

NATIONAL INSTITUTE FOR FUSION SCIENCE

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(Received - Aug. 10, 2000)

NIFS-642

Sep. 2000

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RESEARCH REPORT
NIFS Series

TOKI, JAPAN

Revisit to the helicity and the generalized self-organization theory

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It is clarified that the so-called “helicity conservation law” is never “the conservation equation of the helicity K itself”, but is merely “the time change rate equation of K ”, which is passively and resultantly determined by the mutually independent volume and surface integral terms. It is shown that since the total helicity K can never be conserved in the real experimental systems, the conjecture of the total helicity invariance is not physically available to real magnetized plasmas in an exact sense. The well-known relaxation theory by Dr. J. B. Taylor is clarified to be neither the variational principle nor the energy principle, but be merely a mathematical calculation, using the variational calculus in order to find the minimum magnetic energy solution from the set of solutions having the same value of K . With the use of auto-correlations for physical quantities, it is presented that a novel basic formulation of an extended generalized self-organization theory, which is not based on neither the variational principle nor the energy principle. It is clarified that conservation equations concerning with all physical quantities for the dynamic system of interest are naturally embedded in the formulation of the generalized self-organization theory. The self-organized states of every physical quantities of interest may be realized during their own phases and the dynamical system may evolve repeatedly those out of phase organizations, depending on boundary conditions and input powers. It is shown that the conservation laws can be used to extend conventional methods of plasma current drives by energy injections with use of various types of energies, such as magnetic energies, electromagnetic wave energies, internal energies of plasmoids by plasma guns, which induce the thermal plasma flow velocity, various particle beam energies, and so on.

Keywords : magnetic helicity, equation of change rate, conservation law, variational calculus, self-organization, auto-correlations, minimum change rate auto-correlation

I. INTRODUCTION

Since Dr. J. B. Taylor published his famous theory [1, 2] to explain the appearance of the reversed field pinch (RFP) configuration [3], the magnetic helicity K has been believed to have important role as a global invariant in the self-organization process or the relaxation one of magnetized plasmas [4–12]. On the other hand, one of the authors (Y.K.) has been proposed the partially relaxed state model (PRSM) of the RFP in order to explain experimental data [13, 14], and published some theoretical works to deduce the PRSM by taking account of partial losses of the magnetic flux and the helicity K [15–19]. Without using the concept of the magnetic helicity, an energy integral was derived to deduce the PRSM and the mode transition point of the self-organized state in order to explain experimental data on the RFP [17–19].

In this paper, keeping the authors’ sincere respect for the great scientist Dr. J. B. Taylor, we study again the meaning of the magnetic helicity itself from the thought analytical stand point of view [20–23], because of many evidences showing no invariance of the total helicity in simulations [24, 25] and various experiments [26–36]. In

Section II, the so-called “helicity conservation law” is analyzed to show how it works. In Section III, a novel basic formulation of the generalized self-organization theory is developed that is applicable to various nonlinear dynamic systems and deduces the Taylor state [37–44]. A typical application of the generalized self-organization theory to fusion plasmas is also presented in Section III to lead to some proposal for plasma current drive by using various types of energy injections.

II. THEORETICAL THOUGHT ANALYSIS

We show here that although the energy conservation law is always physically correct, the so-called “helicity conservation law”, which has been believed useful by many scientists in the fusion plasma physics [1, 2, 5–8, 10, 11], is never “the conservation equation of the helicity K itself” but merely “the time change rate equation of K ”, as will be shown below from the thought analytical stand point of view [20–23].

Both of the energy conservation law and the so-called helicity conservation one are derived from the following

axiom set of physical laws of Maxwell's equations written in the MKSA unit used in the usual text books:

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad (1)$$

$$\frac{\partial \mathbf{D}}{\partial t} = -\mathbf{j} + \nabla \times \mathbf{H}, \quad (2)$$

$$\nabla \cdot \mathbf{D} = \rho, \quad (3)$$

$$\nabla \cdot \mathbf{B} = 0. \quad (4)$$

We can get Poynting's energy conservation law by the following processes.

1) After applying the scalar product of \mathbf{H} on both sides of Eq.(1), we integrate both sides of the resultant equation over the volume V .

2) After applying the scalar product of \mathbf{E} on both sides of Eq.(2), we integrate both sides of the resultant equation over the volume V .

3) Adding both sides of the two equations by 1) and 2), using vector formulae and Gauss theorem, we obtain the following familiar Poynting's energy conservation law Eq.(6) concerning with the field energy W_f which is written by

$$W_f = \int_V \left(\frac{\mathbf{E} \cdot \mathbf{D}}{2} + \frac{\mathbf{H} \cdot \mathbf{B}}{2} \right) dV, \quad (5)$$

$$\frac{\partial W_f}{\partial t} = - \int_V \mathbf{j} \cdot \mathbf{E} dV - \frac{1}{\mu_0} \oint_S (\mathbf{E} \times \mathbf{B}) \cdot d\mathbf{S}. \quad (6)$$

It should be emphasized here that using the two dynamic equations of (1) and (2), we can determine the full time evolution of \mathbf{E} and \mathbf{B} , and that Eqs.(3) and (4) are known to play the role of initial conditions of \mathbf{E} and \mathbf{B} , as is shown in usual textbooks. Furthermore, we should notice that there exist complete mathematical connections between Poynting's energy conservation law Eq.(6) and the two dynamic equations of (1) and (2) which give the full time evolution of \mathbf{E} and \mathbf{B} . Because of these physical and mathematical background, the total energy dealt in Eq.(6) are conserved in the dynamic system by changing the types of energy such as the field energy, the kinetic energy of charged particles, the thermal energy of particles, the radiation energy, and so on.

