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Universal Instability of Dust Ion-Sound Waves and Dust-Acoustic Waves

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Abstract

It is shown that the dust ion-sound waves (DISW) and the dust-acoustic waves (DAW) are universally unstable for wave numbers less than some critical wave number. The basic dusty plasma state is assumed to be quasi-neutral with balance of the plasma particle absorption on the dust particles and the ionization with the rate proportional to the electron density. An analytical expression for the critical wave numbers, for the frequencies and for the growth rates of DISW and DAW are found using the hydrodynamic description of dusty plasma components with self-consistent treatment of the dust charge variations and by taking into account the change of the ion and electron distributions in the dust charging process. Most of the previous treatment do not take into account the latter process and do not treat the basic state self-consistently. The critical lengths corresponding to these critical wave numbers can be easily achieved in the existing experiments. It is shown that at the wave numbers larger than the critical ones DISW and DAW have a large damping which was not treated previously and which can be also measured. The instabilities found in the present work on their non linear stage can lead to formation of different types of dust self-organized structures.

Key words: dusty plasmas, waves, instabilities

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1 Introduction

The DISW and DAW were considered earlier in different papers (see [1-7] but without self-consistent definition of the basic state in which the waves propagate. The DISW have the phase velocities larger than both the dust and the ion thermal velocity and less than the electron thermal velocity, the DAW waves have the phase velocities less than both the electron and the ion thermal velocity but larger than the dust thermal velocity. Both of them are usually described by approximately linear dispersion law $\omega_{disw)} = kv_{disw}; \omega_{daw} = kv_{daw}$ where the v_{daw} is the DISW speed (estimated often as $\sqrt{T_e/m_i}$) and v_{daw} is the DAW speed (estimated often as $\sqrt{T_i Z_d P/m_d}$ where T_e, Ti are the electron and ion temperature respectively, m_i, m_d are the ion and dust masses respectively Z_d is the dust charge in units of electron charge and $P = Z_d n_d/n_i$, n_i, n_d are the ion and the dust densities respectively. We will give more detailed expression in the paper from which it will be possible to make a more precise estimates of the phase velocities of these waves. Here we first give some important references on the previous works. First of all the presence of two branches of sound waves in multi-component plasmas was known by years and is described in textbooks [8] and the first papers on this subject [1-3] just uses the multi-component approach. The main difference of the dusty plasma from the multi-component plasma is that the charge on dust particle is not fixed and its perturbations are different for different wave-number. This is usually called as charge variations. Several investigations were devoted to the influence of charge variations on DISW and DAW [9-12]. But there exist more important effect related with dust charging which is also necessary to take into account. In most of investigations of the influence of dust charge variations there was not taken into account that the same process which makes the dust charge to change influences the distributions of electrons and ions taking part in the charging process (first steps to take into account this effect were made in [10][13-16]). Thus to treat the problem self-consistently one need to take into account both the changes in the dust charges and the related with these changes the variations of the electron and ion distributions. In the present paper we do this self-consistent treatment. But there exist also another important difference of dusty plasmas from multi-component plasmas, namely the definition of the ground state on which the disturbances appear which propagate as DISW or DIAW is different for dusty plasmas and for multi-component plasmas. Usually in multi-component plasmas this state is defined by condition of quasi-neutrality which in application for dusty plasmas will look like $n_i = n_e + n_d Z_d$ and this expression was used in many papers devoted to the treatment of DISW and DAW in a way they are treated in multi-component plasmas approach [1-5]. But in dusty plasmas this relation is *not sufficient* for definition of the ground state. Both the electrons and the ions are absorbed on the dust grains in the charging process. Therefore for steady-state conditions there should exist a source of electrons and ions to compensate this absorption and to keep the electron and the ion densities constant[10],[15][17]. This source can be called as ionization source if it works in the whole volume of dusty plasmas. Experiments indeed show that dusty plasma just disappears in very short time scales being absorbed on dust if the electron and ion densities are not supported by some source. In experiments performed in HF low temperature plasmas [18-21] the plasma is absorbed on time-scales of micro-seconds if the HF source is switched off. The necessity of the presence of external source was emphasized and taken into account in formulation of kinetic theory of dusty plasmas [15,22-24]. Without a source it is not possible even to define the ground state of dusty plasmas and to introduce the perturbations of it. This difference of dusty plasmas from multi-component plasmas leads to expressions for DISW and DAW quite different from that of multi-component plasmas. The condition for balance of the parameters in the ground state fixes the other parameters of the system. The first treatment of DISW with source the strength of which is independent on plasma parameters was made in [10,15,25], but in these papers not all processes which influences the DISW were taken into account, namely although the absorption of electrons and ions on dust particles was treated self-consistently, the elastic scattering of electrons and ions on dust particles was not taken into account. It is indeed not important for $\tau = T_i/T_e \gg 1$. The present laboratory experiments are usually performed for opposite condition $\tau \ll 1$ where the elastic collisions dominate. Thus the result of [10] are correct for $\tau \gg 1$ and at least can give qualitatively correct expressions for $\tau \approx 1$ which is often met in astrophysical dusty plasmas. In the present consideration we will deal mainly with the case $\tau \ll 1$ having in mind the laboratory applications. The DISW and DAW where considered in detail [25] by a kinetic approach including the case $\tau \ll 1$ but again as in [10] there was assumed that the external source is independent on the plasma parameters. A substantial differences between the expressions for DISW and DAW waves found in [25] and that of multi-component approach was clearly demonstrated in [25].

