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RADIATIVE CASCADE DUE TO DIELECTRONIC RECOMBINATION

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Abstract

The problem of a determination of two-dimensional (in quantum numbers n,l) atomic level populations under the action of dielectronic recombination (DR) sources and corresponding radiative cascades is considered. A quasiclassical approach is used for calculations both DR rates and radiative cascade transitions which makes it possible to obtain a simple analytical solution of the problem. The populations obtained are expressed in the analytical form allowing estimations of energy level populations for every types of ions having transitions in their cores without change of principle quantum numbers. The results are illustrated by numerical data for energy level populations of *Li*-like ions.

Key words: dielectronic recombination, radiative cascade, two-dimensional level populations

1. Introduction

Dielectronic recombination (DR) is an effective mechanism of highly excited (Rydberg) atomic state populations in plasmas, see [1]. The DR source for Rydberg atomic states is large for transitions in atomic core without change of their principal quantum numbers ($\Delta n=0$ transitions). The DR electron capture results in a strongly nonequilibrium atomic level population with respect to values of orbital quantum numbers l which are much smaller as compared with principle quantum numbers n [1,2]. The every population event is accompanied by radiative-collisional cascade resulting in a change of initial nl-distributions. When the plasma density is relatively low the main mechanism responsible for nl-redistribution is the radiative cascade. Such conditions are usual for atoms and multicharged ions under coronal equilibrium conditions in astrophysical conditions as well as in rarefied laboratory plasmas (storage rings, tokamaks, stellarators, etc). The conditions for separation between domains in n,l-space driven by collisional or radiative transitions are also well known [1,3,4]. The l-mixing collisions can be also taken into account in the frame of quasiclassical basis [5,6].

The problem of radiative cascade in hydrogen atom for radiative recombination (RR) source has been investigated in details [3,7]. The DR source possesses specific properties as compared with RR-source connected with a resonance between the difference of initial and final electron energies and the transition energy in the atomic core. So it is of general interest to investigate the nl-distribution of atomic energy levels with dominant DR-population source resulting in radiative two dimentional cascade in rarefied plasmas. Just this problem is under consideration in the present note.

A direct solution of a system of kinetic equations becomes rather tedious for large values of principle quantum numbers n of order of 10^2 or larger. Really the quantity of matrix elements for a particular radiative transition between Rydberg atomic states is of order of $n^2\cdot n'^2=10^4\cdot 10^4=10^8$ for n-n' transitions between atomic levels with n, $n'\approx 10^2$. This quantity must be multiplied by the n^2 numbers of matrix elements responsible for population sources. So the direct calculation becomes tedious even in the case of hydrogen–like atomic states where all matrix elements are well known. But the most important thing is connected with difficulties related to an extraction of different scaling lows for a distribution of atomic level populations over quantum numbers n and l from the results of quantum calculations. The last ones are tables of particular values in the n,l-space whereas these values follow specific universal dependencies between n and l which can be simply caught from classical description of the processes [7,8].

So we shall use below a quasiclassical description for 2D radiative cascade under the action of DR population source, selective in the n,l space.

2. General relationships

The general formula for DR recombination rates being a source q_{DR} for a radiative cascade takes the form [1]:

$q_{DR} =$

$$\left(\frac{2\pi}{T}\right)^{3/2} \frac{g_j}{g_i} \frac{(2l+1)W_R W_A}{W_R + W_A} exp\left(-\frac{\omega}{T} + \frac{Z^2}{2n^2T}\right) \quad (1)$$

where T is an electron temperature, $g_{i,j}$ are statistical weights for a transition inside an ion core, l,n are the orbital and principle quantum numbers of captured electron, $W_{R,A}$ are radiative and autoionization decay rates, ω is the frequency for a transition with $\Delta n=0$ inside the core.

