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Collective Plasma Corrections to Thermonuclear Reactions Rates in Dense Plasmas

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Abstract

General kinetic equations for nuclear reaction in dense plasmas are obtained. They take into account the first order collective plasma effects. Together with previously known corrections proportional to $Z_i Z_j$, the product of the charges Z_i and Z_j of two interacting nuclei, it is shown that there exist corrections proportional to the squares Z_i^2 and Z_j^2 of the charges. It is shown that the Salpeter's [1] correction due to the plasma screening of the interaction potential is at least r/d smaller(r is the nuclei size and d is Debye screening length) than previously thought and is zero in the approximation when the terms of the order r/d are neglected. But the correlation effects in the first approximation in the parameter $1/N_d$ (where N_d is the number of particle in the Debye sphere) give corrections which often coincide with the first order Salpeter's corrections (found by expansion in another small parameter, the ratio of thermal energy to Gamov's energy). The correlation corrections are $\propto Z_i Z_j$, have a different physical meaning than the corrections [1], can have a different sign and are present for reactions where the Salpeter's corrections are zero. Previously in astrophysical applications it was widely used the interpolation formulas between weak and strong Salpeter's screening corrections. Since the correlation correction take place the previously known Salpeter's corrections and the strong correlation corrections is difficult to describe analytically, the interpolation formulas between the weak and strong correlations cannot be yet found. A new type of corrections are found here which are proportional to the square of the charges. They are due to collective change in electrostatic self-energy of the plasma system during the nuclear reactions. The latter corrections are found by taking into account the changes of plasma particle fluctuations by the nuclear reactions. Numerical evaluation of the plasma corrections for the nuclear reactions of the hydrogen cycle, using the parameters of the present temperature, density and abundance in the solar interior, are performed.

Key words: nuclear reactions, dense plasmas, solar interior, solar neutrino

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1 Introduction

In the well known paper of Salpeter 1954 ,[1] it was shown that for Debye screening Coulomb potential the probability of nuclear reaction is substantially larger in a dense plasmas than for non-screened Coulomb potential. This effect was then called as plasma screening of nuclear reaction rates. It is widely used in most models of stellar evolution [2] and for the nuclear reactions in the centre of the Sun [3] (see the last reviews [4,5]). For reaction in the solar interior it ranges from 5% to 20% for different reactions in the hydrogen cycle, which is appreciable amount both for solar neutrino problem and for the value of the solar sound speed detected with a good precision by means of the present solar seismology methods. Only 35 years later in 1988 it was pointed out in [6] that the static screening of thermonuclear reaction (as considered in [1]) is meaningless from physical point of view. The point of [6[was that the nuclear reactions usually occur at the energies (so called Gamov energies) which substantially exceed the thermal energy where the static screening is absent (see textbooks,for example,[7]). More exactly, for energies at which the nuclear reaction occurs the screening start to be a dynamic screening, vanishing in the limits of very large energies (velocities). After the publication [6], although it contains physical reasonable arguments for screening to be considered as dynamical screening. there appear many serious investigations [8,9], using sophisticated diagram technic in quantum statistic showing that nevertheless the screening of nuclear reaction should be static. We will show here that, although in [8,9] no calculation mistake where made, but from physical point of view the effect calculated in [8,9] is different from that of [1] and therefore the interpretation of the results [8,9] is not correct. The coincidence of the result [8,9] with the result [1] is occasional and not always occurs. It happen only in the first approximation using small parameters which are different in [1] and [8,9]. Apart of that, the present consideration shows that there exist additional plasma corrections to the rate of nuclear reactions which are proportional to the squares of the interacting nuclei charges .

The discussion whether the screening of nuclear reactions is dynamic or static was continued up to the present time. To resolve this problem it was necessary no remove the starting assumption of both [1] and [6] that the interaction of two nuclei is determined by the average potential. First in [10-12] a new approach was used without this assumption deriving the equations for nuclear kinetic of interacting nuclei from first principles by averaging the micro-equations with respect to plasma fluctuations. From previous usage of such technic with nuclear reactions not taken into account it was found that only such approach is appropriate for systems of large number of particles and is able to prove directly that in Coulomb collisions the plasma particle screening is a dynamic one. The approach of [1,6] deals only with two interacting nuclei considering them as two probe particle for which without any direct prove a screened potential is used (with static screening in [1] and dynamic screening in [6]). Here we will make one further step, as compared to [10-12], by taking into account the change of particle fluctuations due to the presence of nuclear reactions. In [10-12] a possible resolution of dilemma of static and dynamic screening was obtained by showing the exact cancellation of all static corrections. Here we give a simple consideration showing that that this cancellation should be absent proving an absence of the one component in this cancellation. namely we show that the effect of Salpeter's screening is absent in first order in parameter r/d where r is the inter-nuclear distance in nuclear reaction and d is Debye screening length. This is the case if one does not in first place assume that the average potential is determined the nuclei interactions but considers the influence of all other plasma particles on nuclei reaction rates as caused by their potential. The latter in the big system of many plasma particles should be treated as a fluctuating potential. We also generalize the fluctuating approach of [10-12] by taking into account the change in plasma particle fluctuations caused by nuclear reactions and will show that the dilemma of dynamic an static screening is resolved by absence of screening effects while the effect of plasma correlation in first approximation in the small parameter $1/N_d$ lead to a result which often (but not always) coincides with the first approximation of the weak static screening result. The existing paradox is resolved since the physical interpretation of the effect is fundamentally changed-the correlation effects can be in principle determined by the static dielectric plasma permittivity (which is well known in plasma for example for electromagnetic wave scattering [7]) while the screening effects cannot be determined by the static dielectric permittivity. For the case where the coincidence between the weak Salpeter screening corrections and the weak correlation corrections corrections occurs, such coincidence exist only in the first approximation and only for effects $\propto Z_i Z_j$. In next approximations the results of two effects are completely different. In the present paper only the weak correlation approximation is considered which will correspond to weak screening in Salpeter's approach.