In this paper we consider the model which is much closer to the experimental situation assuming that the ionization source intensity is proportional to the electron density. This case was also considered in several non-linear treatments but explicit expressions for DISW and DAW were not investigated. These are of interest because of many experiments [26-33] which are going for measurements both the dispersion and the damping of the DISW's and the DAW's. The correct expressions are needed for comparison of the theoretical predictions with results of these experiments. By investigating the dispersion relation for self-consistent treatment of the charge variation and the plasma particle distribution changes with an ionization source which is proportional to the electron density we found here that for a certain wave numbers less or of the order of the critical wave number the DISW and DAW branches are universally unstable and that for wave numbers larger than another critical wave number (usually larger than the first critical wave number) there exist an appreciable damping of both the DISW's and the DAW's, which was previously not recognized. The reason for appearance of both the instabilities and the dissipation is the deviations in the balance of the dissipation on dust and ionization caused by the wave perturbation of the ground state. Without the condition of balance the ground state cannot be defined and therefore one cannot consider the perturbations and cannot treat both the DISW and DAW without treating such a balance in the ground state . At the end of the paper we discuss the experimental possibilities of measuring of the calculated effects.

2 Basic state

We use the dimensionless notations

$$\mathbf{E} = \frac{\mathbf{E}^{ac}\lambda_{Di}^2}{aT_e}; n = \frac{n_i^{ac}}{n_0}; n_e = \frac{n_e^{ac}}{n_0}; P = \frac{n_d Z_d}{n_0}; \mathbf{u} = \frac{u_i^{ac}}{\sqrt{2}v_{Ti}}; \mathbf{u}_d = \frac{\mathbf{u}_d^{ac}}{\sqrt{2}v_{Ti}}; z = \frac{Z_d e^2}{aT_e}; a = \frac{a^{ac}}{\lambda_{Di}}$$

$$\mathbf{k} = \mathbf{k}^{ac}\frac{\lambda_{Di}^2}{a}; \omega = \frac{\omega^{ac}\lambda_{Di}^2}{a\sqrt{2}v_{Ti}}; \tau = \frac{T_i}{T_e}; \tau_d = \frac{T_d}{T_e Z_d}; \mu = \frac{Z_d m_i}{2m_d}; v_{Ti}^2 = \frac{T_i}{m_i}; \lambda_{Di}^2 = \frac{T_i}{4\pi n_0 e^2} \quad (1)$$

where the superscript ^{ac} is used for the actual values, n_0 is the ion density in the basic state, T_i, T_e, T_d are the ion, electron and dust temperatures respectively, assumed to be constant, n_i, n_e, n_d are the ion, electron and dust densities respectively, m_i, m_d are the ion and the dust masses respectively, $\mathbf{E}$ is the strength of the electric field, a is the dust size.

In what follows we will consider only the case applicable to existing dusty plasma experiments and assume that

$$\tau \ll 1 \quad (2)$$

In these notations for the case where the relation (2) is valid the rate Q_{abs} of numbers of ions absorbed on dust particles in a unit volume per unit time can be calculated as

$$Q_{abs} = \alpha_{ch} P n \quad (3)$$

were (for the case where the Orbit Motion Limited(OML) approach is used for the cross-sections) the charging coefficient is simply $\alpha_{ch} = 1/2\sqrt{\pi}$. Having in mind the possibility of using other models for calculation of the charging coefficient, we leave the expression for Q_{abs} with an arbitrary constant coefficient α_{ch}. For small deviations from the basic state one can still use the expression (2) with constant charging coefficient since the corrections to it related with small ion drift motions existing in perturbations is proportional to the square of the drift velocity and therefore can be neglected in the linear approximation.

The ionization rate we assume to be proportional to the electron density and write the power of ionization in the dimensionless units in the form

$$Q_{ion} = \frac{n_e}{\tau_{ion}} \tag{4}$$

where τ_{ion} is the characteristic ionization time in dimensionless units (from (1) it is easy to find its meaning in conventional units - it is the number of electron-ion pairs created by the ionization source per one electron in a unit volume during the characteristic dimensionless time).

The two equations of the basic states will be the charge neutrality and the balance of absorption and ionization powers. The values of the initial basic state we denote by subscript $_0$. We have

$$n_{e0} = 1 - P_0 \tag{5}$$

$$\frac{1}{\tau_i} = \frac{P_0 \alpha_{ch}}{n_{e0}} \tag{6}$$

Using (5) we can calculate the dust charge z_0 in the basic state from OML charging equation

$$\exp(-z_0) = \sqrt{\frac{m_e}{m_i \tau}} \frac{z_0}{1 - P_0} \tag{7}$$

We can use P_0 as a single parameter determining the basic state through which all other parameters n_{e0}, τ_i and z_0 are expressed, n_0 remains as a parameter of the initial state but it enters only in the normalization of all other variables. It is this advantage of used normalization that allows to describe the basic state by a single parameter P_0.