The radiative decay rate in the core is simply expressed in terms of an oscillator strength f_{ij} for the transition in the core (c is the velocity of light):

$$W_R = 2\,\omega^2 f_{ij} (3c^3)^{-1} \quad (2)$$

The quasiclassical approximation is used for the autoionization decay rate W_A, which can be obtained by a direct transition to the classical limit in general quantum formulas for corresponding matrix elements [2,9] or by direct classical calculations [10,11]:

$$W_A = f_{ij}(2l+1)G[\,\omega(l+1/2)^3/3Z^2\,]/2\pi n^3 \quad (3)$$

$$G(u) = u[K_{1/3}^2(u) + K_{2/3}^2(u)], \quad (4)$$

where K are standard MacDonald function. We shall put the values of principal quantum numbers n (or the temperature T) large enough to exclude the member $Z^2/2n^2T$ in the exponent.

Let us transform the eq.(1) for the source q_{DR} introducing an effective value of the principle quantum number n^* equal to (compare with [1,2]):

$$n^*(l,Z) = [3c^3\,(2l+1)\,G(\,\omega(l+1/2)^3/3Z^2)(4\pi\,\omega^2)^{-1}]^{1/3}. \quad (5)$$

Then the source q_{DR} with substitution of n^* takes the form

$$q_{DR} = B(T,Z)\,c^{-3}(2l+1)\,[(n/n^*)^3 + 1]^{-1} \quad (6)$$

$$B(T,Z) = 2\,(2\pi/T)^{3/2} g_j\,\omega^2 f_{ij}\,exp(-\,\omega/T)(3g_i)^{-1} \quad (7)$$

The quasiclassical formula for the radiative decay rate $A(n,l)$ of an n,l atomic state is also used as below:

$$A(n,l) = 16Z^4[\,\pi\sqrt{3}\,c^3n^3\,(2l+1)^2]^{-1} \quad (8)$$

3. Quasiclassical radiative cascade model

The general idea of a classical approach to the radiative cascade is to use classical kinetic equations for electron energy and angular momentum losses in Coulomb field instead of quantum kinetic equations. The quasiclassical radiative cascade model applied for radiative recombination sources results in the very good correspondence with quantum calculations [7,8]. This is the reason for its application for DR population source as well. The classical kinetic equation is simply a continuity equation for a distribution function $f(n,l)$ in n,l-space:

$$\partial(\dot{n}f)/\partial n + \partial(\dot{l}f)/\partial l = q(n,l) \quad (9)$$

where $\dot{n}, \dot{l}$ are radiative losses of classical action variables corresponding to quantum numbers n,l during a fall of the radiating electron on the Coulomb center [9]:

$$\dot{n} = Z^4(1 - l^2/3n^2)(c^3l^5)^{-1}; \qquad \dot{l} = -2Z^4(c^3l^2n^3)^{-1} \quad (10)$$

The general solution of the classical kinetic equation is expressed in term of an integral along characteristics with some boundary conditions corresponding to a solution without any cascade transitions. This last solution is simply the source q divided by the spontaneous decay rate A. So the general solution of the kinetic equation is as follows

$$f(n,l) = q_{DR}(n,l)/A(n,l) + \int_{n+1}^{\infty} dn'\,q_{DR}(n',l)/\dot{n}(n',l) \quad (11)$$

The solution (11) follows also from quantum kinetic equation when $l<<n$, which just corresponds to most strong radiative transition probabilities (classically that means a transition to strongly curved classical trajectories with the most irradiated intensities). So the variable l in the eq.(11) is a parameter under present conditions. Substitution of the eqs. (6-8) into eq.(11) leads to the result

$$f(n,l) = B(Z,T)Z^{-4}l^3\{(\pi\sqrt{3}/2)n^3[(n/n^*)^3+1]^{-1} + 2l^3\,n^*J(n/n^*)\}, \quad (12)$$

where the function $J(z)$ is introduced by Fig. 1 and limiting expressions:

$$J(z) = \int_z^{\infty} \frac{dx}{x^3+1} \approx \begin{cases} 1.21, & z << 1 \\ 1/2z^2, & z >> 1 \end{cases} \quad (13)$$

