_{ij}\widetilde{J}^{t+\Delta t}_{lmj} = R^\dagger_{ij}\widetilde{J}^t_{lmj}, \quad (0 \le i \le N,\ 0 \le l \le L,\ -l \le m \le l) \quad (65)$$

where

$$L^B_{ij} = \begin{cases} L_{ij} & (0 \le i \le N-1,\ 0 \le j \le N), \\ (2j)^2 + k & (i = N,\ 0 \le j \le N), \end{cases} \quad (66)$$

$$L^J_{ij} = \begin{cases} L_{ij} & (0 \le i \le N-1,\ 0 \le j \le N), \\ 1 & (i = N,\ 0 \le j \le N), \end{cases} \quad (67)$$

$$R^\dagger_{ij} = \begin{cases} R_{ij} & (0 \le i \le N-1,\ 0 \le j \le N), \\ 0 & (i = N,\ 0 \le j \le N). \end{cases} \quad (68)$$

This is called the tau method [13]. By preparing the following matrices in advance,

$$\underline{\mathbf{Z}}^B = (\underline{\mathbf{L}}^B)^{-1}\underline{\mathbf{R}}^\dagger, \quad (69)$$

$$\underline{\mathbf{Z}}^J = (\underline{\mathbf{L}}^J)^{-1}\underline{\mathbf{R}}^\dagger, \quad (70)$$

we can renew the potential functions by direct matrix operations,

$$\widetilde{B}^{t+\Delta t}_{lmi} = Z^B_{ij}\widetilde{B}^t_{lmj} \quad (0 \le i \le N,\ 1 \le l \le L,\ -l \le m \le l), \quad (71)$$

$$\widetilde{J}^{t+\Delta t}_{lmi} = Z^J_{ij}\widetilde{J}^t_{lmj} \quad (0 \le i \le N,\ 1 \le l \le L,\ -l \le m \le l). \quad (72)$$

These are the final form for numerical integration of free decay evolution of the magnetic field in a sphere. See Appendix B for procedures to obtain the magnetic field $\mathbf{b}$ from the potential functions $\widetilde{B}_{lmi}$ and $\widetilde{J}_{lmi}$.

5.2 Comparison of Numerical and Analytical Results

In order to check the numerical accuracy of the present spectral method, we compare the minimum damping rates (or the maximum growth rates) for each mode l with analytical solutions. (The damping rates does not depend on m; see (48)–(51).) The minimum damping rates are given by squared roots of the spherical Bessel functions [12]. In our numerical calculations, they are obtained by time integrations till the growth rates have converged. Several Chebyshev modes are excited for each l and m at the initial instant.

The minimum damping rates of the poloidal and toroidal potential functions are shown in tables 1 and 2, respectively. Numerical parameters are set at $c = 2a^2/\eta\Delta t = 2 \times 10^5$, and $N = 2^4$. The agreement between the numerical and the analytical results are excellent, which means that our spectral method is applicable to formulation in the spherical coordinate system with a coordinate singularity at the origin.

6 Summary

We have presented a new spectral method that is applicable to formulations in the spherical coordinate system including the origin. The pole condition at the origin which is the analyticity condition of solutions is satisfied exactly. The problem of unnecessarily enhanced spatial resolution is avoided automatically by employing Chebyshev polynomials of even order for the radial expansion. High-accuracy of this method has been confirmed by applying it to a free decay of magnetic field in a spherical geometry. This method is applicable to quadratic nonlinear problems. We are developing a fluid simulation code in a spherical geometry using the present technique and it will be reported in a forthcoming paper.

Appendix A: Derivation of Matrices $\underline{\mathbf{X}}$ and $\underline{\mathbf{Y}}$

Let even functions $u(r)$, $v(r) \equiv ru'(r)$, and $y(r) \equiv r^2u(r)$ be expanded in terms of Chebyshev polynomials of even order as

$$u(r) = \sum_{i=0}^{N} u_i T_{2i}(r), \tag{73}$$

$$v(r) = \sum_{i=0}^{N} v_i T_{2i}(r), \tag{74}$$

and

$$y(r) = \sum_{i=0}^{N} y_i T_{2i}(r). \tag{75}$$

Let $\underline{\mathbf{X}}$ and $\underline{\mathbf{Y}}$ be $(N+1, N+1)$ operator matrices for $r\,\mathrm{d}/\mathrm{d}r$ and r^2, i.e.,

$$v_i = X_{ij} u_j \qquad (0 \le i \le N), \tag{76}$$

and

$$y_i = Y_{ij} u_j \qquad (0 \le i \le N). \tag{77}$$

Then, it is straightforward from the differentiation formula of Chebyshev polynomials,

$$rT'_{2n}(r) = 2n(T_{2n} + 2T_{2n-2} + 2T_{2n-4} + \cdots + 2T_2 + T_0), \tag{78}$$