3 Dispersion relation for DISW

We consider than a linear perturbations of the ground state. First consider the phase velocities which are in the range of DISW, the range already defined earlier. For electrons we use the balance of the electron pressure and the electric field, since the other forces such as friction on dust and electron inertia are negligible. This gives for linear perturbations (all perturbations will be denoted by adding the symbol δ except those which are zero in the ground state)

$$\delta n_e = (1 - P_0)\frac{ie(\mathbf{k}\cdot\mathbf{E})}{k^2} \tag{8}$$

It describes the Debye screening by electrons.

To find the ion density perturbations we have two equation, namely, the first one, describing the balance of forces in which we take into account the ion inertia, ion pressure, the electric field force and the friction on dust and the second one, describing the ion density changes, which is the continuity equation for ion density with perturbations of the Q_{abs} and Q_{ion}. The first equation describes the change of the ion momenta due to the dust drag by ion flux which appears in perturbations of the ground state. This term is very important since it is one of the elements of the self-consistent treatment describing the reaction of ion distribution on interactions with dust particles and gives a necessary feed-back effect which drives the instability. We have

$$e(\mathbf{E}\cdot\mathbf{k}) = \alpha_{dr} P_0 z_0(\mathbf{k}\cdot\mathbf{u}) + \tau i k^2 \delta n - i2\tau\omega(\mathbf{k}\cdot\mathbf{u}) \tag{9}$$

Here the left hand side is the electric field force, the first term of the right hand side is the important term of momentum transfer to ions in dust drag creating the friction of ions proportional both to dust density and dust charge, the second term of the right hand side describes the ion pressure and the last term of the right hand side describes the ion inertia. Different models can be used for the drag coefficient α_{dr}. The most important for drag is the Coulomb scattering of ions on dust particles which gives $\alpha_{dr} = 2\ln\Lambda/3\sqrt{\pi}$ where $\ln\Lambda$ is the Coulomb logarithm which also could take into account the modifications related with the charging process and described in [16].

The second equation, the continuity equation for ions, is

$$i(\mathbf{k}\cdot\mathbf{u} - \omega) = \frac{\delta n_e}{1 - P_0}\alpha_{ch} P_0 - \alpha_{ch} P_0 \delta n - P_0 \alpha_{ch}\frac{\delta z}{z_0} \tag{10}$$

We assumed here that for DISW the dust is not moving and therefore

$$\frac{\delta P}{P_0} = \frac{\delta z}{z_0} \tag{11}$$

The equation (10) takes into account both the dust charge variations (the last term of the right hand side) and self-consistent change of the ion density due to change of ionization in the perturbations (the first term of the right hand side) and the change of ion absorption in the perturbations (the second term in the right hand side). Thus it takes into account the change in ion distribution due to their absorption on dust particles. This term describes the feedback of the ion distribution related with the charging process. It should be taken into account in any self-consistent treatment. The change in ionization is also important since in different regions of the perturbations the electron-ion pairs will be created differently which will mean an increase or decrease of the other forces acting on ions.

We use the OML charging equation corresponding to

$$\frac{\partial z}{\partial t} = \alpha_{ch}\left(\sqrt{\frac{m_i \tau}{m_e}} e^{-z} n_e - zn\right) \tag{12}$$

with substitution of $n_0 + \delta n$ and $n_{e,0} + \delta n_e$ for n_0 and n_{e0} respectively and find

$$\frac{\delta z}{z_0} = \frac{\alpha_{ch}}{(1+z_0)\alpha_{ch} - i\omega}\left(\frac{\delta n_e}{n_{e0}} - \delta n\right) \tag{13}$$

which converts the equation (13) to

$$i(\mathbf{k}\cdot\mathbf{u} - \omega\delta n) = \frac{\alpha_{ch}^2 P_0 z_0}{(1+z_0)\alpha_{ch} - i\omega}\left(\frac{\delta n_e}{n_{e0}} - \delta n\right) \tag{14}$$

The expressions (14) and (9) give the ion density perturbation

$$\delta n = -\frac{ie(\mathbf{k}\cdot\mathbf{E})}{k^2}\frac{\left[1 - \frac{\alpha_{ch}^2 P_0 z_0(\alpha_{dr} P_0 z_0 - 2i\tau\omega)}{((1+z_0)\alpha_{ch} - i\omega)k^2}\right]}{\left[\tau + \left(\frac{P_0 z_0 \alpha_{ch}^2}{(1+z_0)\alpha_{ch} - i\omega} - i\omega\right)\frac{(\alpha_{dr} P_0 z_0 - 2i\tau\omega)}{k^2}\right]} \tag{15}$$

and finally from Poisson's equation in its dimensionless form

$$ie(\mathbf{E}\cdot\mathbf{k}) = \frac{\tau}{a^2}(\delta n - \delta n_e - \delta P) =$$

$$\frac{\tau}{a^2}\left[\delta n\left(1 + \frac{\alpha_{ch} P_0}{(1+z_0)\alpha_{ch} - i\omega}\right) - \frac{\delta n_e}{n_{e0}}\left(1 - P_0 + \frac{\alpha_{ch} P_0}{(1+z_0)\alpha_{ch} - i\omega}\right)\right] \tag{16}$$