$J(0) = 1.21$

Using the result (12,13) one can obtain the limiting cases for n,l distribution lows:

$$f(n,l) = B(Z,T)Z^{-4}l^3\{(\pi\sqrt{3}/2)n^3 + 2.42\,l^3\,n^*\}, \quad n<<n^*(l) \quad (14)$$

$$f(n,l) = B(Z,T)Z^{-4}l^3\,n^{*3}\{(\pi\sqrt{3}/2) + l^3/n^2\}, \quad n>>n^*(l) \quad (15)$$

In estimations of contributions of cascade and direct population terms one must take into account that the value of n^* is very large (of order of 10^2) for relatively small values of orbital momentum l but it decreases sharply with increase of l in accordance with eq.(5)

The effective values of $l=l_{ef}$, when the sharp decrease of $n^*(l)$ becomes of importance is estimated from the argument of G-function in eq (5) as

$$l_{ef}=(3Z^2/\omega)^{1/3},\ \omega \approx \alpha Z\ , \qquad (16)$$

where the approximate scaling for ω is substituted (values α are constants tabulated for every isoelectronic sequences of ions [12]). In the case of *Li*-like ion cores the value of α is equal to *0.07*. Putting for example Z=30 one obtains from eq.(16) $l_{ef}=10{,}86$. It follows from eqs. (14,15) that the cascade term is of importance in the case up to $n=40$.

4. Results and conclusion

The dependence of atomic level populations (arbitrary units) on orbital quantum number l for different values of principle quantum number n for the *Li*-like ion ZnXXVIII with nuclear charge $Z=30$ is presented on Figs.2-6. The dashed line presents the contribution of cascade population. One can see the essential contribution of cascade transitions in the broad domain of atomic quantum numbers.

So the simple formulas (6,12,13) solve the problem of the radiative cascade under the action of DR source. The precision of quasiclassical solution is near *10-20%* even for relatively small values of principal and orbital quantum numbers as it has been demonstrated in [7,8] by direct comparison with solution of quantum kinetic equations. The results obtained make it possible to estimate intensities of spectral lines in multicharged ions spectra under coronal plasma conditions.

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Figure captions

Fig.1. The function $J(z)$.

Fig.2 The dependence of atomic level populations f(n,l) (solid line) (arbitrary units) on orbital quantum number l for principle quantum number $n=100$ for the *Li*-like ion ZnXXVIII with nuclear charge $Z=30$. The dashed line presents the contribution of cascade population.

Fig. 3 The same as Fig. 2 but for n=70.

Fig. 4 The same as Fig. 2-3 but for n=30.

Fig. 5 The same as Fig. 2-4 but for n=20.

Fig. 6 The same as Fig. 2-5 but for n=10.