A qualitatively new result obtained in the present investigation is a self-consistent consideration of the the corrections proportional to the squares of the nuclei charges Z_i^2 and Z_j^2. This type of corrections were first found in [10-12]. Such corrections are specific for systems in which the inverse nuclear reaction are not developing. Such systems are open systems. Nuclear reactions in star interiors with neutrino emission are of such kind, since neutrino freely leave the region of nuclear reactions. By taking into account this effect in the collision integral (which was obviously previously neglected) we found that it leads to a corrections which will be proportional to the time derivatives of nuclear distributions and therefore will be depending on the nuclear reaction rates. This leads to the renormalization of distribution functions of interacting nuclei and to the corrections proportional to squares of the nuclei charges. Similarly to the corrections caused by correlations these effects are related with changes in nuclei distributions but they are not related with the nuclei reactions themselves and can be regarded only as some effective change of nuclear reaction probabilities. It is well known that the renormalization of particle distribution is a standard procedure in any kinetic theory [13-14] and it seems to be obvious that such procedure should be used for kinetic description of nuclear reactions in plasmas.

The processes of time evolution of nuclear reactions are treated in the present paper by the procedure natural for any temporal problem, namely, it is assumed that nuclear reaction did not operate at $t < 0$, that at initial moment $t = 0$ the nuclear reactions start to operate and the asymptotic behaviour at large t is investigated.

This procedure lead to the explicit expressions for the corrections which are square in nuclei charges. Additional to the effects already taken into account in [10-12] in the present investigation it is taken into account that the plasma fluctuations are modified by nuclear reactions.The final result for these corrections depends on the whole cycle of nuclear reactions and is different than that found in [10-12].

Thus the present work is aimed to give the best available result for weak plasma corrections both proportional to the product of nuclei charges and proportional to the squares of nuclear charges.

The general expressions are used for explicit numerical calculations of plasma corrections of nuclear reaction rates for the hydrogen cycle for parameters that are presently accepted for the solar interior.

2 Absence of Salpeter's screening corrections

It is desirable to remind the arguments first given in [1]. It is assumed that the potential of interaction of nuclei is a Debye screened Coulomb potential:

$$\phi(r) = \frac{Z_i Z_j e^2}{r} \exp\left(-\frac{r}{d}\right) \approx \frac{Z_i Z_j e^2}{r} - \frac{Z_i Z_j e^2}{d} \tag{1}$$

where r is the relative distance between two nuclei, which is assumed to be much less than the distance of Debye screening d. Notice that the correction to the Coulomb potential is described as a constant in the interaction energy and is in [1] included in the energy of relative motion E_r, the only value on which the probability of nuclear reaction depends.

$$w_{ij} = w_{ij}(E_r + Z_i Z_j e^2/d) \approx w_{ij}(E_r) + \frac{Z_i Z_j e^2}{d} \frac{\partial}{\partial E_r} w_{ij}(E_r) \tag{2}$$

The screening potential is just assumed. We will show later that without this assumption some constant in energy indeed appears but it should be attributed to the center of mass motion but not to the relative motion of nuclei. In [1] the corrections to the thermonuclear reaction rates are obtained by integration of the probability times thermal Maxwellian distribution where the derivative with respect to the relative energy (using the integration by parts in relative energy) is converted to the factor $1/T$ for zero approximation in the parameter $T/E_r \approx T/E_G << 1$, where E_G is the Gamov energy. The derivative of the phase volume factor has the smallness T/E_G as compared to the derivative of the Maxwell distribution and are neglected. By this procedure in [1] for the rate of nuclear reaction R_{ij} it was obtained

$$R_{ij} = R_{ij}^{(0)} \left(1 + \Lambda_{ij}^{(S)}\right) \tag{3}$$

$$\Lambda_{ij}^{(S)} = \frac{Z_i Z_j e^2}{dT} = \frac{Z_i Z_j e^2}{2\pi^2 T} \int \left(1 - \frac{1}{\epsilon_{\mathbf{k},0}}\right) d\mathbf{k} \tag{4}$$

where $R_{ij}^{(0)}$ is the rate of nuclear reaction without plasma corrections and $\epsilon_{\mathbf{k},0}$ is the static dielectric permittivity. As was previously noticed the presence of the latter in the final result is physically not reasonable.