we get the dielectric permittivity

$$\epsilon_{\mathbf{k},\omega} = 1 + \frac{\tau}{k^2 a^2}\left\{\left(1 - P_0 + \frac{\alpha_{ch} P_0}{(1+z_0)\alpha_{ch} - i\omega}\right) + \right.$$

$$\left(1+\frac{\alpha_{ch}P_0}{(1+z_0)\alpha_{ch}-i\omega}\right)\frac{\left[1-\frac{\alpha_{ch}^2 P_0 z_0(\alpha_{dr}P_0 z_0-2i\tau\omega)}{((1+z_0)\alpha_{ch}-i\omega)k^2}\right]}{\left[\tau+\frac{1}{k^2}\left(\frac{P_0 z_0\alpha_{ch}^2}{(1+z_0)\alpha_{ch}-i\omega}-i\omega\right)(\alpha_{dr}P_0 z_0-2i\tau\omega)\right]}\Bigg\} \tag{17}$$

Mention that in actual (dimensional) value of $1/k^2a^2$ is $1/k^2\lambda_{Di}^2$ and in the conditions of quasi-neutral disturbances $k^2\lambda_{Di}^2 \ll 1$ one can neglect the first term 1 in the expression (15) and the dispersion relation $\epsilon_{\mathbf{k},\omega} = 0$ for DISW becomes independent on the dust size in the dimensionless units. This is another advantage of the use of these units. In absence of dust $P_0 = 0$ we get the standard expression for ion-sound waves for $k^2\lambda_{Di}^2 \ll 1$ in dimensionless units in the case the phase velocity is much larger than the ion thermal velocity (which in dimensionless units is just $\omega \gg k$)

$$\omega_{is}^2 = k^2 v_{is}^2 = k^2\frac{1}{2\tau} \tag{18}$$

where $v_{is} = 1/\sqrt{2\tau}$ is the dimensionless ion-sound speed.The influence of dust is determined by the parameter P_0 which enters both in the first term and in the second term. In the first one $1-P_0$ describes the charge neutrality condition while the other term proportional to α_{ch} and the similar term in the first bracket of the second term are describing the charge variation and these effects are usually taken into account in description of DISW, while the all terms in the square brackets of the second term are related with the perturbation of the absorption and ionization balance of the ground state and are rarely taken into account (at least the change of ionization was not taken into account in any previous investigations). An exception are the papers [10,15] in which the source term was considered as independent on plasma parameters and therefore the change of ionization was not considered. The form (16) for the second term corresponds to the conditions which are the most realistic for the existing experiments where the ionization source depends on plasma parameters and is proportional to the electron density. Certainly the other models of the source can be used and will give somewhat different expressions for the second term, but in the frame of simple hydrodynamic description used here the form of the source term is the simplest one and the closest to the existing experiments. The contribution by the perturbations of the ground state are in most cases more important than the contribution due to change in quasi-neutrality condition and due to simple charge variations. Indeed in the case we neglect the latter contributions we can get the expression for DISW with only quasi-neutrality change and charge variations taken into account

$$\omega_{disw}^2 = k^2 v_{disw}^2 = k^2\frac{1+\frac{\alpha_{ch}P_0}{(1+z_0)\alpha_{ch}-i\omega}}{2\tau\left(1-P_0+\frac{\alpha_{ch}P_0}{(1+z_0)\alpha_{ch}-i\omega}\right)} \approx \frac{k^2}{2\tau(1-P_0)} \tag{19}$$

which differs from the (17) by a factor $(1 - P_0)$ in the denominator appearing from contribution of dust in the charge quasi-neutrality condition of the ground state. We wrote the first relation (19) to show that these terms which are due to charge variations are small since the condition of validity of (19) is more rigid that $\omega \gg \alpha_{ch}(1 + z_0)$. The latter means that the relation (19) will be valid only for frequencies much larger than the charging frequency. Indeed, the contribution of the other terms of (17)not taken into account in (19) can be neglected only if

$$\omega \gg \frac{\alpha_{dr} P_0 z_0}{2\tau}, \alpha_{ch}(1 + z_0); k \gg \alpha_{ch}\sqrt{P_0 z_0 \tau}; \omega \gg \alpha_{ch}(1 + z_0) \tag{20}$$

For ω given by (19) and for the coefficients α_{ch},and α_{dr} of the order of 1 and for $\tau \ll 1$, z_0 of the order of 1 and $P_0 \gg \tau$ the fulfillment of the first inequality leads to fulfillment of other two. Therefore we write down the first inequality in the dimensional units

$$k\lambda_{Di} \ll \sqrt{\frac{2}{\pi}} \ln \Lambda P_0 z_0 \sqrt{(1 - P_0)} \frac{a}{\lambda_{Di}\sqrt{\tau}} \tag{21}$$