The two physical laws of Eqs.(1) and (4) are rewritten equivalently by the following two equations with the use of the scalar and the vector potentials;

$$\frac{\partial \mathbf{A}}{\partial t} = -\nabla \phi - \mathbf{E}, \quad (7)$$

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad (8)$$

We now discuss and check physically the derivation process of the so-called "helicity conservation law" along its process step by step. The magnetic helicity K is basically defined by the following equation [1, 2]:

$$K \equiv \frac{1}{\mu_0} \int_V \mathbf{A} \cdot \mathbf{B} dV. \quad (9)$$

Here, we emphasize that even if we include "the external helicity", taking account of the gauge invariance [2], the following argument is still essentially correct and applicable. After the partial derivative of the definition Eq.(9) with respect to t , and using only two physical laws of Eqs. (1) and (4) with their equivalent Eqs. (7) and (8), the vector formulae $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = (\nabla \times \mathbf{a}) \cdot \mathbf{b} - \mathbf{a} \cdot (\nabla \times \mathbf{b})$ and $\nabla \cdot (\alpha \mathbf{a}) = (\nabla \cdot \mathbf{a})\alpha + \mathbf{a} \cdot (\nabla \alpha)$, and Gauss theorem, we obtain the following equation for "the time change rate of K ",

$$\begin{aligned} \frac{\partial K}{\partial t} &= \frac{1}{\mu_0} \int_V \left(\frac{\partial \mathbf{A}}{\partial t} \cdot \mathbf{B} + \mathbf{A} \cdot \frac{\partial \mathbf{B}}{\partial t} \right) dV \\ &= -\frac{1}{\mu_0} \int_V \{ \mathbf{B} \cdot \nabla \phi + \mathbf{E} \cdot \mathbf{B} \\ &\quad + (\nabla \times \mathbf{E}) \cdot \mathbf{A} \} dV \\ &= -\frac{2}{\mu_0} \int_V \mathbf{B} \cdot \mathbf{E} dV \\ &\quad + \frac{1}{\mu_0} \oint_S (\mathbf{E} \times \mathbf{A} - \phi \mathbf{B}) \cdot d\mathbf{S}, \end{aligned} \quad (10)$$

where $\nabla \cdot (\phi \mathbf{B}) = \mathbf{B} \cdot (\nabla \phi)$ is used. Using the same derivation process for the energy conservation law Eq.(6), we can also obtain "the time change rate of K , Eq.(10)" from only one dynamic Eq.(1) and its equivalent Eq.(7), as follows: a) Applying the scalar product of \mathbf{A} on both sides of Eq.(1), we integrate both sides of the resultant equation over the volume V . b) After applying the scalar product of \mathbf{B} on both sides of Eq.(7), we integrate both sides of the resultant equation over the volume V . c) Adding both sides of the two resultant equations by a) and b), using vector formulae and Gauss theorem, we come to "the time change rate of K , Eq.(10)".

It should be emphasized here that "the time change rate of K , Eq.(10)" is derived from merely one physical law of Eq.(1), which can never lead to any deterministic time evolutions of \mathbf{A} and \mathbf{B} without another physical law of Eq.(2). The quantity of K composed only by \mathbf{A} and \mathbf{B} does have no physical grounds to restrict the time evolution of the magnetized plasma system. Even though the value of K always exists along the time variable t by the definition Eq.(9), the value of $\partial K / \partial t$ is passively and resultantly determined by the mutually independent volume and surface integral terms in Eq.(10). This fact indicates definitely that Eq.(10) is not "the conservation equation of the helicity K itself", but is merely an equation for "the time change rate of K ". Because of these physical and mathematical background, the total helicity K can never be conserved in the dynamic system, due to the volume integral term of $\mathbf{B} \cdot \mathbf{E}$ in Eq.(10), which

arbitrarily changes its value between positive and negative ones. We cannot find any associate quantities for the changed part of the total helicity K . This is because that the helicity K is not the physical quantity, but only represents the topological property of the magnetic field lines "at each instant".