There should be an error in this derivation which seems to be not quite simple to recognize. It is hidden in assumption that the screening potential is due to a fixed polarization charge around the nuclei, while in fact it is formed by plasma particle fluctuations created by the fields of all other plasma particles except ones taking part in the nuclear reactions. In presence of plasma the nuclear reactions between the two pare of nuclei are occurring in an external fluctuating potential ϕ created by all other plasma particles and therefore the average value of this potential does not determine the nuclear reaction rate, since the time of averaging is much larger than the rate of nuclear tunneling. One can take into account the fluctuating potential ϕ as a constant in the expression for the nuclear reaction probability. But the subsequent procedure of expansion in this potential up to quadratic terms and averaging on fluctuations, although it recovers (4), is also not correct. We will nevertheless present such calculations. For terms $\propto Z_i Z_j$ the change of the average probability $\delta < w_{ij} >$ is

$$\delta < w_{ij} >= Z_i Z_j e^2 < \phi^2 > \frac{\partial^2}{\partial E_r^2} w_{ij}(E_r). \tag{5}$$

From this expression it can be seen that the fluctuating potential ϕ is determined by the dynamically screened particles but using the fluctuation dissipation theorem in integrating with respect to frequencies and particle velocities one again finds expression (4) containing the static dielectric constant. There are two points in this derivation which are correct. First is the presence of a small parameter, the ratio of nuclei size to the Debye length which allows to consider the fluctuation potential to be approximately constant during the nuclear tunneling. The second is the presence of another small parameter, the ratio of tunneling time to the characteristic time of fluctuations. Therefore the averaging on all possible configurations of other plasma particles occurs on time-scale much larger than the tunneling time and the averaged potential of other particles is zero. Opposite case will correspond to polarization to be established during the tunneling which will give the screening interaction potential and than the consideration [1] could be applied. The real conditions are opposite. Still the mistake of the previous calculation exist and it appears at the point where one assumes that the additional fluctuating potential, which is constant during nuclear reaction, influences the relative nuclei motion. In fact only the space derivative of it is entering in equation of the relative motion, or, in other word the nuclear reaction rate can be influenced only by the fluctuating field strengths.

Using the mentioned two small parameters one can obtain the correct answer. By introducing the coordinate of the center of mass of two interacting nuclei $\mathbf{R}$ and the relative coordinate $\mathbf{r}$ as well as the coordinates of each of interacting nuclei $\mathbf{r}_i$ and $\mathbf{r}_j$ one can notice that in nuclear reaction the coordinates of two nuclei almost coincide with each other and that $r \ll R$. Then for additional energy related with the fluctuating potential one can use two terms of expansion

$$eZ_i\phi(\mathbf{r}_i) + eZ_j\phi(\mathbf{r}_j) \approx e(Z_i + Z_j)\phi(\mathbf{R}) + e(\mathbf{r}\cdot\mathbf{E})\frac{m_j Z_i - m_i Z_j}{m_i + m_j} \tag{6}$$

The first term in the last expansion indeed is constant for nuclear reactions, but it depends only on the coordinate of the center of mass and in the case the second term of (6) can be neglected we can see that it will determine only the center of mass motion but not the relative motion and thus does not change the probability of the nuclear reaction. The second term which depends on the relative coordinate can be estimated and can be found to be small at least of the order of the ratio of the tunneling size to the fluctuation size. Thus the constant which was appearing in [1] should be in fact included in translational motion but not in the relative motion which means that Salpeter's screening is absent. The Salpeter's result is valid only if the time of tunneling is larger that the time of averaging which in turn is larger than the time of fluctuations. The central point in this consideration is the fact established in modern plasma physics that the screening is produced in fluctuations which was not taken into account both in the approach [1] and in the approach [6].

