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A periodic motion embedded in plane Couette turbulence

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A flow between two parallel plates which move with a constant velocity in opposite directions becomes turbulent at the Reynolds number above some critical value if it starts with a strongly disturbed state [1, 2]. This is called the plane Couette turbulence [3, 4], the fluid motion in which is chaotic and never repeated. Nevertheless, it is known that the regeneration cycle [5, 6] is present to sustain near-wall coherent structures such as streamwise vortices and low-velocity streaks though its theoretical description has not been established. Here we report a periodic motion, discovered by solving the Navier–Stokes equation iteratively, which describes a full cycle of repetition of a series of dynamical processes including the formation and breakdown of coherent structures. Since it is unstable, this periodic motion is not attained in reality. However, the turbulent state spends most of the time around it. As a result, the mean velocity profile as well as the root-mean-squares of velocity fluctuations of the Couette turbulence coincide remarkably well with the temporal averages of the corresponding quantities of the periodic motion.

KEYWORDS: Periodic motion, Couette turbulence

Since the monumental experiments on transition to turbulence performed by Reynolds [7] in 1883 many efforts have been devoted to understanding and controlling of fluid turbulence. However, the lack of a simple spatiotemporal characterization of such a large irregular system with strong nonlinearity has been making it difficult to elucidate the structural and dynamical properties of turbulent flows. Indeed complex and chaotic behaviour in both space and time is the primary characteristics of fully-developed turbulence, but on other hand, there is accumulation of experimental and numerical evidence of the existence of striking coherent motions and structures [8, 9]. Coherent structures are known, at least qualitatively, to play key roles in the transport [10] of passive materials, momentum and energy, the maintenance [11] of turbulence activity, the building of intermittent fluctuations [12], and so on and so forth. The presence of the coherent structures can be quite helpful to understanding of turbulent flows because they exhibit simpler behaviour than turbulence itself.

The number of active modes in any turbulent motions of a viscous fluid in a finite domain is always finite, since the small-scale motions are smoothed out and attenuated by viscosity [13]. Therefore, a turbulent flow may be regarded as a dynamical system of finite dimension. In terms of dynamical systems theory, coherent structures in turbulence may be thought [14] of as being a lower-dimensional manifold, in the neighborhood of which the dynamical system spends a substantial fraction of time. In chaotic systems [15, 16] of lower degrees of freedom the infinite number of unstable periodic orbits, which are known [17] to be embedded in a chaotic attractor, have been shown to provide a useful measure of characterization of the structure and dynamics of the attractor. In this context, the possible simplest description in phase space of coherent structures in turbulence should be given by a periodic saddle orbit embedded in a turbulent attractor, though it is much more difficult to find an unstable periodic orbit in higher-dimensional systems such as fluid turbulence.

In their numerical simulations of plane Poiseuille turbulence, which is a pressure-driven turbulent flow between parallel plates, Jiménez & Moin [18] minimized the streamwise and spanwise dimensions of a computational periodic box so that turbulence could be sustained, and they succeeded in reducing the degrees of freedom of the flow while maintaining the turbulence activity. By using the same numerical technique, Hamilton, Kim & Waleffe [5] decreased the Reynolds number further to reduce the degrees of freedom of plane Couette turbulence which is driven by two parallel plates moving with different velocities. In this highly constrained plane Couette turbulence they reported the recurrent formation and breakdown of near-wall coherent structures, such as streamwise vortices and low-velocity streaks, in a qualitative sense.

An unstable periodic orbit, if exist, may be relatively easily obtained in the above-mentioned constrained turbulence of lower degrees of freedom. We have therefore performed direct numerical simulations of the incompressible Navier–Stokes equations by using a spectral method for the same constrained Couette turbulence. The Fourier expansions are employed in the streamwise (x) and spanwise (z) directions, and the Chebyshev-polynomial expansion in the wall-normal (y) direction. Numerical computations are carried out on 8,448 grid points ($16 \times 33 \times 16$ in x, y, and z) at Reynolds number $Re \equiv Uh/\nu = 400$, where U stands for half the difference of the two wall velocities, h is half the wall separation, and ν is the kinematic viscosity of fluid. The streamwise and spanwise computational periods are $L_x = 5.513h$ and $L_z = 3.770h$, respectively. The grid resolution is taken to be sufficiently fine to resolve the smallest active scales of motions. The energy is injected through the frictional force on the moving plates and consumed at small scales over the whole flow field by viscous dissipation. The energy input I and dissipation D per unit time vary in a complicated way in time, but their temporal averages, which are substantially larger than the corresponding ones in a laminar state, are the same since the turbulence is statistically stationary.