We next derive the so-called familiar "helicity conservation law" and the familiar energy conservation one used in the fusion theory. Assuming $(\mathbf{E} \cdot \mathbf{D})/2 \ll (\mathbf{H} \cdot \mathbf{B})/2$ in the plasma confinement experiments, we obtain the following equation for the field energy W_f ,

$$W_f = \int_V \left(\frac{\mathbf{E} \cdot \mathbf{D}}{2} + \frac{\mathbf{H} \cdot \mathbf{B}}{2} \right) dV \cong \int_V \frac{\mathbf{H} \cdot \mathbf{B}}{2} dV \equiv W_m, \quad (11)$$

where W_m is the magnetic field energy component of W_f . From Eqs.(11) and (6), we obtain

$$\frac{\partial W_f}{\partial t} \cong \frac{\partial W_m}{\partial t} = - \int_V \mathbf{j} \cdot \mathbf{E} dV - \frac{1}{\mu_0} \oint_S (\mathbf{E} \times \mathbf{B}) \cdot d\mathbf{S}. \quad (12)$$

Using the following simplified Ohm's law of Eq.(13), Eqs.(12) and (10) are rewritten to the following familiar energy conservation law of Eq.(14) and the so-called final familiar "helicity conservation law", respectively:

$$\text{Ohm's law: } \mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j}. \quad (13)$$

$$\begin{aligned} \frac{\partial W_f}{\partial t} &\cong \frac{\partial W_m}{\partial t} \\ &= - \int_V \{ \eta \mathbf{j} \cdot \mathbf{j} + (\mathbf{j} \times \mathbf{B}) \cdot \mathbf{v} \} dV \\ &\quad - \frac{1}{\mu_0} \oint_S (\mathbf{E} \times \mathbf{B}) \cdot d\mathbf{S}. \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{\partial K}{\partial t} &= - \frac{2}{\mu_0} \int_V \eta \mathbf{j} \cdot \mathbf{B} dV \\ &\quad + \frac{1}{\mu_0} \oint_S (\mathbf{E} \times \mathbf{A} - \phi \mathbf{B}) \cdot d\mathbf{S}. \end{aligned} \quad (15)$$

The misunderstanding on the so-called "helicity conservation law" has been established from the following argument, using "the time change rate equation of K written by Eq.(15)" [1, 2].

(A) At first, we consider "the ideal case" where the whole region of plasmas inside the boundary is filled with the ideally conducting plasma and the boundary surface is the ideally conducting wall, i.e., $\eta = 0$ and $\mathbf{E} = 0$ and $\mathbf{B} \cdot d\mathbf{S} = 0$ at the ideally conducting wall. We then get from Eq.(15) in this "ideal case" that "the time change rate of K " becomes as $\partial K / \partial t = 0$.

(B) From this result, we may conclude followings. Since the value of K is constant along the time variable t , the total helicity K is conserved and therefore it must be "the time invariant in the dynamical system in the case of ideal plasmas".

However, the part of (A) declares only that the value of K defined by Eq.(8) does not change along the time variable t in "the special or the trivial case" of $\eta = 0$ plasmas filling fully within the ideally conducting wall. From the following simple thought experiment, we can easily find that the total helicity K is never "the time invariant inside the ideally conducting wall". We consider a case, where there exists some vacuum field region, i.e., $\eta = \infty$, near the ideally conducting wall, and the other region is still filled with the ideally conducting plasma. We then have to come back to Eq.(10), and we can put $\mathbf{E} = 0$ in the plasma but have to leave \mathbf{E} in the vacuum field region. In this simple case, the value of $\partial K / \partial t$ is passively and resultantly determined by the volume integral of $\mathbf{B} \cdot \mathbf{E}$ in Eq.(10) along the time variable t , as was discussed after Eq.(10). The decrement of K in this simple case is by no means "the resistivity loss of K ", because of no current in the vacuum field region. On the other hand, we definitely know that the changed part of W_m transfers to the other type of energy, such as the kinetic energy, inside the ideally conducting wall by Eq.(12). However, the total helicity K can never be conserved in the dynamical system in this simple case. This is because that Eq.(10) is merely an equation for "the time change rate of K ", and the helicity K is not the physical quantity but merely represents the topological property of the magnetic field lines at each instant.

The simple thought experiment shown above may lead us to a conclusion that the total value of K is never "the time invariant in the dynamical system", like as it has been believed by many scientists from misunderstanding on the meaning of "the time change rate equation of K , Eqs.(10) and (15)".

On the other hand, the energy conservation law of Eq.(14) declares that even if $\eta = 0$ and $\mathbf{E} = 0$ at the ideally conducting wall, the left-hand side $\partial W_m / \partial t$ always balances with the volume integral term of $(\mathbf{j} \times \mathbf{B}) \cdot \mathbf{v}$, which is called the dynamo term, in the right-hand side. Furthermore, when $\eta \neq 0$, then $\partial W_m / \partial t$ still balances with the two of the Joule's heating and the dynamo terms. The energy conservation law has also no power to determine the time evolution of the dynamic system only by using itself.