3 Correlation effects

The correlation effects describe the correlation of the states of two reacting nuclei and in the case where these nuclei appear to be more often in the states where the reaction rate is higher the average rate of reactions increase. This effect is different from [1,6], where the rate itself is increasing independent on the states in which are the reacting nuclei. The correlations are determined by the kinetics of fluctuations in the system of reacting nuclei. To investigate this effect we will use, as in [10-12], the micro-equations but in the form somewhat more exact than that used in [10-12]. Final result of this improvement appears to be necessary only for detailed and general theory of correlations, which was developed by the author but will be not presented here. The reason for the latter is that the exact theory of correlations in first approximation used below give the result coinciding with that obtained in [10-12]. Thus the more exact treatment of correlations is needed only for a precise foundation of the corrections already found in [10-12]. Only perturbations approach was used in exact correlation equation and thus the exact equation can give in this case only the criterion of validity of the first approximation, namely, the small parameter of expansion, which appears to be $1/N_d << 1$. where N_d is the number of particles in the Debye sphere. We will give here the basic equations used for exact theory of correlations,namely

$$\frac{\partial}{\partial t}f_i + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}f_i + Z_i e\mathbf{E}\cdot\frac{\partial}{\partial \mathbf{p}}f_i = -\int w_{ij} f_{ij}\frac{d\mathbf{p}}{(2\pi)^3} \tag{7}$$

where f_i is the one-particle distribution function, and f_{ij} is the two-particle distribution function. The first is obtained by integrations of many particle distribution with respect to the phase volume of all particle except one particle i and the second is obtained by integration of the many particle distribution on the phase volume of all particles except two particles i and j. In [10,12] there was used an approximation $f_{ij} \approx f_i f_j$. The equation (7) is more exact than that used in [10,12]. The analysis of the equation (7) should be performed by deriving and solving the equation for f_{ij}: the latter is obtained by integration of basic equation on all variables except i and j, but not, as in deriving of the equation (7), by integration with respect to all variables except i. Such analysis is exact but cumbersome and only the perturbation approach is effective for that. The correlation effects already exist in the approach used in [10,12] since average value of the product of one-particle distributions is not equal to the product of the average distributions. We will give only the result of the more sophisticated treatment of the correlation problem by the equation (7): in the first approximation in parameter $1/N_d$ the correlation corrections coincide with that obtained in [10,12] by assuming that $f_{ij} \approx f_i f_j$. For the further approximations which we do not use here this coincidence is absent. We remind here that to obtain the correlation corrections, as performed in [10-12], one need to introduce the distributions averaged on fluctuations and the fluctuating part of the distributions

$$f_i = \Phi_i + \delta f_i; \langle f_i \rangle = \Phi_i. \tag{8}$$

Than by averaging the starting equation and subtracting the average equation from the starting equation and taking into account that the probability of nuclear reaction does not depend on fluctuating potential (as shown here earlier but this effect was taken into account in [10-12]) we find in the right hand side of the equation (7) the following expression

$$\langle f_i f_j \rangle = \Phi_i \Phi_j + \langle \delta f_i \delta f_j \rangle \tag{9}$$

The second term in the right hand of equation (9) (as the detailed investigation described earlier shows) correctly describes the correlation effects in the first non-zero approximation. For explicit evaluation of the correlation contribution one uses another small parameter the ratio of the nuclear reaction rate to the characteristic time-scale of fluctuations (notice that the latter has opposite relation with tunneling time which is much less than the characteristic time of fluctuations) and uses then in the first approximation the known expressions for fluctuations of particle distributions in absence of nuclear reactions. This procedure leads to correction for arbitrary particle distributions. Assuming then that the distributions are thermal and using in the fluctuation dissipation theorem one expresses finally the correlation corrections through the static dielectric permittivity:

$$\langle \delta f_i \delta f_j \rangle = \frac{Z_i Z_j e^2}{2\pi^2 T} \Phi_i \Phi_j \int \frac{d\mathbf{k}}{k^2} \left(1 - \frac{1}{\epsilon_{\mathbf{k},0}} \right) \tag{10}$$

In the last expression it is assumed that the Coulombb field of two interacting nuclei should not be taken into account in the fluctuating potential created by all other plasma particles. In [10,12] not only the correlations were taken into account but also the change in the probability of nuclear reaction which was in [10,12] interfering with correlation effects. Since here we proved that the change of the probability is absent, the only effect left (later we describe additional effects) is the correlation effect. Since the correlation corrections are proportional to the product of the average distributions it is possible to introduce some effective probabilities which lead to the same result in the equations for the nuclear reactions as the effects of correlations.

$$w_{ij}^{eff} = w_{ij}\left[1 + \frac{Z_i Z_j e^2}{2\pi^2 T}\int \frac{d\mathbf{k}}{k^2}\left(1 - \frac{1}{\epsilon_{bfk,0}}\right)\right] = w_{ij}(1 + \Lambda_{ij}^{(C)}) \tag{11}$$

Formal coincidence of the expression (11) for correlation corrections, $\Lambda_{ij}^{(C)}$ with expression (4) for the Salpeter's corrections, $\Lambda_{ij}^{(S)}$ should not lead to any miss-understandings since the corrections (11) and the corrections (4) have not only a different physical meaning but also differ from each other quantitatively. An example of such difference is the reaction of nuclei ^7Be with electrons where the nuclear barrier is absent and the Salpeter's corrections are zero while the correlation corrections are not zero and are even negative describing the suppression but not enhancement of the nuclear reaction rate.