In the present numerical scheme the dependent variables are 31 Chebyshev coefficients for the mean streamwise and spanwise components of velocity, $7,424$ ($= 16 \times 29 \times 16$) Fourier-Chebyshev-Fourier coefficients for the wall-normal velocity and $7,936$ ($= 16 \times 31 \times 16$) Fourier-Chebyshev-Fourier coefficients for the wall-normal vorticity. The resulting number N of degrees of freedom of the present dynamical system is therefore 15,422. An instantaneous state of the flow field and its temporal evolution should be represented respectively, in principle, as a point and its trajectory in the N-dimensional phase space spanned by all the independent variables. Unfortunately, however, it is not possible to represent the entire phase space on a single paper but only a two-dimensional subspace. In Fig. 1, we plot, with a grey

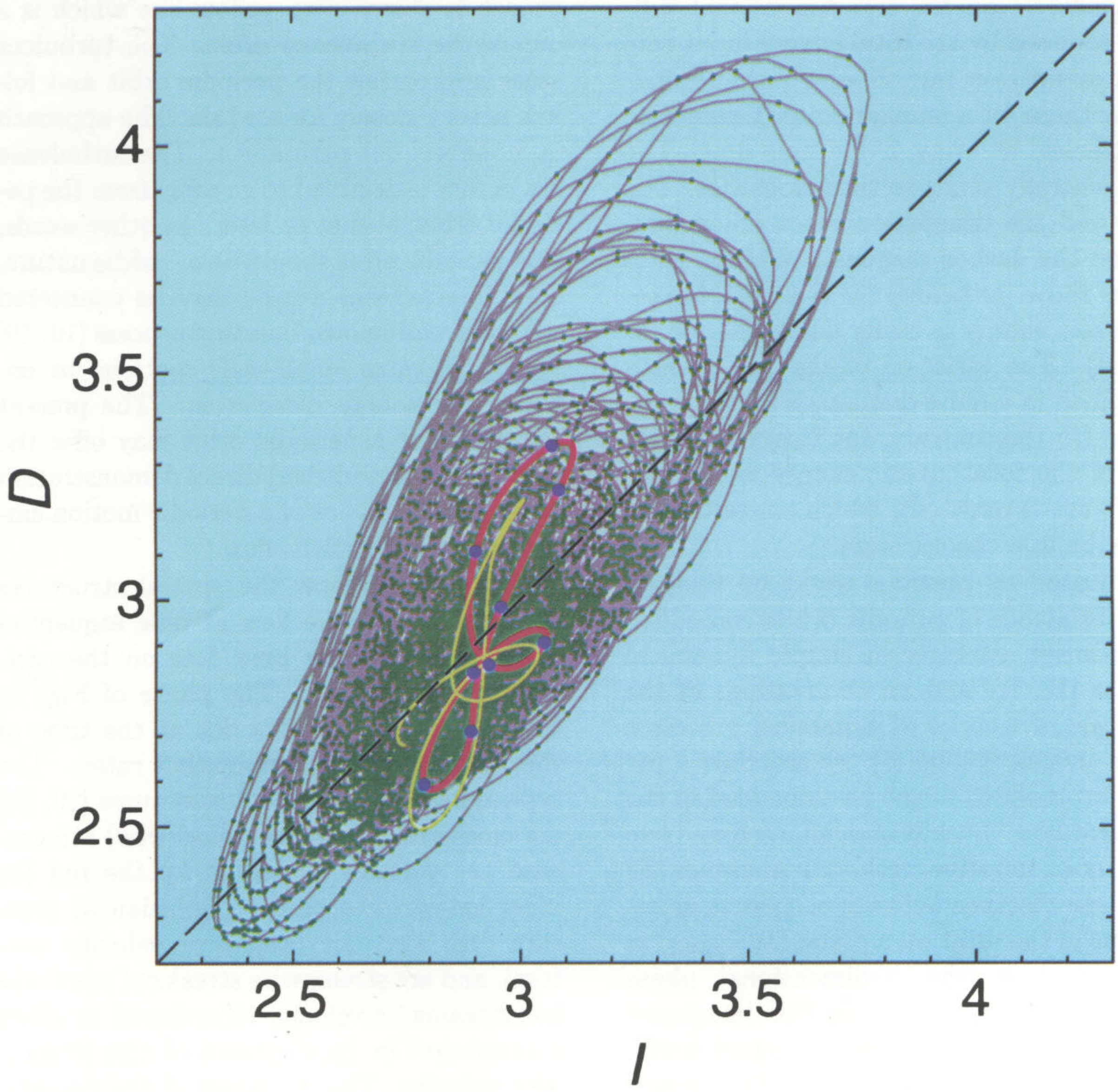

Figure 1: Two-dimensional projections of a turbulent and a periodic orbits. The horizontal and vertical axes respectively represent total energy input rate I and dissipation rate D normalized by those for a laminar state. The grey line stands for the turbulence trajectory, to which green dots are attached at $2h/U$ time intervals.. A closed red line denotes the periodic orbit. A cut of the turbulence trajectory is coloured yellow to show a typical approach to the periodic orbit. Nine blue dots on the periodic orbit indicate the phases of panels **a**–**i** in Fig. 2. The dashed diagonal denotes the equilibrium between the energy input and dissipation.

line, a projection of the orbit over a period of $10,000h/U$ on the two-dimensional subspace spanned by the total energy input rate I and dissipation rate D which are normalized by those for a laminar state. Green dots are attached at every $2h/U$ time unit. The orbit generally tends to turn clockwise. The input and the dissipation rates are in balance on the dashed diagonal. While a state locates above (or below) the diagonal, the total kinetic energy is being decreased (or increased). The variation of the orbit, which is confined in a finite domain, is far from periodic. On the contrary, the frequency spectrum of the total kinetic energy is continuous (figure is omitted), which suggests that it may be in a chaotic state.

Motivated by previous works on findings of the existence of periodic orbits embedded in a strange attractor in simple dynamical systems [15, 16] and on observations of the repetition of a series of dynamical processes in the present system [5], we searched a periodic orbit which might be embedded in the turbulent flow. Luckily enough, we found one by use of an iterative method to minimize the Euclidean distance between successive cross-sections of the orbit on a plane $\mathrm{Im}(\tilde{\omega}_{y\,0,0,1}) = -0.1875U/h$ in the N-dimensional phase space, where $\mathrm{Im}(\tilde{\omega}_{y\,0,0,1})$ is the imaginary part of the Fourier-Chebyshev-Fourier coefficient of the wall-normal vorticity for the zero streamwise wavenumber, the zero-th order Chebyshev polynomial, and the $2\pi/L_z$ spanwise wavenumber. A state in a period during which the turbulence trajectory is travelling more or less periodically in the phase space was chosen as the first guess for the iteration. The iteration was continued until the distance between successive cross-points fell within 1% of the distance from the origin of the earlier cross-point. A periodic orbit thus obtained is drawn with a closed red line, the period of which is $64.7h/U$. Green dots on the turbulence trajectory crowd much densely near the periodic orbit, implying that the turbulent state often approaches the periodic orbit. An example of such close approach is shown with yellow line which is a cut of the turbulence orbit. The turbulent state approaches the periodic orbit and follow it very closely for a while. The approach is, however, not permanent. The turbulence trajectory is destined to go away from the periodic orbit, sooner or later. In other words, this periodic orbit should be of saddle nature. This intermittent escape may be connected with the well-known bursting process [10, 19] which activates small-scale motions to enhance the energy dissipation. The present extraction of a periodic orbit may offer the first (to our knowledge) direct demonstration of the real existence of a periodic motion embedded in a turbulent flow.

Figures 2a–i show the spatial structures of the time-periodic flow at nine sequential phases indicated by blue dots on the periodic orbit in Fig. 1. The phase of Fig. 2a corresponds to the blue dot at the time of the least input and dissipation rates. The typical near-wall coherent structures [20, 21] are clockwise (or counter-clockwise) streamwise (x) vortices visualized by the red (or blue) iso-surfaces of the Laplacian of pressure (see also the cross-flow velocity vectors), and are streamwise streaks of relatively low streamwise velocity represented by lifted iso-contours in (y, z)-planes of the streamwise velocity. The dynamics of the periodic flow is characterized by a cyclic sequence of events which is composed of (i) the formation and development of the low-velocity streak through the advection of streamwise velocity in the cross-flow induced by decaying streamwise vortices (Fig. 2a-d), (ii) the bending along the streamwise direction and tilting in the spanwise (z) direction of the streak followed by the regeneration of streamwise vortices (Fig. 2e-g), and (iii) the breakdown of streaks and the violent development of streamwise vortices (Fig. 2h, i). This cyclic sequence is completely consistent with the previously reported regeneration cycle [5, 22] of coherent structures in turbulent plane