In most existing experiments except those with very small dust particles $a/\lambda_{Di} \approx 1/7, 1/10$ and $\tau \approx 0.02 - 0.01$, the parameter P_0, although it should be always less than 1, is in experiments close or of the order of 1. Thus the right hand side is of the order or even larger than 1 not leaving any appreciate range in wave numbers to DISW to exist in their conventional form (19). Of course the condition (20) can be fulfilled for very small $P_0 \ll 1$ to create a certain interval of wave numbers where the DISW can exist but than those waves (19) do not differ from usual ion-sound waves (18). All these arguments should be taken into account in comparing the experiments with the theory. If in experiments the value of the parameter P_0 is not very small and the dust can influence the spectra of DISW than to fulfill the relation (20) one should use a rather small dust sizes. Even in the limit (20)(21) where the approximate expression for DISW coincides with conventional ones (19), the exact dispersion relation found from (17) gives additional real and imaginary contributions to the frequency of the DISW'es. The most important is the imaginary parts creating a rather large damping. We write the expression for it in conventional dimensional units

$$\gamma_{disw} = Im(\omega)_{disw} = -\frac{P_0}{2\sqrt{2}}\omega_{pi}\frac{a}{\lambda_{Di}}\left[\frac{\alpha_{dr} z_0}{\tau} + 2P_0\alpha_{ch}^2 + 4\frac{\alpha_{ch}^3 \tau(1 + z_0)}{k^2\lambda_{Di}^2}\frac{a^2}{\lambda_{Di}^2}\right] \tag{22}$$

The second and the third term of (22) are negligibly small as compared to the first one and we write them only to demonstrate that the damping due to the charging process (both the charge variations, described by the second term, and the change in the ground state due charging process, described by the third term) being related with coefficient α_{ch} can be neglected. The damping described by the first term of (22) can easily exceed the collision less Landau damping (taken often into account in kinetic description of DISW) and becomes of the order of the frequency of DISW when the wave number k reaches its limit determined by the right hand side of (21). This damping is collective (proportional to P_0), it is increasing with the dust charge proportional to the square of the dust charge and it is large due to presence of small factor τ in the denominator of (22).

In the limit opposite to the last inequality (20)

$$\omega \ll \frac{\alpha_{dr} P_0 z_0}{2\tau} \tag{23}$$

we find the solution of the dispersion equation relation

$$\omega = i \frac{1}{\alpha_{dr} P_0 z_0} \frac{(\alpha_{dr}\alpha_{ch}^2 P_0^3 z_0^2 - k^2((1+z_0)\alpha_{ch} + P_0\alpha_{ch} - i\omega))}{((1-P_0)(1+z_0)\alpha_{ch} + P_0\alpha_{ch} - i(1-P_0)\omega)} \tag{24}$$

The damping is changed to *instability* which can be found by simple solution of the quadratic equation (24) and give a critical value of the wave number

$$k_{cr,disw}^2 = \frac{\alpha_{dr} P_0^3}{1 + z_0 + P_0} \tag{25}$$

which also can be found directly from (24). The critical wave number is determined only by the drag coefficient and is lower than the upper limit (23), i.e. it is inside the range (23).

The range (23) covers the most important values of wave numbers in existing experiments and the instability found can be detected experimentally. Thus the DISW *are universally unstable.* This instability can play an important role in creating short scale dust structures. Similar to the gravitational instability the growth rate of this instability reaches a constant value at $k = 0$. The reason for the instability to be present is the possibility of creation of ion collective flux similar to the effect of collective attraction of dust particles found in [34] also created by the collective flux. The ionization source creates both electron and ions between two interacting ions but the electrons can be accumulated their but not ions which are repelled from the region between two interacting ions. Since for large dust sizes the allowed range of wave numbers for DISW is rather narrow it is very improbable that these waves on their nonlinear stage can form the shock waves.

4 Dispersion relation for DAW

For DAW one can neglect the ion inertia but should take into account the dust inertia and the change of dust density. The variation of the dust density appears both in the Poisson equation (often taken into account), in the disturbance of ion absorption, in the ion continuity equation (the effect which is the subject of present consideration). Both effects in dimensionless equation depend only on the perturbation of the parameter P. Here we will use

$$\frac{\delta P}{P_0} = \frac{\delta z}{z_0} + \frac{\delta n_d}{n_d} \tag{26}$$

instead of (11).Thus the ion continuity equation will therefore contain an additional term depending on $\delta n_d/n_d$ and describing the change in ion absorption in DAW. Since the frequency of the DAW is much less than the charging frequency we will use

$$\frac{\delta z}{z_0} = \frac{1}{1+z_0}\left(\frac{\delta n_e}{n_{e0}} - \delta n\right) \tag{27}$$

instead of (13)

Another additional equations are the dust continuity equation and the dust force balance equation. In the latter we take into account the dust pressure force, the dust inertia force and the ion drag force. We find (see the definitions (1) for the dimensionless values)

$$ie(\mathbf{E}\cdot\mathbf{k}) = \left(\tau_d k^2 - \omega^2\frac{\tau}{\mu}\right)\frac{\delta n_d}{n_d} + i\alpha_{dr} z_0(\mathbf{k}\cdot\mathbf{u}) \tag{28}$$