The logical and mathematical process to derive the so-called helicity conservation law described above is logically the same as the following simple example: We consider here two functions of $f(x, t)$ and $g(x, t)$, which are determined by their dynamic equations and further have the following relation

$$\partial f(x, t) / \partial x = g(x, t). \quad (16)$$

Here, the relation between $f(x, t)$ and $g(x, t)$ is corresponding to that between \mathbf{A} and \mathbf{B} . Using the two functions of $f(x, t)$ and $g(x, t)$, we can define a ghost helicity Kg , that corresponds to the helicity K , and get an equation of "the time change rate of Kg " with the use of the time derivative of Kg , as follows,

$$Kg \equiv \int_V f(x,t)g(x,t) dV, \quad (17)$$

$$\begin{aligned} \frac{\partial Kg}{\partial t} = \int_V & \left[\frac{\partial f(x,t)}{\partial t} g(x,t) \right. \\ & \left. + f(x,t) \frac{\partial g(x,t)}{\partial t} \right] dV. \end{aligned} \quad (18)$$

Next, we use following two “Someone’s laws”, which are corresponding to the simplified Ohm’s law of Eq.(13),

$$\text{Someone's law - 1 : } \frac{\partial f(x,t)}{\partial t} = \eta_1 h_1(x,t), \quad (19)$$

$$\text{Someone's law - 2 : } \frac{\partial g(x,t)}{\partial t} = \eta_2 h_2(x,t). \quad (20)$$

Substituting Eqs.(19) and (20) into Eq.(18), we obtain the final equation of “the time change rate of Kg ”, as follows,

$$\begin{aligned} \frac{\partial Kg}{\partial t} = \int_V & \left[\eta_1 h_1(x,t) g(x,t) \right. \\ & \left. + \eta_2 h_2(x,t) f(x,t) \right] dV. \end{aligned} \quad (21)$$

We now use Eq.(21) instead of Eq.(15), and follow the arguments at (A) and (B). (C) We consider here “the ideal case” that $\eta_1 = 0$ and $\eta_2 = 0$, then we obtain from Eq.(20) that the time change rate of Kg becomes as $\partial Kg/\partial t = 0$.

(D) From this result, we may conclude followings. Since the value of Kg is constant, the ghost helicity Kg is conserved and therefore it must be “the time invariant in the dynamical system”.

Can we say that the above (C) and (D) arguments are logically correct? No, we logically and definitely know that the value of Kg defined by Eq.(17) does not change in “this ideal or the trivial case”, but if $\eta_1 = \infty$ or $\eta_2 = \infty$, we have to come back to Eq.(18). We know from Eq.(18) that the value of Kg changes along the time variable and “the total value of Kg is never the time invariant in this dynamical system”. We also know that if $\eta_1 \neq 0$ and $\eta_2 \neq 0$, then Eq.(21) merely declares that the time change rate of the ghost helicity Kg is “passively and resultantly determined” by the two mutually independent terms of the right-hand sides of Eq.(21), because $\eta_1 h_1(x,t)$ and $\eta_2 h_2(x,t)$ can have mutually independent values. We should notice here again that the value of the ghost helicity Kg is always resultantly determined by its definition Eq.(18) with the values of $f(x,t)$ and $g(x,t)$ at each instant. Furthermore, we should notice that Kg itself does never control the dynamic system to determine the time evolution of $f(x,t)$ and $g(x,t)$. This is because there are no mathematical connections between the dynamic equations, which determine the time evolution of $f(x,t)$ and $g(x,t)$, and the definition of Kg itself.

This analogical explanation shown above may give us clearly the fact that the so-called “helicity conservation law” of Eq.(15) is never “the conservation equation of

the helicity K itself” but merely “the time change rate equation of K ”.

The value of the helicity K has never been conserved in the computer simulations by R. Horiuchi and T. Sato [24, 25], and also in all experiments on the reversed field pinch (RFP) by many authors [3, 26–30], on the toroidal Z-pinch by K. Sugisaki [31–34] and on merging two spheromacs into one field reversed configuration (FRC) or one spheromac by Y. Ono, Katsurai et. al. [35, 36]. Especially, in the case of the toroidal Z-pinch experiments, the total helicity K increases to finite values from zero initial value within a few tens of μs [31–34]. These experimental results have been demonstrated that the conjecture of the total helicity invariance by Dr. J. B. Taylor is not physically available to real magnetized plasmas in an exact sense.

Even if they believe the so-called helicity injection as a technical method, the assumption of the steady state $\partial K/\partial t = 0$ should be changed to the assumption of the steady state $\partial W_m/\partial t = 0$ in real experiments. In other words, the so-called helicity injection should be physically the magnetic energy injection itself. If the steady state of $\partial W_m/\partial t = 0$ is realized by the so-called helicity injection without no energy injection against the plasma current decay due to the Ohm’s loss, then we have to face a strange conclusion that the process “violates the more important physical law of the energy conservation!”

As is well known, the variational principle and the related or resultant dynamic equations are physically equivalent, i.e., we can start with either the variational principle or the related set of dynamic equations. This fundamental physical thought is also the same for the energy principle and the related dynamic equations. Physically and mathematically important point is that the set of related or resultant dynamic equations give us all time evolutions of the dynamic system itself, including not only self-organized states of equilibria but also the relaxation processes and all time change processes.

The relaxation theory by Dr. J. B. Taylor [1, 2] has neither been the variational principle nor the energy principle, but his calculation is merely the variational calculus with global constraint with respect to the value of K . That is to say, his calculation is the mathematical calculation to find the minimum energy solution from the set of solutions, which have the same value of K . The so-called Taylor state of $\nabla \times \mathbf{B} = \lambda_T \mathbf{B}$ has never been satisfied in the boundary region of experimental RFP-, spheromack-, and toroidal Z-pinch plasmas, i.e., the experimental spatial profiles of λ_T have never been constant from the central region of plasmas to the wall. Without using the concept of the helicity K , we can derive the equation of the relaxed state of MHD plasmas as $\nabla \times (\eta \nabla \times \mathbf{B}) = \lambda \mathbf{B}$, which includes the solution of $\nabla \times \mathbf{B} = \lambda_T \mathbf{B}$ for a special case where η is spatially uniform [38, 39]. In other words, experimental results of relaxed plasmas can be explained without using the concept of magnetic helicity, and the relaxed states depend on the resistivity profile η [39].