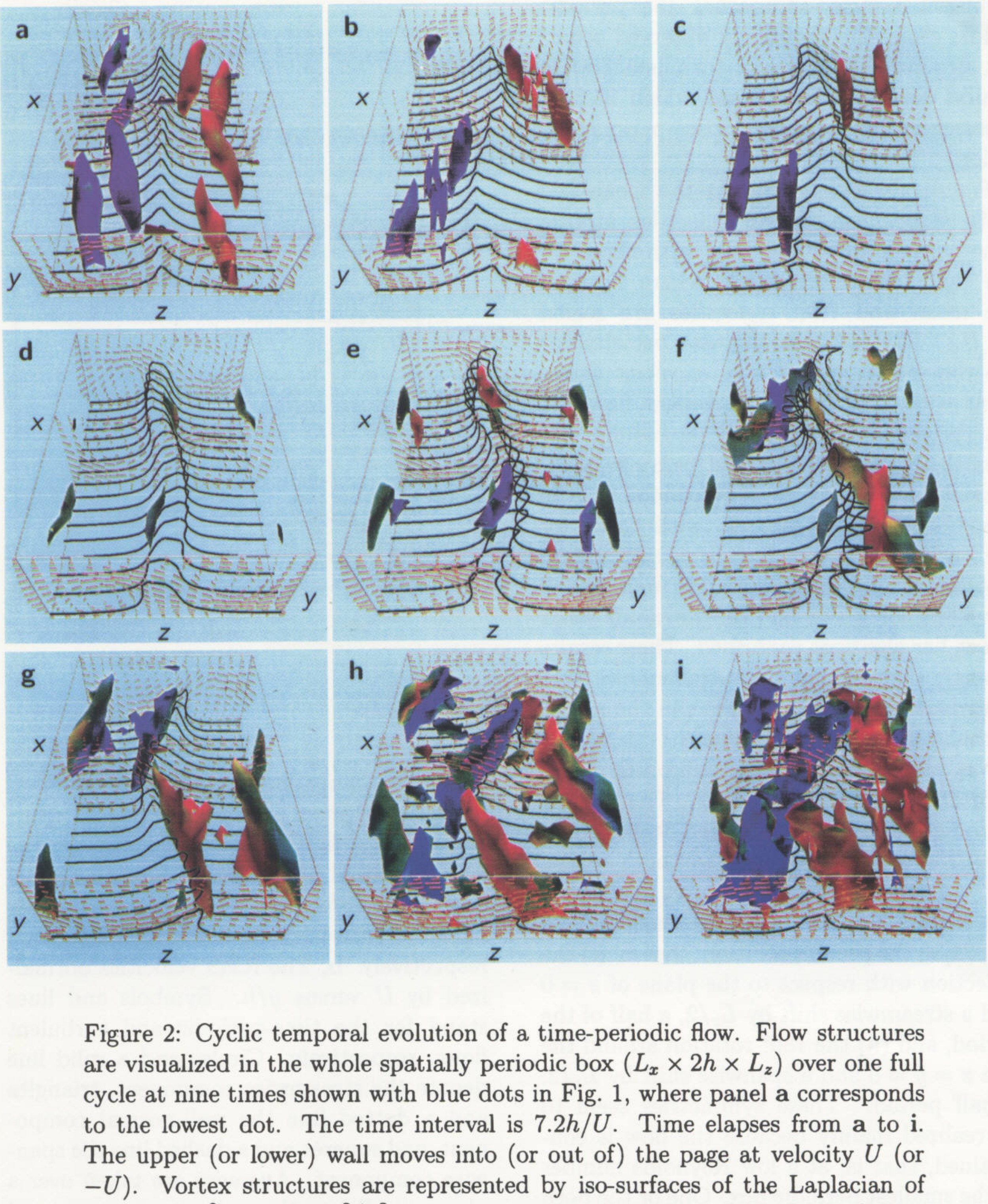

Figure 2: Cyclic temporal evolution of a time-periodic flow. Flow structures are visualized in the whole spatially periodic box ($L_x \times 2h \times L_z$) over one full cycle at nine times shown with blue dots in Fig. 1, where panel **a** corresponds to the lowest dot. The time interval is $7.2h/U$. Time elapses from **a** to **i**. The upper (or lower) wall moves into (or out of) the page at velocity U (or $-U$). Vortex structures are represented by iso-surfaces of the Laplacian of pressure, $\nabla^2 p = 0.15\rho U^2/h^2$, where ρ is the mass density of the fluid. Colour on the iso-surfaces of $\nabla^2 p$ denotes the direction of the streamwise (x) vorticity: red is positive (clockwise) and blue is negative (counter-clockwise). Cross-flow velocity vectors and contours of the streamwise velocity at $u = -0.3U$ are also shown on planes $x = const.$