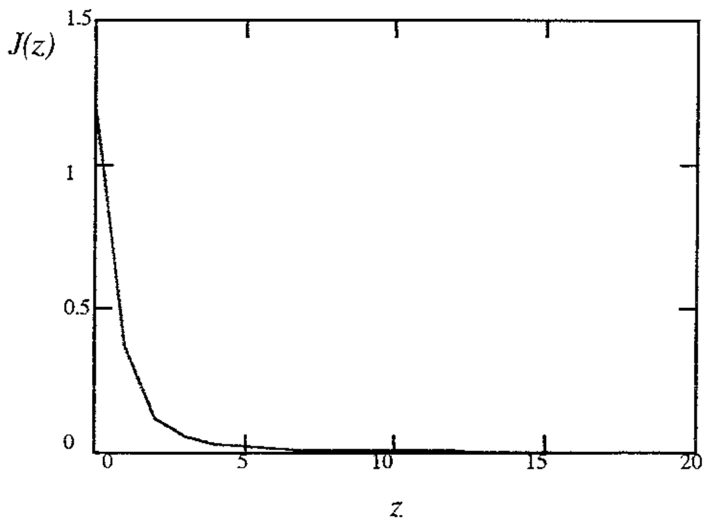

Fig. 1

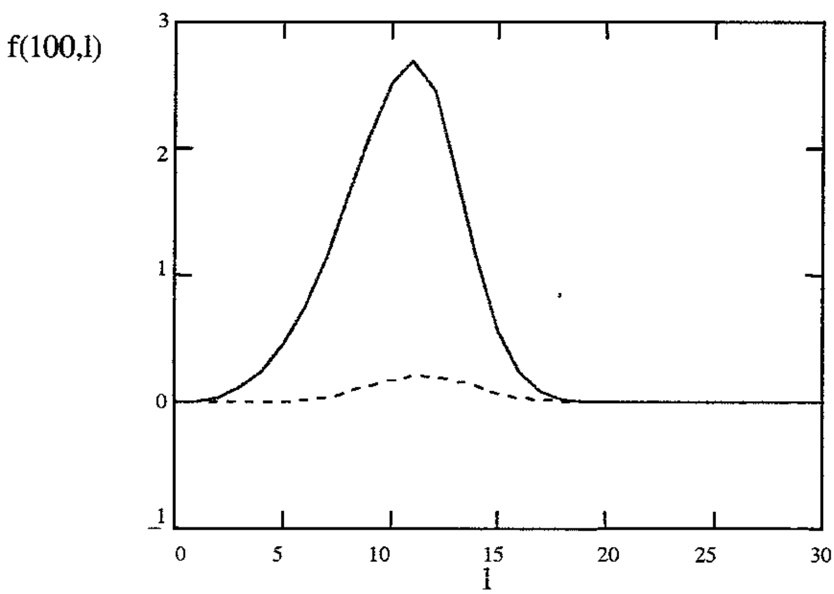

Fig.2

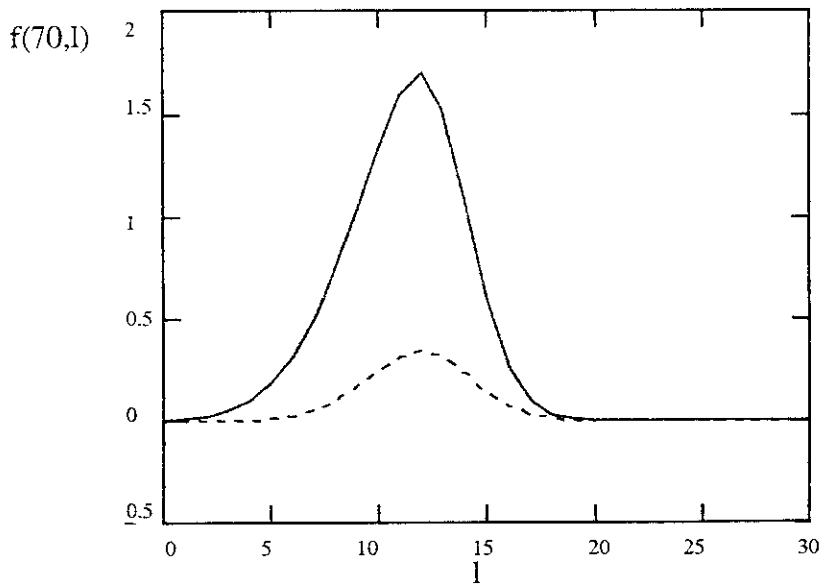
f(70,l)
2
1.5
1
0.5
0
-0.5
0
5
10
15
20
25
30
l

Fig.3

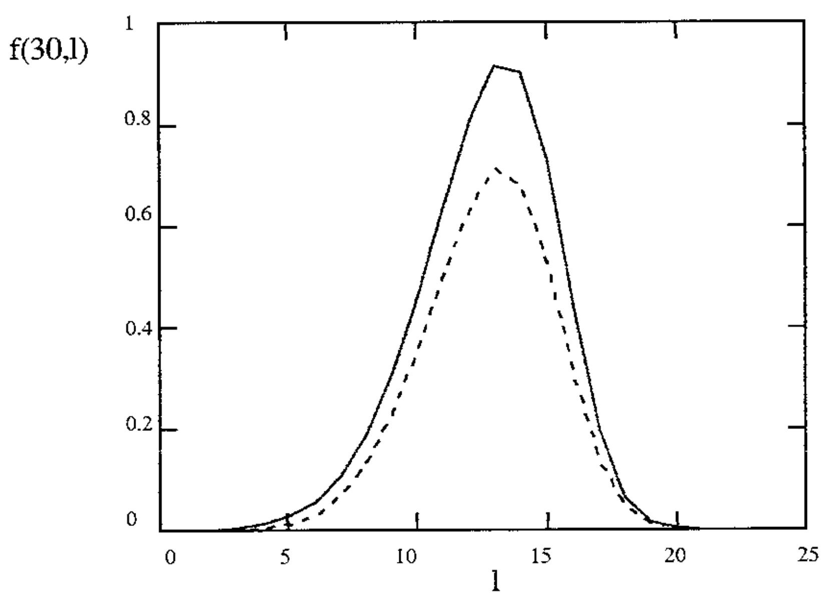
f(30,l)
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
l