Using (28), the ion continuity equation with (26)(27) we find $\delta n_d/n_d$ and δn and then using the Poisson equation we find the dielectric permittivity for low frequencies corresponding to the range of the DAW

$$\epsilon_{\mathbf{k},\omega} = 1 + \frac{\tau}{a^2k^2}\frac{1}{\left(\tau + \frac{\alpha_{dr}\alpha_{ch}P_0^2 z_0^2}{k^2(1+z_0)}\right)}\left\{1 + \frac{P_0}{1+z_0} + \tau\left(1 - \frac{P_0 z_0}{1+z_0}\right) - \frac{\alpha_{ch}\alpha_{dr}P_0^3 z_0^2}{k^2(1+z_0)} + \right.$$
$$\left. + \frac{P_0 k^2\left(\tau + \frac{\alpha_{ch}\alpha_{dr}z_0^2 P_0(P_0-\tau)}{k^2(1+z_0)}\right)\left(\tau + \frac{\alpha_{dr}\alpha_{ch}P_0 z_0(1+P_0)}{k^2}\right)}{\left[\left(\tau_d k^2 - \omega^2\frac{\tau}{\mu}\right)\left(\tau + \frac{\alpha_{dr}\alpha_{ch}P_0^2 z_0^2}{k^2(1+z_0)}\right) - P_0 z_0 \alpha_{ch}\alpha_{dr}\tau\right]}\right\} \tag{29}$$

Mention that this expression contains only the product of α_{ch} and α_{dr} and therefore all changes related with perturbations of the background are important only if both the drag and the charging processes are included. Also the friction of the ions described by α_{dr} can be related with ion-neutral collisions if their rate exceeds the drag force. The drag coefficient both describes the friction of ions on dust and the dust drag. In conditions where the ion-neutral collisions dominate in the equation for the ion motion the friction of ions on dust should be substituted for the drag coefficient. The neutral dust collisions change the dielectric permittivity in a way that one should substitute $\omega(\omega + i\nu_{di})$ for ω^2 in the expression (29). In the case we neglect all effects related with the perturbations of the balance of the ground state and neglects the charge variations we find the standard expression for DAW

$$\omega_{daw}^2 = k^2 \mu P_0 \frac{1}{1+\tau(1-P_0)} + k^2 \frac{\tau_d}{\tau}\mu \approx k^2 v_{daw}^2; v_{daw}^2 = \mu P_0 \tag{30}$$

where the second term which describes the dust pressure can be usually neglected. In conventional dimensional units the (30) has the standard form

$$\omega^2 = k^2 \frac{P_0 Z_d T_i}{m_d(1+\tau(1-P_0))} + k^2 \frac{T_d}{m_d} \approx k^2 \frac{P_0 Z_d T_i}{m_d} \tag{31}$$

In the case one does not neglect the charge variations but still neglects the changes in the ground state one get for $\tau \ll 1$ neglecting the dust temperature effects the relation

$$\omega_{daw}^2 = k^2 \mu P_0 \frac{1+z_0}{1+z_0+P_0+\tau(1+z_0-P_0 z_0)} \approx k^2 P_0 \mu \frac{1+z_0}{1+z_0+P_0} \tag{32}$$

The latter approximate expression is written for $\tau \ll 1$. The linear dispersion (the frequency proportional to the wave number) is valid for wave numbers less than the ion Debye length, which in dimensionless units corresponds to $k^2 \ll 1/a^2$, for $k^2 \approx 1/a^2$ (32) (31) give $\omega^2 \approx \omega_{pd}^2 = \mu P_0/a^2$. The dust plasma frequency ω_{pd} is here in dimensionless units and when used the conventional dimensional units is $\omega_{pd}^2 = 4\pi e^2 n_d Z_d^2/m_d$. The linear wave will exist only if the rigid restriction, described by relation (22), (see (20)) is fulfilled which requires $a^2 \ll \tau$ or in dimensional units

$$\frac{a^2}{\lambda_{Di}^2} \ll \tau \tag{33}$$

In the conditions where (21) is still valid the additional terms neglected in derivation of (32) gives that the spectra of the waves differs from the linear law (where the frequency of the wave is proportional to the wave number). These corrections will compete with that related with deviations from quasi-neutrality

$$\omega^2 \approx \omega_{daw}^2 \left(1 + \frac{\alpha_{dr}\alpha_{ch}P_0 z_0(1+P_0)}{k^2\tau} - k^2 a^2\right) \tag{34}$$

the curvature of the dispersion curve introduced by the first correction of (34) is opposite to the curvature of the curve introduced by the second correction. In the present experiments one can easily distinguish the two deviations from the linear law and that already detected in [26] corresponds to the first type of corrections. This is an indications that the effect described here is already observed and can be investigated experimentally in more details. The condition that the dispersion corrections due to the perturbations of the basic state dominate is

$$\frac{1}{a^2} \gg k^2 \gg \frac{1}{a\sqrt{\tau}} \tag{35}$$

which is consistent with the inequality (33).