Relaxations, the magnetic field generation, and the transformation between the toroidal- and the poloidal magnetic fields are due to the dynamo term of $(\mathbf{j} \times \mathbf{B}) \cdot \mathbf{v}$ in the energy conservation law of Eq.(14), where the velocity \mathbf{v} comes from the Lorentz force and/or the thermal convection of the conducting fluids. The earth dynamo to induce the polar magnetic field are originating from the dynamo term of $(\mathbf{j} \times \mathbf{B}) \cdot \mathbf{v}$ with the thermal convection velocity \mathbf{v} . In order to realize the steady state $\partial W_m / \partial t = 0$ in experimental plasmas, the energy conservation law of Eq.(14) may suggest us various methods of the plasma current drive by external energy injections with use of various type of energies.

III. A GENERALIZED SELF - ORGANIZATION THEORY

III - 1. BASIC FORMULATION

We develop here a novel basic formulation of the generalized self-organization theory that is an extension of the last report in [38]. It should be emphasized here that the generalized self-organization theory with the use of auto-correlations for physical quantities is not fundamentally based on neither the variational principle nor the energy principle, and the auto-correlations is never time invariants.

Quantities with n elements in general dynamic systems of interest shall be expressed as $\mathbf{q}(t, \mathbf{x}) = \{q_1(t, \mathbf{x}), q_2(t, \mathbf{x}), \dots, q_n(t, \mathbf{x})\}$. Here, t is time, \mathbf{x} denotes m -dimensional space variables, and \mathbf{q} represents a set of physical quantities having n elements, some of which are vectors such as the velocity \mathbf{u} , the magnetic field \mathbf{B} , the current density \mathbf{j} , \dots , and others are scalars such as the mass density, the energy density, the specific entropy and so on. We consider a dissipative nonlinear dynamic system which may be generally described by

$$\frac{\partial q_i}{\partial t} = G_i[\mathbf{q}], \quad (22)$$

where $G_i[\mathbf{q}]$ denotes linear or nonlinear dynamic operators, which may include nondissipative and/or dissipative terms. In some cases, the operator $G_i[\mathbf{q}]$ may include negative dissipation terms such as energy input terms. After taking the product of $q_i(t, \mathbf{x})$ and both sides of Eq.(22), and integrating both sides of the resultant equation over the volume V , we obtain the conservation equations concerning with the quantities $q_i(t, \mathbf{x})$ for the dynamic system of interest, as follows,

$$\begin{aligned} \int_V \left\{ \frac{\partial}{\partial t} \frac{1}{2} [q_i(t, \mathbf{x}) q_i(t, \mathbf{x})] \right\} dV \\ = \int_V \{ q_i(t, \mathbf{x}) G_i[\mathbf{q}] \} dV. \end{aligned} \quad (23)$$

When the dynamic system has some unstable regions, the nondissipative terms in the dynamic operator $G_i[\mathbf{q}]$ may become dominant and lead to the rapid growth of perturbations there and further to turbulent phases. This

may yield spectrum transfers or spectrum spreadings toward both the higher and the lower wave number regions in q_i distributions, as in the normal energy cascade and also the inverse cascade shown by 3-D MHD simulations in [24, 25]. When the higher wave number becomes a large fraction of the spectrum, the dissipative terms in the dynamic operator $G_i[\mathbf{q}]$ may become dominant to yield higher dissipations for the higher wave number components. In this rapid dissipation phase, which is far from equilibrium, the unstable regions in the dynamic system are considered to vanish to come to a relaxed and quasi-steady configuration again.

If we trace over all time evolution of the dynamic system, we may find that the dynamic system may go through various phases, some of which the spatial profiles of $q_i(t, \mathbf{x})$ [$i = 1, 2, 3, \dots$] has changeable structures and other phases its profiles does have unchangeable ones. Here, we should expect that every spatially unchangeable structure of $q_i(t, \mathbf{x})$ [$i = 1, 2, 3, \dots$] may occur repeatedly at different peculiar time for every $q_i(t, \mathbf{x})$, as have been observed in most of all experiments, such as the sawtooth oscillations, the transport barrier by the shear flows and so on. From this standpoint of observation on over all time evolution of the dynamic system, we can identify or define "the self-organized state" as "the self-similar state in the phase with the most unchangeable structure". In order to describe quantitatively those most unchangeable structure, we inevitably introduce the auto-correlations as a suitable measure. The definition may be mathematically expressed by using auto-correlations, $q_i(t, \mathbf{x}) q_i(t + \Delta t, \mathbf{x})$, between the time, t , and slightly transferred time, $t + \Delta t$, with a small Δt , as follows

