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Spin dependent transport in magnetically confined plasma

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Abstract

The transport in tokamaks with internal transport barrier and H-mode is described in terms of emission and absorption of quasi-particles with spin. The process is equivalent of the magnetic reconnection in the wave picture. The transport rate depends on the magnetic helicity profile. The experimental results appear to support the key features of the model.

Key words: Tokamak, transport, ITB, H-mode, quasiparticles, helicity, magnetic reconnection

1] Introduction

The transport phenomena in magnetically confined plasma are often dominated by the turbulence present in the plasma. The transport rates far exceed the classical rate, namely the rate due to Coulomb collisions. For magnetic fusion, the thermal transport has been the central issue and it is important to understand the nature of the turbulent transport.

Various attempts have been made to improve the tokomak confinement by varying the magnetic configurations. Discoveries of H-mode and ITB [Internal Transport Barrier] have shown that the thermal transport rate can indeed be reduced to near the classical values at least locally.

Experimentalists have various knobs at their disposal to inject energy, helicity and angular momentum into the plasma, with neutral beams, ohmic drive, r-f heating and current drive. The locations in the plasma where power, current and momentum are injected vary from experiment to experiment. The heating power is naturally injected near the center of the plasma, while a large fraction of the helicity is provided by the ohmic voltage from outside. It is important to recognize that the turbulent transport of the energy may not be independent of the turbulent transport of the helicity and the angular momentum. The process that the experimentalists use to select the magnetic configurations involves the helicity transport. Therefore the experimental procedure may only measure the relationship between the helicity transport and the energy transport.

Suppose that the experimentalists succeed in creating an extremum of the helicity in the plasma by balancing inner and outer sources of the helicity against the resistive dissipation of the helicity. There will be no need for the transport of the helicity across the extremum to maintain a steady state. If the turbulence responsible for the transport of the helicity is also responsible for the transport of the energy and the angular momentum, the configuration may create a large temperature gradient and a large shear of the rotational velocity across the extremum. The discovery of ITB in tokamaks may be just this process.

It is instructive to view the transport due to the turbulence as the process where the quasi-particles, i.e. quanta of the turbulent wave are emitted and absorbed. The mean free path between the emission and the absorption are small so that the transport is diffusive. In the quasi-particle model, each quasi-particle carries the energy, the helicity and the angular momentum. For example the quanta of the electrostatic turbulence have no spin because the wave is described by scalar potential. On the other hand, the quanta of the mhd turbulence have spin 1 because the wave is described by vector potential. The turbulence due to zero spin quanta transports energy but neither helicity nor angular momentum. The turbulence with finite spin could transport energy, helicity and angular momentum. For this to happen, the transport must be done by spin polarized quanta. Helical magnetic flux tubes have helicity and thus spin.

When we consider the mhd perturbations with the poloidal and the toroidal mode numbers m an n, there may be singular surfaces where the wave number vector is perpendicular to the magnetic field line. It has been well known that the modes with singular surface are more dangerous in terms of the mhd instabilities. If the mode with the mode number m and n has a singular surface, the mode with $-m$ and n does not. With the reasonable assumption that the turbulence is produced by the modes with the singular surface the sign of the helicity or the spin of the quasi-particles is selected.

The mhd modes on the singular surface create the magnetic island. The transport across the surface may be viewed as the result of the re-connection of the magnetic flux lines near the saddle points on the separatrices. Energy, helicity and angular momentum are transported through the re-connection process. We examine the island structures near the singular surface with and without shear reversal. The structural change of the islands in the case of the shear reversal leads to the inhibition of the re-connection transport. In the quasi-particle model, the transport occurs through the exchange of the quasi-particles across the singular surfaces. The reversal of the shear leads to the absence of the exchangeable quasi-particles across the singular surfaces and the transport is inhibited.

This model may also explain the H-mode. Actually the characteristics of the H-mode gives a strong support of this model.