Mention that the approach applied in [8,9] was using the statistical averaging of the undisturbed probability on the electron and ion and electron disturbances which corresponds to the correlation approach and therefore the presently obtained result is in agreement with the result of [8,9].

4 Fluctuations in a system evolving in time

Due to absence of inverse processes with neutrino absorption, the system is an open system evolving in time. The plasma fluctuations are then non-stationary (as usual in absence of nuclear reactions) and the rate of the change of fluctuation in time is determined by the nuclear reaction rates. Although the rate of change of the fluctuations is small as compared to the fluctuation time but in calculating of all effects linear in the nuclear reaction rate one should take into account the effects related with the time evolution of the fluctuations. These effects were previously neglected (also in [10-12]). The fluctuations consist of two parts, first one which is not related with collective fluctuating field and the second one which is proportional to the fluctuating electric field. The second one leads to collective plasma corrections, but one need to know the first one through which the second one is expressed. The effects related with electric fields will be treated by perturbation which is sufficient for weak corrections in which we are here interested. Therefore we consider first the changes of fluctuations by nuclear reactions in the part independent on the fluctuating electric field. We denote this zero approximation by superscript $^{(0)}$. Then the starting equation for linear fluctuations will have the form

$$\frac{\partial \delta f_i^{(0)}(\mathbf{p})}{\partial t} + \mathbf{v}\cdot\frac{\partial \delta f_i^{(0)}(\mathbf{p})}{\partial \mathbf{r}} = -\int w_{ij}(\mathbf{p},\mathbf{p}')(\delta f_i^{(0)}(\mathbf{p})\Phi_j(\mathbf{p}') + \delta f_j^{(0)}(\mathbf{p}')\Phi_i(\mathbf{p}))\frac{d\mathbf{p}'}{(2\pi)^3}. \tag{12}$$

Similar equation should be written for $f_j^{(0)}(\mathbf{p}')$:

$$\frac{\partial \delta f_j^{(0)}(\mathbf{p}')}{\partial t} + \mathbf{v}\cdot\frac{\partial \delta f_j^{(0)}(\mathbf{p}')}{\partial \mathbf{r}} = -\int w_{ij}(\mathbf{p},\mathbf{p}')(\delta f_i^{(0)}(\mathbf{p})\Phi_j(\mathbf{p}') + \delta f_j^{(0)}(\mathbf{p}')\Phi_i(\mathbf{p}))\frac{d\mathbf{p}}{(2\pi)^3}. \tag{13}$$

By writing these equations for Fourier components

$$\delta f_{i,j} = \int \delta f_{i,j,\mathbf{k},\omega}\exp(i\mathbf{k}\cdot\mathbf{r} - \omega t)d\mathbf{k}d\omega,$$

it is possible to obtain an equation containing the fluctuation function of only one of the interacting nuclei

$$(\omega - \mathbf{k}\cdot\mathbf{v} + i\nu_i(\mathbf{v})\delta f_{i,\mathbf{k},\omega}^{(0)}(\mathbf{v}) = -\Phi_i(\mathbf{v})\int \frac{w_{ij}(\mathbf{p},\mathbf{p}')w_{ij}(\mathbf{p}'',\mathbf{p}')}{\omega - \mathbf{k}\cdot\mathbf{v}' + i\nu_j(\mathbf{v}')}\Phi_j(\mathbf{v}')\delta f_{i,\mathbf{k},\omega}^{(0)}(\mathbf{v}'')\frac{d\mathbf{p}'d\mathbf{p}''}{(2\pi)^6} \tag{14}$$

where

$$\nu_i(\mathbf{p}) = \int w_{ij}(\mathbf{p},\mathbf{p}')\Phi_j(\mathbf{p}')\frac{d\mathbf{p}'}{(2\pi)^3} \quad \nu_j(\mathbf{p}') = \int w_{ij}(\mathbf{p},\mathbf{p}')\Phi_i(\mathbf{p})\frac{d\mathbf{p}}{(2\pi)^3} \tag{15}$$

describes the damping due to presence of nuclear reactions. The right hand side of (14) describes another (than that described above) correlations of fluctuations due to the presence of nuclear reactions. These correlations can be simply estimated by taking into account that the characteristic frequency of fluctuations is of the order of $kv_{Ti} \approx \omega_{pi}$ for k of the order of inverse Debye length $1/d$, and $\omega_{pi}$is the ion plasma frequency. The right hand side of (14) is ν_i/ω_{pi} times less than the damping entering in the left hand side of (14) and can be therefore neglected. We will write down the correspondent equation for spatial Furier components of the distribution function $\delta f_i^{(0)}(\mathbf{v},\mathbf{r},t) = \int \delta f_{i,\mathbf{k}}^{(0)}(\mathbf{v},t)\exp(i\mathbf{k}\cdot\mathbf{r})d\mathbf{k}$:

$$\left(\frac{\partial}{\partial t} + i\mathbf{k}\cdot\mathbf{v} + \nu_i(\mathbf{v})\right)\delta f_{i,\mathbf{k}}^{(0)}(\mathbf{v},t) = 0 \tag{16}$$