Self - organized state \equiv

$$\min \left| \frac{\int q_i(t, \mathbf{x}) q_i(t + \Delta t, \mathbf{x}) dV}{\int q_i(t, \mathbf{x}) q_i(t, \mathbf{x}) dV} - 1 \right| \text{ state}. \quad (24)$$

Substituting the Taylor expansion of $q_i(t + \Delta t, \mathbf{x}) = q_i(t, \mathbf{x}) + [\partial q_i(t, \mathbf{x}) / \partial t] \Delta t + (1/2) [\partial^2 q_i(t, \mathbf{x}) / \partial t^2] (\Delta t)^2 + \dots$ into the definition Eq.(24), and taking account of the arbitrary smallness of Δt , we obtain the following equivalent definition for of the self-organized state from the first order of Δt in the definition Eq.(24):

Self - organized state \equiv

$$\min \left| \frac{\int q_i(t, \mathbf{x}) [\partial q_i(t, \mathbf{x}) / \partial t] dV}{\int q_i(t, \mathbf{x}) q_i(t, \mathbf{x}) dV} \right| \text{ state}. \quad (25)$$

Substituting the original dynamic equation, Eq.(22), into Eq.(25), we obtain the following final condition for the self-organized state

Self - organized state $=$

$$\min \left| \frac{\int q_i(t, \mathbf{x}) G_i[\mathbf{q}] dV}{\int q_i(t, \mathbf{x}) q_i(t, \mathbf{x}) dV} \right| \text{ state}. \quad (26)$$

We find from the definition of Eq.(25) that realization of the self-similar coherent structures in dissipative dynamical systems is equivalent to that of self-organized states

with "the minimum change rate of auto-correlations for their instantaneous values". Furthermore, since we have substituted the original dynamic equations into the definition of the self-organized state, we can recognize that "whole properties of the dynamic system is essentially embedded in the process of calculations to derive the self-organized state from the final condition of Eq.(26)".

The mathematical expressions with the use of the variational calculus for the definition of Eq.(25) and further the final condition Eq.(26) are written as follows, defining a functional F with use of a Lagrange multiplier λ_i :

$$\begin{aligned} F &\equiv \int_V \left\{ \frac{\partial}{\partial t} \frac{1}{2} [q_i(t, \mathbf{x}) q_i(t, \mathbf{x})] \right. \\ &\quad \left. + \lambda_i q_i(t, \mathbf{x}) q_i(t, \mathbf{x}) \right\} dV \\ &= \int_V \left\{ q_i(t, \mathbf{x}) G_i[\mathbf{q}] \right. \\ &\quad \left. + \lambda_i q_i(t, \mathbf{x}) q_i(t, \mathbf{x}) \right\} dV. \end{aligned} \quad (27)$$

$$\delta F = 0, \quad (28)$$

$$\delta^2 F > 0, \quad (29)$$

where δF and $\delta^2 F$ are respectively the first and the second variations of F "with respect to the variation $\delta \mathbf{q}(\mathbf{x})$ only for the spatial variable \mathbf{x} ". Comparing Eqs.(23) and (27), we can find that the conservation equations concerning with the quantities $q_i(t, \mathbf{x})$ for the dynamic system of interest are naturally included in the present formulation of the generalized self-organization theory. We should remind here that the global auto-correlation $\int q_i(t, \mathbf{x}) q_i(t, \mathbf{x}) dV$ is never the time invariant but strictly the global constraint. The implicit assumption in this theory is that the dynamical system evolves all possible area in state phases.

III - 2. APPLICATION TO PLASMAS

We apply here the generalized self-organization theory shown above to fusion plasmas.

According to the general type of the dynamic equations, Eq.(22), we rewrite Eqs.(1) and (2) as follows

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}. \quad (30)$$

$$\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{H} - \mathbf{j}. \quad (31)$$

We use here the following three more physical laws, i.e., the conservation laws of the mass, Eq.(32), and the momentum, Eq.(33), and the generalized Ohm's law, Eq.(34).

$$\frac{\partial \rho_m}{\partial t} = -\nabla \cdot (\rho_m \mathbf{v}). \quad (32)$$

$$\begin{aligned} \rho_m \frac{\partial \mathbf{v}}{\partial t} &= -\rho_m (\mathbf{v} \cdot \nabla) \mathbf{v} + [\rho_e \mathbf{E} + \mathbf{j} \times \mathbf{B} \\ &\quad - \nabla (P_e + P_i)]. \end{aligned} \quad (33)$$

$$\begin{aligned} \frac{\partial \mathbf{j}}{\partial t} &= \frac{e^2 n_e}{m_e} \{ \mathbf{E} + \mathbf{v} \times \mathbf{B} - \eta_{ei} \mathbf{j} - \frac{1}{en_e} (\mathbf{j} \times \mathbf{B}) \\ &\quad + \frac{1}{en_e} [\nabla P_e - (m_e/m_i) Z_i \nabla P_i] \}. \end{aligned} \quad (34)$$

These three equations come from the Boltzmann kinetic equations for electrons and ions. Therefore, we start with an axiom set of seven physical laws, i.e., Eqs.(1),(2),(3), (4), (32), (33) and (34), where Eqs.(1) and (2) are rewritten to Eqs.(30) and (31), respectively, and the charge conservation law is included in Maxwell's equations, Eqs.(1) - (4). We can get Poynting's energy conservation law by the following processes.