The heating power threshold as a function of the plasma density and the toroidal magnetic field is interpreted as the condition that the β value at the edge of the plasma exceeds the ratio of the electron and the ion mass. The electrostatic modes become the electromagnetic modes and the quanta with 0 spin change to the quanta with spin 1. Secondly the local magnetic shear reverses at the outboard side of the torus. The structure of the flux tube parallel to the perturbation changes due to the local shear reversal similar to the ITB case. Since the barrier is present only in the outboard side, the confinement is not as good as that of ITB.

In the following we discuss mhd quanta, magnetic island structure, reconnection process and localized shear reversal.

2] Mhd quanta

We consider two types of the quanta, the Alfven quanta analogous to photons and the magnetic flux quanta analogous to that in superconductors.

Photons carry the energy $\hbar\,\omega$ and the spin $\hbar$ where ω is the angular frequency and $\hbar$ is $h / 2\pi$. When photons are emitted and absorbed, the spin as well as the energy is transported. In general the direction of the spin is parallel or anti-parallel with respect of the direction of the propagation and the effect of the spin transport cancels out averaged over many photons. When light wave is circularly polarized, the spin points either parallel or anti-parallel depending on the polarization is right handed or left handed. The polarized light wave has helicity and can transport the helicity and the angular momentum.

We consider circularly polarized Alfven wave in the direction of uniform magnetic field B_0. The vector potential A is given by

$$A_x = A \cos[-\omega t + k z]$$

$$A_y = \pm A \sin[-\omega t + k z] \qquad [2\text{-}1]$$

where z is in the direction of the uniform static field.

The energy flow, namely Poynting vector P, the helicity H and the angular momentum p are given by

$$P \equiv \mu_0^{-1} E \times B = E^2 k / [\mu_0\, \omega] \qquad [2\text{-}2]$$

$$H \equiv A * B = \mp E^2 k / \omega^2 = \mp \mu_0 P / \omega \qquad [2\text{-}3]$$

$$p \equiv \rho\,[\omega/k]\, r \times [\, E \times B_0]/\, B_0^{\,2} = \mp E^2 k \,/\, \mu_0\, \omega = \mp P\,/\omega \qquad [2\text{-}4]$$

where ρ is the mass density and r is the displacement. The above relationship shows that the spin of the quasi-particle is unity.

To represent vibrating mhd fluid, we superpose Alfven waves propagating in the opposite direction. The standing wave may be produced either placing reflectors or closing the flux lines. The total spin and helicity depends on the combination of the spin relative to the direction of the propagation. An example is

$$A_x = A \cos k z \cos \omega t$$

$$A_y = A \sin k z \cos \omega t \qquad [2\text{-}5]$$

The helicity and the angular momentum are given by

$$H = - A^2 k \cos^2 \omega t$$

$$p = 0 \qquad [2\text{-}6]$$

The time averaged helicity is finite.

In toroidal configurations, mhd perturbations may be viewed as vibrating magnetic flux lines. On the singular surfaces, the mode with the mode numbers n and m is essentially the vibration of the closed flux line. Since we are interested in the radial transport, it is more convenient to express the vibrating flux lines as the flux quanta rather than the Alfven wave quanta. It is equivalent of considering the wave packet of the magneto-sonic wave. We define the flux Φ by

$$\Phi \equiv \int B \; dS \qquad [2\text{-}7]$$

where B is the magnetic field and dS is the cross-sectional area perpendicular to the magnetic field. The flux tube contains the physical quantities λ integrated over the volume of the flux tube V,

$$\Lambda = \int \lambda\, d\, V = \int \lambda\, L\, d\, S \qquad [2\text{-}8]$$

where L is the length of the flux tube.

In the limit of the ideal mhd, the flux lines are frozen in the plasma. The motion of the flux tube will carry Λ with it. The reconnection process is interpreted as the exchange of the flux tubes between the adjacent radii. The exchange will leave the magnetic configuration largely undisturbed but the plasma quantities may be transported radially.