Couette flows. It turns out that when the turbulent system approaches the periodic saddle orbit, the commonly observed coherent structures appear in a turbulent flow to exhibit recurrent behaviour, which is fully represented by the present time-periodic solution.

Figures 3a and b compare the mean and RMS (root-mean-square) velocities for the time-periodic flow (symbols) with those for the turbulent flow (lines), where both of the mean and RMS velocities are scaled by U. The mean streamwise velocity for the time-periodic flow is in excellent agreement with that for the turbulent flow. It is surprising and remarkably interesting that even the RMS velocities of streamwise, wall-normal, and spanwise components for the time-periodic flow coincide with those for the turbulent flow. Excellent coincidence in other second-order statistics, namely, all the RMS vorticities and the Reynolds shear stress, has also been confirmed. These results strongly suggest that the turbulent state actually spends most of the time in the neighborhood of the periodic orbit.

Not only spatial but also temporal coherence exists in turbulent flows. The present periodic solution to the Navier–Stokes equation gives a concrete example of both spatial and temporal coherent structures in a turbulent flow. There are two spatial symmetries [23, 24] in the present solution, that is, (i) the reflection with respect to the plane of $z = 0$ and a streamwise shift by $L_x/2$, a half of the period, and (ii) the 180° rotation around the line $x = y = 0$ and a spanwise shift by $L_z/2$, a half period. These symmetries seem to be realized mainly because the flow is constrained, that is, at a low Reynolds number in the smallest periodic box. One or the both will be broken at a higher Reynolds number or in a larger periodic box. Nevertheless, the present periodic solution may provide us with a simple spatiotemporal characterization of turbulence, which is a crucial base for understanding of turbulent flows, at least

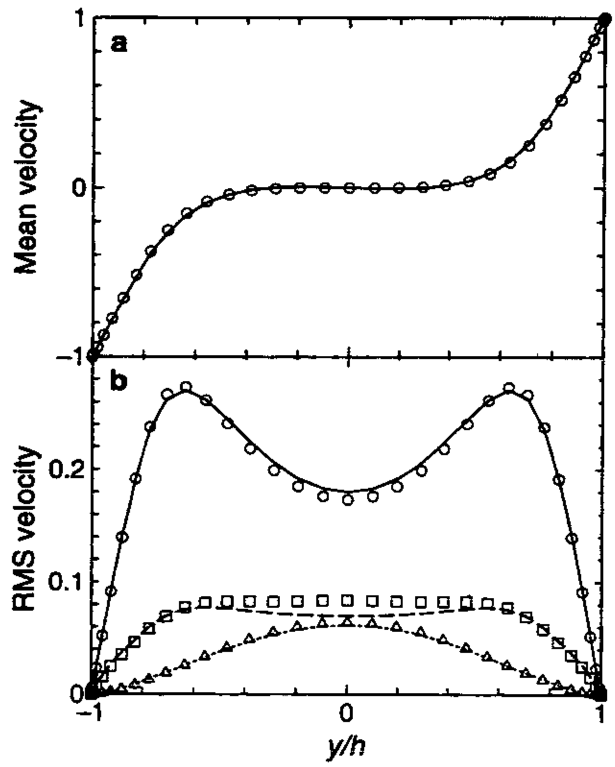

Figure 3: Comparison of the mean and RMS velocities between a time-periodic and a turbulent flows. **a**, The mean streamwise velocity normalized by U versus the wall-normal coordinate y/h. Circles and a solid line denote the time-periodic and turbulent flows, respectively. **b**, The RMS velocities normalized by U versus y/h. Symbols and lines stand for the time-periodic and turbulent flows, respectively. Circles and a solid line denote the streamwise component, triangles and a dotted line the wall-normal component, and squares and a dashed line the spanwise component. Averages are taken over a plane parallel to the walls, $y = const.$, and over one time period $64.7h/U$ for the time-periodic flow or time $60,000h/U$ for the turbulent flow.

in the constrained plane Couette flow.

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