In the opposite case for $k^2 \ll 1/\tau$ and for $\tau \ll 1$ we find

$$\omega^2 = \frac{\mu}{\tau} \frac{\alpha_{ch}\alpha_{dr} z_0 P_0^2 (1+P_0)(1+z_0)}{\left[1 + P_0 + z_0 - \frac{\alpha_{ch}\alpha_{dr}P_0^3 z_0^2}{k^2}\right]} \tag{36}$$

The instability appears at wave numbers close to

$$k_{cr,daw}^2 \approx \frac{\alpha_{ch}\alpha_{dr}P_0^3 z_0^2}{1 + z_0 + P_0} \tag{37}$$

which depends both on the charging and on the drag coefficients, and also is proportional to the square of the dust charges. For $\alpha_{ch} = 1/2\sqrt{\pi}$ and the $z_0 \approx 2$ the critical wave numbers for DISW and DAW are of the same order of magnitude and do not differ much. For $k \gg k_{cr,daw}$ we find a new mode

$$\omega^2 = \frac{\mu}{\tau} \frac{\alpha_{ch}\alpha_{dr} z_0 P_0^2 (1+P_0)(1+z_0)}{[1 + P_0 + z_0]} \tag{38}$$

The charging damping of this mode can be easily found by substituting $1 + z_0 - i\omega/\alpha_{ch}$ for $1 + z_0$. The damping of this mode will be

$$\gamma = -\frac{\mu}{\tau} \frac{\alpha_{dr} z_0 P_0^3 (1+P_0)}{2(1+z_0)(1+z_0+P_0)} \tag{39}$$

This damping rate is approximately $\sqrt{\mu/\tau}$ less than the frequency. The instability is set on at the wave numbers close the critical wave number with the growth rate the maximum of which is relatively large as compared to that at $k \ll k_{cr,daw}$ and as compared to the damping at $k \gg k_{cr,daw}$. Indeed the infinity in (33) at $k = k_{cr,daw}$ is not reached if one takes into account the imaginary part related with the charging process. We find then the shift in wave numbers from critical value and the value of the maximum growth rate

$$\gamma_{max} = \frac{\sqrt{3}}{2}\left(\frac{\mu}{\tau}\alpha_{ch}\alpha_{dr}P_0^2(1+P_0)(1+z_0)\right)^{1/3} \tag{40}$$

The threshold of instability corresponds to wave numbers somewhat less that the $k_{cr,daw}$,namely at $k = k_{cr,daw} - \Delta k_{thr}$. From (37) we find

$$\frac{\Delta k_{thr}}{k_{cr.daw}} \approx \gamma_{max}\frac{k_{cr}^2}{2\alpha_{ch}\alpha_{dr}P_0^3 z_0^2} = \frac{\sqrt{3}}{4}\frac{\alpha_{ch}\alpha_{dr}P_0^3 z_0^2}{(1+z_0+P_0)^2}\left(\alpha_{ch}\alpha_{dr}P_0^2(1+P_0)(1+z_0)\right)^{1/3}\left(\frac{\mu}{\tau}\right)^{1/3} \tag{41}$$

This consideration illustrates that large growth rates occurs at very narrow wave number range and that the system will mainly excite only one mode, the most unstable mode, specially it occurs in the case the systems has a finite size. This mode can create structures among which could be the modes turning the system to a crystal state. The stabilization of this instability by the dust pressure can be found by including the dust pressure terms $k^2\tau_d$ (see (29)) in the dispersion equation. The criteria of stabilization should include the maximum growth rate γ_{max}.We find

$$\tau_d > \frac{2(1+z_0+P_0)}{\sqrt{3}\alpha_{ch}\alpha_{dr}P_0^3 z_0^2}\left(\alpha_{ch}\alpha_{dr}P_0^2(1+P_0)(1+z_0)\right)^{2/3}\left(\frac{\mu}{\tau}\right)^{2/3} \tag{42}$$

By assuming that the charging and the drag coefficients and z_0 are of the order of 1 and using the notations (1) we get the dependence of the critical temperature on the parameters of the system $T_d > T_{d,cr}$

$$T_{d,cr} \approx T_i\frac{Z_d^{4/3}}{\tau^{4/3}P_0^{5/3}}\left(\frac{m_i}{m_d}\right)^{2/3} \tag{43}$$

By using the data which are usually found in the present experiments [18-21] $\tau = 0.02, Z_d = 10^3, P_0 \approx 1; m_d/m_i \approx 10^{10}$ we get from (39) $T_d > (1.5) \times T_i$. One should have in mind that a factor of the order of 1 is missing in the estimate (43). The obtained criterion is close to that observed in experiments for phase transition of dusty to the plasma dust crystal state. Therefore one can hope that the instability considered here can be relevant or can be the most probable candidate for the start of formation of the plasma dust crystal structures. For this treatment of course the nonlinear approach is necessary. The linear consideration can give some orientations in the problem.

5 Discussions

We start by emphasizing the main points of the present investigation

- The DISW and DAW becomes damped or not existing for rather large wave numbers, therefore the experiments for detecting them and not affected by strong damping effects should be performed for small size dust particles.
- New modes appear at wave numbers lower than the critical wave numbers.
- The critical wave numbers restricting the range of DISW and DAW can be easily achieved in experiments [26-33].
- The universal instability of both DISW and DAW occurs at critical wave numbers of the same order of magnitude which corresponds to the values approximately $\sqrt{\tau}$ less than that which restricts the range of existence of DISW and DAW.
- The critical sizes for the universal instability can be also reached in existing experiments if the ratio of dust sizes to Debye length is not very small.
- The DISW have at large wave numbers have a rather big damping which can be measured in the present experiments [26-33].
- The curvature of the DAW should change with increasing of the wave numbers being positive at wave k^2 less than $1/a\sqrt{\tau}$ and negative in the opposite case; this effect can be checked experimentally [26–33].
- For investigations of the universal instability found here it is better to use the dust particles with larger sizes than that used presently.
- The instability found is similar to the gravitational instability.
- The physical reason for the instability to develop is the *collective attraction* of equally signed particles, the collective attraction is related with collective flux created by dust particles was investigated in [34].