3] Singular surface and magnetic island

We approximate the torus with a straight cylinder with the length 2π R. The helicity H_0 and the rotational transform q^{-1} are given by

$$H_0 = A_{\theta 0} B_{\theta 0} + A_{z0} B_{z0}$$

$$q^{-1} = R B_{\theta 0} / r B_{z0} \qquad [3\text{-}1]$$

where the subscript 0 denote the unperturbed quantities.

The radial gradients become

$$d H_0 / d r = r A_{\theta 0} d [B_{\theta 0} / r] / d r + A_{z0} d B_{z0} / d r \qquad [3\text{-}2]$$

$$d q^{-1} / d r = [R / B_{z0}^2] \{ B_{z0} d [B_{\theta 0} / r] / d r - d B_{z0} / d r B_{\theta 0} / r \}$$

$$= [R H_0 / r A_{\theta 0} B_{z0}] [H_0^{-1} d H_0 / dr - B_{z0}^{-1} d B_{z0} / dr] \qquad [3\text{-}3]$$

The last term in eq[3-3] is very small because of small β. The location of the extrema of the helicity and the rotational transform are very close.

The plasma is perturbed by helically symmetric vector potential A_1 given by

$$A_1 = A_1[r,t] \exp [i \phi]$$

$$\phi = m \theta + k z, \qquad k = n / R \qquad [3\text{-}5]$$

At the radius where

$$m B_{\theta 0} + k r B_{z0} = 0$$

the perturbation becomes parallel to the magnetic flux lines. The flux surface is called the singular surface and it plays the important role in determining the mhd stabilities.

The perturbed flux surfaces may be labeled by the helical flux function ψ, given by

$$\psi \equiv k r A_\theta - m A_z \qquad [3\text{-}6]$$

Under this perturbation the helical canonical momentum p_h of a particle is a constant of motion.

$$p_h = M [kr v_\theta - m v_z] + e \psi \qquad [3\text{-}7]$$

For slowly varying perturbations, the plasma temperature and the density are functions of ψ.

We first consider the case where the singular surface has a finite magnetic shear. The helical flux function ψ_0 of the unperturbed state has an extremum at the surface. With the helical perturbation, the magnetic islands are formed as shown in Fig-1. The global flux surfaces across the singular surface may have the same helical flux function values. However the physical quantities on the flux surfaces with the same value of the helical flux function are not necessarily the same since the surfaces are not connected. In other words, if a physical quantity is written as a function of the helical flux function, two separate functions must be used for the regions across the singular surface.

Now small non-symmetric perturbations are added to the symmetric configuration. The effects are greatest near the saddle point between the islands. The reconnection of the flux lines occurs most readily between the flux surface with the same value of the helical flux function, namely between the global flux surfaces. Since the helical flux function values inside the islands differ from the values of the global flux surfaces the islands are little disturbed. The physical quantities on the flux surfaces with the same flux function value that are described by different functions are averaged by the reconnection. The transport occurs between the global flux surfaces.

In the quasi-particle description, the reconnection process is the exchange of the flux quanta. At the saddle points the flux quanta may be exchange with minimal disturbance to the magnetic energy because the helical flux function has extremum there. The exchange of the flux quanta that contains different physical quantities leads to the transport.

When the singular surface coincides with the shear-reversal surface, namely the extremum of the rotational transform, the situation is quite different. The second derivative of ψ_0 vanishes and the island structure is different as shown in Fig-2. Every other islands have the opposite value of the helical flux function relative to ψ_0. As a result the saddle points connect the global flux surfaces to the island flux surfaces. The reconnection and the transport are between the global flux surfaces to the island flux surfaces. There is no reconnection between the global flux surfaces across the singular surface. Every other islands equilibrate with either the plasma inside or out side the singular surface. The radial transport is thus inhibited.