to obtain

$$\underline{\mathbf{X}} = \begin{bmatrix} 0 & 2 & 4 & 6 & 8 & \cdots & 2N \\ & 2 & 8 & 12 & 16 & \cdots & 4N \\ & & 4 & 12 & 16 & \cdots & 4N \\ & & & 6 & 16 & \cdots & 4N \\ & & & & 8 & \cdots & \vdots \\ & & & & & \vdots & 4N \\ & \mathbf{0} & & & & & 4N \\ & & & & & & 2N \end{bmatrix}, \tag{79}$$

and from the multiplication formula,

$$r^2 T_0 = \frac{1}{2}T_0 + \frac{1}{2}T_2, \tag{80}$$

$$r^2 T_n = \frac{1}{4}T_{n-2} + \frac{1}{2}T_n + \frac{1}{4}T_{n+2} \quad (n \ge 2), \tag{81}$$

to obtain

$$\underline{\mathbf{Y}} = \frac{1}{4}\begin{bmatrix} 2 & 1 & & & & & \\ 2 & 2 & 1 & & & \mathbf{0} & \\ & 1 & 2 & 1 & & & \\ & & 1 & 2 & \ddots & & \\ & & & 1 & \ddots & 1 & \\ & \mathbf{0} & & & \ddots & 2 & 1 \\ & & & & & 1 & 2 \end{bmatrix}. \tag{82}$$

Appendix B: Calculation of Magnetic Field from Potential Functions

The Chebyshev series (56) and (57) of potential functions are summed up by the fast Fourier cosine transform (25). It follows from (45)-(47) that

$$b_r(r,\theta,\phi;t) = \sum_{l=1}^{L}\sum_{m=-l}^{l} l(l+1)r^{l-1}\,\widetilde{B}^t_{lm}(r)\,Y_{lm}(\theta,\phi), \tag{83}$$

$$\sin\theta\; b_\theta(r,\theta,\phi;t) = \sum_{l=1}^{L}\sum_{m=-l}^{l}\Big[r^{l-1}\Big\{(l+1) + r\frac{\partial}{\partial r}\Big\}\widetilde{B}^t_{lm}(r) \times \sin\theta\frac{\partial}{\partial\theta}Y_{lm}(\theta,\phi) + imr^l\widetilde{J}^t_{lm}(r)Y_{lm}(\theta,\phi)\Big] \tag{84}$$

$$\sin\theta\; b_\phi(r,\theta,\phi;t) = \sum_{l=1}^{L}\sum_{m=-l}^{l}\Big[im\,r^{l-1}\Big\{(l+1) + r\frac{\partial}{\partial r}\Big\}\widetilde{B}^t_{lm}(r)\;Y_{lm}(\theta,\phi) - r^l\,\widetilde{J}^t_{lm}(r)\,\sin\theta\,\frac{\partial}{\partial\theta}Y_{lm}(\theta,\phi)\Big]. \tag{85}$$

Since

$$\sin\theta\frac{\partial}{\partial\theta}Y_{lm} = l\sqrt{\frac{(l-m+1)(l+m+1)}{(2l+1)(2l+3)}}Y_{l+1,m} - (l+1)\sqrt{\frac{(l+m)(l-m)}{(2l+1)(2l-1)}}Y_{l-1,m} \tag{86}$$

the right-hand sides of (84) and (85) are written by a linear combination of $\{Y_{lm}\}$. The term $\{(l+1) + r\partial/\partial r\}\,\widetilde{B}^t_{lm}(r)$ in (84) and (85) is calculated by the operator matrix $\underline{\mathbf{X}}$ derived in Appendix A.

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Table 1: Minimum damping rates for poloidal modes $\tilde{B}_{lm}$.

mode l	numerical	analytical	relative error
1	-9.869604405E+00	-9.869604401E+00	4E-10
2	-2.019072862E+01	-2.019072856E+01	3E-09
3	-3.321746222E+01	-3.321746191E+01	9E-09
4	-4.883119461E+01	-4.883119364E+01	2E-08
5	-6.695431443E+01	-6.695431193E+01	4E-08
6	-8.753122584E+01	-8.753122026E+01	6E-08
7	-1.105197196E+02	-1.105197083E+02	1E-07
8	-1.358864204E+02	-1.358863995E+02	2E-07
9	-1.636041333E+02	-1.636040968E+02	2E-07
10	-1.936501795E+02	-1.936501189E+02	3E-07

Table 2: Minimum damping rates for toroidal modes $\tilde{J}_{lm}$.

mode l	numerical	analytical	relative error
1	-2.019072863E+01	-2.019072856E+01	3E-09
2	-3.321746222E+01	-3.321746191E+01	9E-09
3	-4.883119462E+01	-4.883119364E+01	2E-08
4	-6.695431443E+01	-6.695431193E+01	4E-08
5	-8.753122585E+01	-8.753122026E+01	6E-08
6	-1.105197196E+02	-1.105197083E+02	1E-07
7	-1.358864205E+02	-1.358863995E+02	2E-07
8	-1.636041333E+02	-1.636040968E+02	2E-07
9	-1.936501794E+02	-1.936501189E+02	3E-07
10	-2.260052955E+02	-2.260051993E+02	4E-07

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