We will consider the time evolution problem with initial conditions at time $t = 0$. Such treatment is necessary in an open system which cannot reach an equilibrium state due to absence of inverse processes which includes neutrino absorption. Assume that the nuclear reactions are switched on at $t = 0$ an then find their rate asymptotically at large t. Thus it is assumed that at $t < 0.\, \nu_i = 0$. By the way, such formulation of the problem is close to the real situations in stars where the nuclear burning starts on certain stage of contraction of the proto-star cloud. For $t < 0$ the solution of (16) will be

$$\delta f_{i,\mathbf{k}}^{(0)}(\mathbf{p},t) = \delta f_{i,\mathbf{k}}^{(0)}(\mathbf{p})\exp(-i\mathbf{k}\cdot\mathbf{v}t), \tag{17}$$

while for $t > 0$:

$$\delta f_{i,\mathbf{k}}^{(0)}(\mathbf{p},t) = \delta f_{i,\mathbf{k}}^{(0)}(\mathbf{p})\exp(-i\mathbf{k}\cdot\mathbf{v}t - i\nu_i(\mathbf{v})t). \tag{18}$$

Then we consider only the spatially homogeneous problem where average values of $\delta f_{i,\mathbf{k}}(\mathbf{p})$ should be the same as for the stationary (in average) system:

$$\langle \delta f_{i,\mathbf{k}}^{(0)}(\mathbf{p})\delta f_{j,\mathbf{k}'}^{(0)}(\mathbf{p}')\rangle = \Phi_i(\mathbf{p})\delta_{i,j}\delta(\mathbf{p}-\mathbf{p}')\delta(\mathbf{k}+\mathbf{k}') \tag{19}$$

This leads to the following law for averaging of the fluctuations in presence of nuclear reactions which we will use in what follows

$$\langle \delta f_{i,\mathbf{k},\omega}^{(0)}(\mathbf{p})\delta f_{j,\mathbf{k}',\omega'}^{(0)}(\mathbf{p}')\rangle = -\frac{1}{4\pi^2}\Phi_i(\mathbf{p})\delta_{i,j}\delta(\mathbf{p}-\mathbf{p}')\delta(\mathbf{k}+\mathbf{k}')\times$$

$$\left[\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}-i0}-\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i\nu_i(\mathbf{v})}\right]\left[\frac{1}{\omega'-\mathbf{k}'\cdot\mathbf{v}'-i0}-\frac{1}{\omega'-\mathbf{k}'\cdot\mathbf{v}'+i\nu_j(\mathbf{v}')}\right] \tag{20}$$

In the limit $\nu_{i,j} \to 0$ the result (20) coincides with the well known law for averaging in a stationary system (we denote the correspondent distributions for the latter case by superscript $^{(0.0)}$)

$$\langle \delta f_{i,\mathbf{k},\omega}^{(0,0)}(\mathbf{p})\delta f_{j,\mathbf{k}',\omega'}^{(0,0)}(\mathbf{p}')\rangle = \Phi_i(\mathbf{p})\delta_{i,j}\delta(\mathbf{p}-\mathbf{p}')\delta(\mathbf{k}+\mathbf{k}')\delta(\omega+\omega')\delta(\omega-\mathbf{k}\cdot\mathbf{v}). \tag{21}$$

Using (20) it is necessary to find the change in time of the nuclei distributions averaged on fluctuations which gives the change of the rates of nuclear reactions due to collective plasma effects. Apart from the change of fluctuations one should take into account also the time evolution effects in the averaged distribution function.

5 Influence of the evolving in time fluctuations on the nuclear reaction rates

The starting equation for calculation of collective corrections to the nuclear reaction rates is the equations obtained by averaging of (7) with respect to plasma fluctuations

$$\frac{\partial}{\partial t}\Phi_i = Z_i e\frac{\partial}{\partial \mathbf{p}}\langle \delta f_i \nabla\phi\rangle - \int w_{ij}(\Phi_i\Phi_j + \langle \delta f_i \delta f_j\rangle)\frac{d\mathbf{p}'}{(2\pi)^3} \tag{22}$$

we have here assumed, as in calculation of fluctuations, that the average distribution is homogeneous but time-dependent. The latter is due to nuclear reactions. The first term in the right hand side of (22) in absence of the time dependence leads to the known collision integral which is approaching zero on the binary collision time-scale converting the particle distributions to the thermal distributions. But up to the present time it was not recognized that in presence of time variations (which in the present consideration are due to the nuclear reactions) this term gives an additional non-zero contribution proportional either to the change of fluctuations in time or to the change in time of the average distribution of nuclei. We will take into account only the effects of the first order in the nuclear reaction rates, i.e. only effects linear in time-derivatives of the averaged distribution and linear in w_{ij}. The last term of (22) contains the correlation effects and both the correlation effects and the effects of temporal evolution will be considered in linear in w_{ij} when these two effects should be simply added in the final result and therefore in the present part we can neglect the correlation effects. In the case we also neglect the effect of temporal evolution of fluctuations, the equation (22) is converted to the following one

$$\frac{\partial}{\partial t}\Phi_i^{(0)} = -\Phi_i^{(0)}\int w_{ij}\Phi_i' \frac{d\mathbf{p}'}{(2\pi)^3} = -\nu_i\Phi_i^{(0)}. \tag{23}$$