1) After taking the scalar product of \mathbf{B}/μ_0 with both sides of Eq.(30), we integrate both sides of the resultant equation over the volume V .

2) After taking the scalar product of \mathbf{E} with both sides of Eq.(31), we integrate both sides of the resultant equation over the volume V .

3) Adding both sides of the two equations by 1) and 2), using vector formulae and Gauss theorem, we obtain Poynting's energy conservation law concerning with the field energy W_f , which is the same as Eq.(7), as follows

$$\frac{\partial W_f}{\partial t} = - \int_V \mathbf{j} \cdot \mathbf{E} dV - \frac{1}{\mu_0} \oint_S (\mathbf{E} \times \mathbf{B}) \cdot d\mathbf{S}. \quad (35)$$

In the same way, after taking the scalar product of \mathbf{v} with both sides of Eq.(33), and integrating both sides over the volume V , we obtain the conservation law of the kinetic energy $W_k = \int_V (\rho_m/2) \mathbf{v} \cdot \mathbf{v} dV$, as follows

$$\begin{aligned} \frac{\partial W_k}{\partial t} &= \int_V \left\{ -\frac{1}{2} \mathbf{v} \cdot \mathbf{v} \nabla \cdot (\rho_m \mathbf{v}) - \rho_m \mathbf{v} \cdot [(\mathbf{v} \cdot \nabla) \mathbf{v}] \right. \\ &\quad \left. + [\rho_e \mathbf{E} \cdot \mathbf{v} + (\mathbf{j} \times \mathbf{B}) \cdot \mathbf{v} \right. \\ &\quad \left. - \rho_m \mathbf{v} \cdot \nabla (P_e + P_i)] \right\} dV. \end{aligned} \quad (36)$$

Similarly, after taking the scalar product of \mathbf{j} with both sides of Eq.(34), and integrating both sides of the resultant equation over the volume V , we obtain another conservation law on the current by defining $W_c = \int_V (1/2) \mathbf{j} \cdot \mathbf{j} dV$, as follows

$$\begin{aligned} \frac{\partial W_c}{\partial t} &= \int_V \frac{e^2 n_e}{m_e} \{ \mathbf{j} \cdot \mathbf{E} - (\mathbf{j} \times \mathbf{B}) \cdot \mathbf{v} - \eta_{ei} \mathbf{j} \cdot \mathbf{j} \\ &\quad + \frac{1}{en_e} \mathbf{j} \cdot [\nabla P_e - (m_e/m_i) Z_i \nabla P_i] \} dV. \end{aligned} \quad (37)$$

According to Eq.(27), we obtain the functional for the field energy F_f , for the kinetic energy F_k , and for the current F_c , respectively, as follows,

$$\begin{aligned} F_f &= \int_V \left\{ -\mathbf{j} \cdot \mathbf{E} + \lambda_f \left(\frac{\epsilon_0 \mathbf{E} \cdot \mathbf{E}}{2} + \frac{\mathbf{B} \cdot \mathbf{B}}{2\mu_0} \right) \right\} dV \\ &\quad - \frac{1}{\mu_0} \oint_S (\mathbf{E} \times \mathbf{B}) \cdot d\mathbf{S}, \end{aligned} \quad (38)$$

$$F_k = \int_V \left\{ -\frac{1}{2} \mathbf{v} \cdot \nabla (\rho_m \mathbf{v}) - \rho_m \mathbf{v} \cdot [(\mathbf{v} \cdot \nabla) \mathbf{v}] \right. \\ \left. + [\rho_e \mathbf{E} \cdot \mathbf{v} + (\mathbf{j} \times \mathbf{B}) \cdot \mathbf{v} - \rho_m \mathbf{v} \cdot \nabla (P_e + P_i)] + \lambda_v \frac{\rho_m}{2} \mathbf{v} \cdot \mathbf{v} \right\} dV, \quad (39)$$

$$F_c = \int_V \left\{ \frac{e^2 n_e}{m_e} \{ \mathbf{j} \cdot \mathbf{E} - (\mathbf{j} \times \mathbf{B}) \cdot \mathbf{v} - \eta_e \mathbf{j} \cdot \mathbf{j} \} \right. \\ \left. + \frac{e}{m_e} \mathbf{j} \cdot [\nabla P_e - (m_e/m_i) Z_i \nabla P_i] + \lambda_c \mathbf{j} \cdot \mathbf{j} \right\} dV. \quad (40)$$

In general, we take variations with respect to $\delta \mathbf{E}$, $\delta \mathbf{B}$, $\delta \mathbf{v}$, $\delta \mathbf{j}$, $\delta \rho_m$, $\delta \rho_e$, δP_e , δP_i , δn_e , δn_i , and $\delta \eta_{ei}$. From the Euler-Lagrange equations for the solutions of Eq.(28), we will get new various equilibrium configurations of the self-organized states with the plasma flow, the shear flow, the space charge, the space potential, the deviation between the ion and the electron density profiles, the resistivity profile, and so on, depending on the boundary conditions and the external input sources such as the various energy injections and the particle beams. The resultant equilibrium configurations are far beyond the conventional MHD equilibrium ones by the Grad-Shafranov equation based on the equation of $\mathbf{j} \times \mathbf{B} = \nabla p$. The results of the present calculation will appear elsewhere.