In the quasi-particle description, the exchange of the flux quanta is limited to between the global flux surfaces and the island flux surfaces. We might loosely interpret the structural inhibition of the transport as equivalent of the statement that the exchange of the flux quanta requires the gradient of the helicity or the rotational transform.

4] Local shear reversal and H-mode

In toroidal geometry, the outboard and the inboard side have different characteristics. The outboard side tends to be more unstable as evidenced by the ballooning modes and the trapped particle modes. As poloidal β-value increases, Shafranov shift or equivalently Pfersch-Schlueter current tends to increase the poloidal magnetic field in the outboard side. The local shear s_L defined by

$$s_L = r\, d\, [\, r\, B_t / R\, B_p\,] / d\, r \qquad [4\text{-}1]$$

where B_t and B_p are the local toroidal and poloidal fields, may reverse when the gradient of the poloidal β becomes large.

On the singular surface where the local shear reversal occurs the island structure in the outboard side looks like the structure shown in Fig-2. In the inboard side the shear reversal does not occur and the pattern is the one shown in Fig-1. Every other islands, namely the ones with the saddle points at the outer radius become small and disappear going from the outboard to the inboard side.

When the magnetic reconnection process takes place, the reconnection between the global flux surfaces is limited to the inboard side as described in the cylindrical case. Thus the transport in the outboard side is inhibited. Furthermore the turbulence level is expected to be weaker in the inboard side compared to the outboard side where the ballooning and the trapped particle effects are prominent. The size of the islands and the frequency of the reconnection are expected to be smaller in the inboard side. It is likely that the transport through only the inboard side is much less than that of the outboard side without the local shear reversal.

We compare the above model of the H-mode with the experimental observations. First, the process in the model is magnetic [spin one quasi-particle], not electrostatic [spin zero]. The criterion that the turbulence becomes electromagnetic is

$$\beta > m_e / m_i \qquad [4\text{-}2]$$

where m_e and m_i are the electron and the ion masses. The above criterion is equivalent of the condition that Alfven wave velocity is smaller than the thermal velocity of the electron.

The threshold of the transition from L-mode to H-mode is usually stated in terms of the required heating power as a function of the plasma density and the toroidal magnetic field. The threshold power is proportional to the toroidal magnetic field strength and $-1/2$ power of the density. The relationship between the heating power to the edge temperature is not clear. Nevertheless the measured β values at the edge of the H-mode plasma appear to just satisfy the above criterion. L-mode edge plasma does not satisfy the criterion.

Second, the edge plasma in H-mode is close to or exceeds the threshold to enter the second stability regime for the ballooning mode. The local shear reversal is the necessary condition for the second stability. A rough estimate of the condition for the local shear reversal is given by

$$R\, d\beta_p / dr > R / r \qquad [4\text{-}3]$$

where β_p is the poloidal β. Because of very sharp pressure gradient, the edge plasma of the H-mode appears to satisfy the above condition.

The both conditions are self re-enforcing. When the confinement improves, β and grad β_p increase and vice versa. The bifurcation of the edge region between H-mode and L-mode occurs. Since the condition on β and grad β_p are related but different, the transitions between H and L modes may behave differently depending on one condition is violated or both are. The observation of different types of the edge localized modes [ELM] might be a manifestation of these phenomena.

Third, The H-mode confinement is better than the L-mode confinement but the scaling of the confinement time on the plasma parameters is similar. In the model, the transport barrier is not present in the inboard side. It is expected that the transport is reduced but the scaling remains unchanged. Also the confinement of the H-mode is not as good as ITB confinement. It may be explained as the difference between the complete barrier and the partial barrier. The other characteristics such as large shear in the rotational velocity with the associated gradient of the radial electric field are not unique for the model. Nevertheless the experiments support the model both in ITB and H-mode.

5] Stellerators

The question is whether the phenomena such as ITB and H-mode are possible in stellerators according to this model.