In this equation the frequency ν_i should be considered as constant since by taking into account its time -dependence one takes into account the corrections of higher order in w_{ij}. For fluctuations we have instead of (16)

$$-i(\omega - \mathbf{k}\cdot\mathbf{v} + i\nu_i)\delta f_{i,\mathbf{k},\omega} = Z_i e\left(\nabla\delta\phi\cdot\frac{\partial}{\partial\mathbf{p}}\Phi_i\right)_{\mathbf{k},\omega}, \tag{24}$$

where ϕ is the potential of the fluctuating electric field. The solution of this equation will be

$$\delta f_{i,\mathbf{k},\omega} = \delta f_{i,\mathbf{k},\omega}^{(0)} - \frac{Z_i e}{\omega - \mathbf{k}\cdot\mathbf{v} + i\nu_i}\int \phi_{\mathbf{k},\omega-\omega'}\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}\Phi_{i,\omega'}\right)d\omega', \tag{25}$$

where $\delta f_{i,\mathbf{k},\omega}^{(0)}$ is the solution of a homogeneous equation (24), describing the fluctuations of the system evolving in time which was discusses in the previous section. Since the average distribution function evolves in time much slower than the plasma fluctuations $\omega' \ll \omega$ one uses an expansion with respect to $\omega'$and also expands in ν_i

$$\phi_{\mathbf{k},\omega-\omega'} \approx \phi_{\mathbf{k},\omega} - \omega'\frac{\partial}{\partial\omega}\phi_{\mathbf{k},\omega};\ \int \omega'\Phi_{i,\omega'}d\omega' \approx i\frac{\partial}{\partial t}\Phi_i \tag{26}$$

Using (23)we obtain

$$\delta f_{i,\mathbf{k},\omega} = \delta f_{i,\mathbf{k},\omega}^{(0)} - \left(1 - i\frac{\partial}{\partial t}\frac{\partial}{\partial\omega}\right)\frac{Z_i e}{\omega - \mathbf{k}\cdot\mathbf{v} + i0}\phi_{\mathbf{k},\omega}\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}\Phi_i\right) \tag{27}$$

We use the Poisson equation to find the fluctuating potential in which the terms with a time-derivatives we consider by perturbation approach

$$\phi_{\mathbf{k},\omega} \approx \phi_{\mathbf{k},\omega}^{(0)} + \phi_{\mathbf{k},\omega}^{(1)} + \ldots : \quad \phi_{\mathbf{k},\omega}^{(0)} = \frac{4\pi}{k^2\epsilon_{\mathbf{k},\omega}^{(0))}}\sum_\alpha Z_\alpha e\int \delta f_{\mathbf{k},\omega}^{(0)}\frac{d\mathbf{p}}{(2\pi)^3} \tag{28}$$

$$\phi_{\mathbf{k},\omega}^{(1)} = \frac{i}{\epsilon_{\mathbf{k},\omega}^{(0)}}\frac{\partial}{\partial\omega}\left(\phi_{\mathbf{k},\omega}^{(0)}\frac{\partial}{\partial t}\epsilon_{\mathbf{k},\omega}^{(0)}\right) \tag{29}$$

$$\epsilon_{\mathbf{k},\omega}^{(0)} = 1 + \frac{4\pi}{k^2}\sum_\alpha Z_\alpha^2 e^2\int \frac{1}{\omega - \mathbf{k}\cdot\mathbf{v} + i0}\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}\Phi_\alpha\right)\frac{d\mathbf{p}}{(2\pi)^3}, \tag{30}$$

where the sum in α is performed on all types of plasma particles,namely on electrons and all ions including that which have nuclear reactions.

$$\delta f_{i,\mathbf{k},\omega} = \delta f_{i,\mathbf{k},\omega}^{(0)} - \frac{Z_i e}{\omega - \mathbf{k}\cdot\mathbf{v} + i0}\phi_{\mathbf{k},\omega}^{(0)}\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}\Phi_i\right) + \delta f_{i,\mathbf{k},\omega}^{(1)} \tag{31}$$

$$\delta f^{(1)}_{i,\mathbf{k},\omega} = i\frac{\partial}{\partial t}\frac{\partial}{\partial \omega}\frac{Z_i e}{\omega - \mathbf{k}\cdot\mathbf{v} + i0}\phi^{(0)}_{\mathbf{k},\omega}\left(\mathbf{k}\cdot\frac{\partial}{\partial \mathbf{p}}\Phi_i\right) - \frac{Z_i e}{\omega - \mathbf{k}\cdot\mathbf{v} + i0}\phi^{(1)}_{\mathbf{k},\omega}\left(\mathbf{k}\cdot\frac{\partial}{\partial \mathbf{p}}\Phi_i\right) \tag{32}$$