- Previously the role on non-collective shadow attraction in formation of the gravitation-like instability was investigated in [35][14] and the structurization instability related to collective attraction in [34].

- In the present paper an explicit expression were found for the instability caused by the collective attraction for DISW's and DAW's.

- The instability discussed should lead to formation of dust structures and probably the instability of DAW's can be considered as one of the best candidate for transition to plasma dust liquid or plasma dust crystal states.

In Future it is desirable to investigate the full development of perturbations starting from the linear instability up to its nonlinear stage when the dust structures are formed. Several stationary nonlinear structures in the gaseous state of dusty plasmas where already investigated previously [36-37].

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REFERENCES

[1]N.N.Rao,P.K.Shukla and M.Y.Yu *Planet. Space Sci.* **38**, 543 (1990)

[2]N.N.Rao,P.K.Shukla and M.Y.Yu *Planet. Space Sci.* **38**, 543 (1990)

[3]P.K.Shukla and V.P.Silin *Physica Scripta* **45**, 508 (1992)

[5]M.Rosenberg *J. Vac.Sci.Technol.A* **14**,631 (1996)

[6] M.Rosenberg *Planet. Space Sci.* **41**,229 (1993);

[7]M.Rosenberg and V.W.Chow *Planet.Space Sci.* **46**,103(1998)

[8]A.Mikhailovskii it Plasma instabilities Pergamon Press (London ,N.Y.)(1975)

[9]F.Melandso,T.K.Aslaksen and O.Havnes *J.Geophys.Res.* **98**,13315 (1993)

[10]V.N.Tsytovich and O.Havnes *Comments Plasma Phys.Contr. Fusion* **18**, 267 (1993)

[11] A.V.Ivlev and G.E.Morfill *Phys.Plasmas* **7**,1094 (2000)

[12] K.Ostrikov,S.Vladimirov,M.Yu and G.E.Morfill *Phys.Rev.E* **61**,4315 (2000)

[13]S.Benkadda,P.Gabai,V.N.Tsytovich and A.Verga *Phys.Rev. E* **53**, 2717 (1996)

[14] V.N.Tsytovich and D.Resendes *Fiz.Plasmy* **24**,71 (1998)

[15]V.N.Tsytovich,U.de Angelis,R.Bingham and D.Resendes *Phys.Plasmas* **4**, 3882 (1997)

[16]V.N.Tsystovich and U.de Angelis *Phys.Plasmas* **6**,1093(1999)

[17]G.E.Morfill and V.N.Tsytovich *Plasma Physics Reports* **26**,727 (2000)

[18] Thomas H, Morfill G et all *Phys.Rev.Lett.* **73** 622 (1994)

[19] Fortov V Nefedov A et all *Phys. Let.* **A 219** 89 (1996)

[20] Chu J H and Lin I *Physica* , **A 205**, 183, (1994)

[21] Melzer A et all *Phys.lett.*, **A 191**, 301, (1994)

[22]V.N.Tsytovich and U.de Angelis *Phys.Plasmas* **7**,554 (2000)

[23]V.N.Tsystovich and U.de Angelis *Phys.Plasmas* **8**,1141 (2001)

[24]P.Ricci,G.Lapenta,U.de Angelis and V.N.Tsytovich *Phys.Plasmas* **8**,769 (2001)

[25]Bingham R. U.de Angelis and V.N.Tsytovich *Phys.Plasmas* **8**,in press (2001)

[26] N.D'Angelo *Phys.Plasmas* **4**,3422 (1997)

[27]S.Hamaguchi and R.T.Farouki *Phys.Plasmas* **1**,2110 (1994)

[28]R.L.Merlino,A.Barkan,C.Thompson and N.D'Angelo *Phys.Plasmas* **5**,1607 (1998)

[29] T.Takaaishi,M.Yabuhara and K.Nishikawa *J.Phys.Soc.Japan* **70**,138 (2001)

[30]A.Barkan,N.D'Angelo and R.I.Merlino ,*Advances in Dusty plasmas* (World Scientific,Singapore, New Jersey,London,Hong Kong, 1996) p.30

[31]C.Thompson,A.Barkan,N.D'Angelo and R.I.Merlino *Phys.Plasmas* **4**, 2331 (1997)

[32]A.Barkan,R.I.Merlino and N.D'Angelo *Phys.Plasmas* **2**, 3563 (1995)

[33]J.E.Pieper and J.Goree *Phys.Rev.Lett.* **77**,3137 (1996)

[34]G.Morfill and V.N.Tsytovich *Fiz.Plasma* in press ((2001)

[35]Tsytovich V.N.,Khodataev Ya. K., Bingham R.and Resendes D.*Comments in plasma physics and Controlled Fusion* , (1996)

[36]V.N.Tsytovich *Comments in Plasma Physics and Controlled Fusion, Comments in Modern Physics* **1, Part C**, 41-63 (2000)

[37]G.Morfill and V.N.Tsytovich *Phys.Plasmas* **6**, 2110 (2001)

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