The rotational transform in current free stellerators is imposed by external vacuum magnetic field. The helicity created by the external field is more rigid compare to the helicity of tokamak configurations. As a result the extremum of the helicity is difficult to obtain by knob twisting of the plasma conditions.

The creation of the extremum by external field requires two sets of helical windings with the opposite handedness. The loss of quasi-helical symmetry may be troublesome in constructing robust flux surfaces. An alternative is the configurations with a helical magnetic axis. The transform by the helical winding is made to oppose the transform due to the helical magnetic axis. The robustness of the flux surface in this type of the configurations remains to be seen.

As for H-mode, it is unlikely to occur in stellerators. Shafranov shift increases the local transform in the outboard side. Since the rotational transform increases with plasma radius in stellerators, a large gradient of poloidal beta tends to increase local shear. Therefore the reversal of the local shear is unlikely.

6] Summary

A model of the transport in ITB, H-mode and L-mode of tokamaks is presented. The transport is caused by the reconnection of the magnetic flux lines on the singular surface. In terms of the quasi-particle description, the transport occurs due to exchange of mhd quanta [spin 1] across the singular surfaces. The transport rate is dependent on the magnetic helicity profile and the extrema of the helicity inhibit the transport. In contrast electrostatic turbulence [spin zero] do not have the helicity dependence. The observations in ITB and H-mode experiments appear to support the model.

7] Acknowledgement

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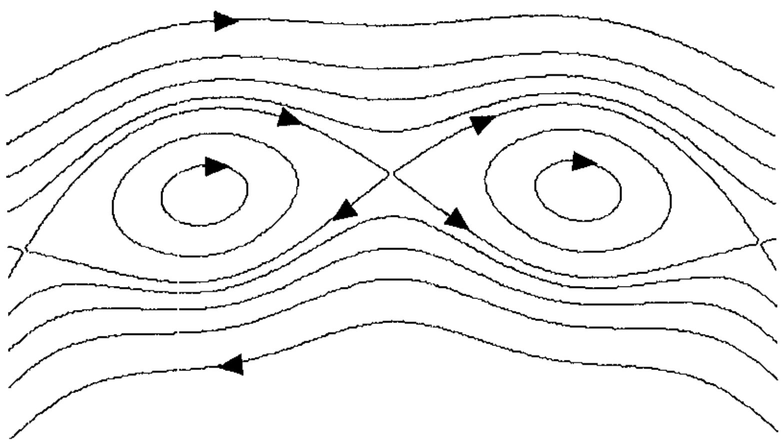

Fig.1 Magnetic islands with a finite shear

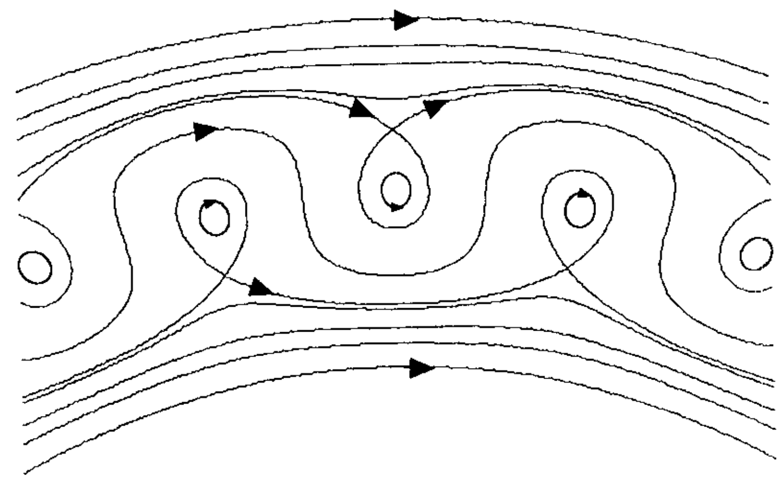

Fig.2 Magnetic islands with shear reversal

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