Dividing Eq.(37) by $e^2 n_e / m_e$, we find the resultant equation to become a power balance equation. In order to realize the steady state of the confinement system of plasmas, we can extend conventional methods of plasma current drives with the use of the three conservation laws of Eqs.(35),(36) and (37), using energy injections with use of various types of energies, such as magnetic energies, electromagnetic wave energies, internal energies of plasmoids by plasma guns, which induce the thermal plasma flow velocity, various particle beam energies, and so on.

IV. CONCLUDING REMARKS

Analyzing the logical and mathematical structures of the derivation process for the so-called "helicity conservation law", and using the simple thought experiment for the case, where there exists some vacuum field region, i.e., $\eta = \infty$, near the ideally conducting wall, and the other region is still filled with the ideally conducting plasma, we have clarified the followings in Section II:

1. Even though the value of K always exists along the time variable t by the definition Eq.(19), the value of $\partial K / \partial t$ is passively and resultantly determined by the mutually independent volume and surface integral terms in Eqs.(10) and (15). This fact indicates definitely that Eqs.(10) and (15) are not "the conservation equations of the helicity K itself", but are merely the equations for "the time change rate of K ".

2. The total helicity K can never be conserved in the dynamic system, due to the volume integral term of $\mathbf{B} \cdot \mathbf{E}$

in Eq.(10) or $\eta \mathbf{j} \cdot \mathbf{B}$ in Eq (15), which arbitrarily change their values between positive and negative ones

3. Since the total helicity K can never be conserved in the real experimental dynamic systems, as was observed in the toroidal Z-pinch experiments [31-34], the conjecture of the total helicity invariance by Dr. J. B. Taylor is not physically available to real magnetized plasmas in an exact sense.

4. We cannot find any associate quantities for the changed part of the total helicity K shown by the second terms of Eqs.(10) and (15). This is because that the helicity K is not the physical quantity, but only represents the topological property of the magnetic field lines "at each instant".

These facts shown above indicate definitely that the so-called "helicity conservation law" is never "the conservation equation of the helicity K itself" but merely "the time change rate equation of K ". From the present logical analysis, we have clarified that the relaxation theory by Dr. J. B. Taylor is neither the variational principle nor the energy principle, but is merely a mathematical calculation, using the variational calculus in order to find the minimum magnetic energy solution from the set of solutions having the same value of K .

The so-called helicity conservation law is derived from merely one physical law of Eq.(1), which can never lead to any deterministic time evolutions of \mathbf{A} and \mathbf{B} without another physical law of Eq.(2). The quantity of K composed only by \mathbf{A} and \mathbf{B} does have no physical grounds to restrict the time evolution of the magnetized plasma system.

In Section III, we have presented the basic formulation of the generalized self-organization theory that is the extension of the last report in [23]. It is emphasized here that the generalized self-organization theory with the use of auto-correlations for physical quantities is not fundamentally based on neither the variational principle nor the energy principle, and the auto-correlations is never time invariants.

We have clarified from the definition of the self-organized state given by Eq.(25) that the realization of the self-similar coherent structures in dissipative dynamical systems is equivalent to that of self-organized states with "the minimum change rate of auto-correlations for their instantaneous values". Furthermore, we have shown that since the original dynamic equations are substituted into the definition of the self-organized state, "whole properties of the dynamic system is essentially embedded in the process of calculations to derive the self-organized state from the final condition of Eq.(26)". From the comparison between Eqs.(23) and (27), the conservation equations concerning with the quantities $q_i(t, \mathbf{x})$ for the dynamic system of interest have been shown to be naturally included in the formulation of the present generalized self-organization theory. It should be emphasized here that the generalized self-organization theory can deduce the Taylor state without using the concept of the

helicity, and further be applicable for any nonlinear dynamical systems [37–44].

It is important to point out that the self-organized states of every physical quantities of interest may be realized during their own peculiar phases and the dynamical system may evolve repeatedly, having those out of phase organizations due to boundary conditions and input powers, as have been observed in most of all experiments, such as the sawtooth oscillations.

In order to realize the steady state of the confinement system of plasmas, we can extend conventional methods of plasma current drives with the use of the three conservation laws of Eqs.(35),(36) and (37), using energy injections with use of various types of energies, such as magnetic energies, electromagnetic wave energies, internal energies of plasmoids by plasma guns, which induce the thermal plasma flow velocity, various particle beam energies, and so on.

ACKNOWLEDGMENTS

The authors would like to thank Professor T. Sato at the National Institute for Fusion Science, Toki, Japan, Drs. Y. Hirano, Y. Yagi, and T. Shimada at ETL, Tsukuba, Japan, and Professor S. Shiina at Nihon University, Tokyo, for their valuable discussion and comments on this work. He appreciates Mr. M. Plastow at NHK, Tokyo, for his valuable discussion on the thought analysis for the method of science. This work was carried out under the collaborative research program at the National Institute for Fusion Science, Toki, Japan.

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