Fig. 4

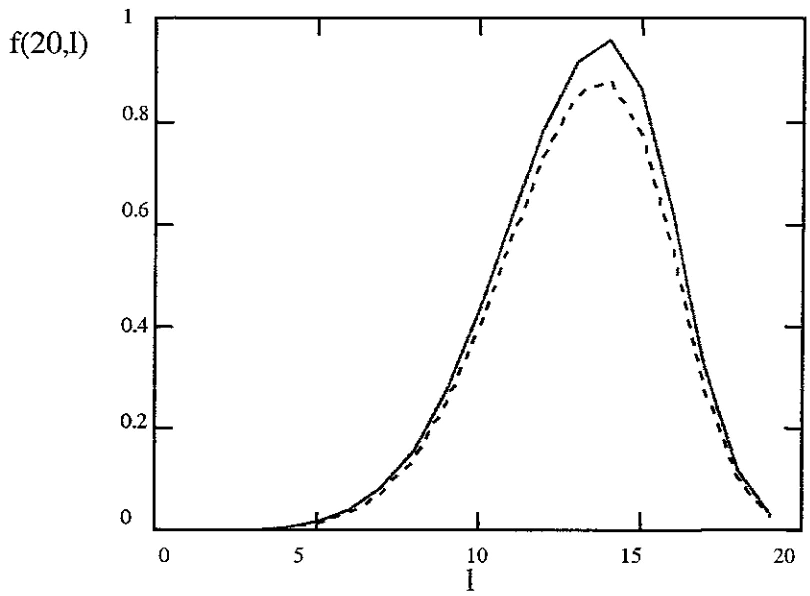

Fig.5

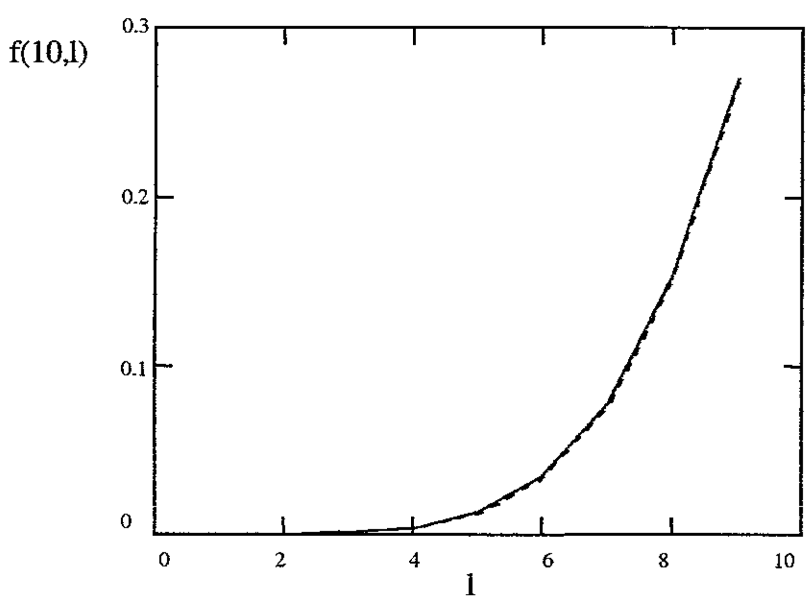

Fig. 6

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