The effects related with temporal evolution of the system are described by an equation averaged on fluctuations. By neglecting the correlation corrections we have from (22)

$$\frac{\partial}{\partial t}\Phi_i = Z_i e\frac{\partial}{\partial \mathbf{p}}\cdot\int i\mathbf{k}'\langle\delta f_{i,\mathbf{k},\omega}\phi_{\mathbf{k}',\omega'}\rangle e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}-i(\omega+\omega')t}d\mathbf{k}d\mathbf{k}'d\omega d\omega' - \int w_{ij}\Phi_i\Phi_j\frac{d\mathbf{p}'}{(2\pi)^3}$$

$$= I^{(0)}_i + I^{(t)}_i - \int w_{ij}\Phi_i\Phi_j\frac{d\mathbf{p}'}{(2\pi)^3}, \tag{33}$$

where $I^{(0)}$ is determined by the fluctuations evolving in time $\delta f^{(0)}_{i,\mathbf{k},\omega}$ while

$$I^{(0)}_i = -Z_i e\frac{\partial}{\partial \mathbf{p}}\cdot\int i\mathbf{k}\langle\left[\delta f^{(0)}_{i,\mathbf{k},\omega} - \frac{Z_i e}{\omega - \mathbf{k}\cdot\mathbf{v} + i0}\left(\mathbf{k}\cdot\frac{\partial\Phi_i}{\partial \mathbf{p}}\right)\phi^{(0)}_{\mathbf{k},\omega}\right]\phi^{(0)}_{\mathbf{k}',\omega'}\rangle\times$$

$$e^{-i(\omega+\omega')t}d\mathbf{k}d\mathbf{k}'d\omega d\omega' \tag{34}$$

and $I^{(t)}_i$ is determined by the variation in time of the average distributions of the reacting nuclei

$$I^{(t)}_i = Z_i e\frac{\partial}{\partial \mathbf{p}}\cdot\int \mathbf{k}d\mathbf{k}d\mathbf{k}'\left\{\left\{\frac{\partial}{\partial t}\frac{\partial}{\partial \omega} - \left[\frac{1}{\epsilon^{(0)}_{\mathbf{k}',\omega'}}\left(\frac{\partial}{\partial \omega'}\frac{\partial}{\partial t}\epsilon^{(0)}_{\mathbf{k}',\omega'}\right) + \frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\left(\frac{\partial}{\partial \omega}\frac{\partial}{\partial t}\epsilon^{(0)}_{\mathbf{k},\omega}\right)\right]\right\}\times\right.$$

$$\left.\frac{Z_i e}{\omega - \mathbf{k}\cdot\mathbf{v} + i0}\left(\mathbf{k}\cdot\frac{\partial\Phi_i}{\partial \mathbf{p}}\right)\langle\phi^{(0)}_{\mathbf{k},\omega}\phi^{(0)}_{\mathbf{k}',\omega'}\rangle + \frac{1}{\epsilon^{(0)}_{\mathbf{k}',\omega'}}\frac{\partial}{\partial \omega'}\frac{\partial\epsilon^{(0)}_{\mathbf{k}',\omega'}}{\partial t}\langle\delta f^{(0)}_{i,\mathbf{k},\omega}\phi^{(0)}_{\mathbf{k}',\omega'}\rangle\right\}e^{-(\omega+\omega')t}d\omega d\omega' \tag{35}$$

We have taken into account that due to (20) (homogeniety of fluctuations in average) $\mathbf{k}' = -\mathbf{k}$. In the case the time evolution of fluctuations is taken into account in the expression(34) then in the expression (35), already containing a time-derivative of the averaged distribution, it is sufficient to use an approximate relations (21). We denote in the latter case the correlation functions by superscript ^{0,0} as for the distribution function in (21). We find then from (21)

$$\int\langle\phi^{(0,0)}_{\mathbf{k}',\omega'}\phi^{(0,0)}_{\mathbf{k},\omega}\rangle d\mathbf{k}' = -\frac{T}{2\pi^3k^2\omega}Im\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\right)\delta(\omega+\omega') \tag{36}$$

$$\int\langle f^{(0,0)}_{i,\mathbf{k}',\omega'}\phi^{(0,0)}_{\mathbf{k},\omega}\rangle d\mathbf{k}' = \frac{Z_i e}{2\pi^2k^2\epsilon^{(0)}_{\mathbf{k},\omega}}\Phi_i\delta(\omega - \mathbf{k}\cdot\mathbf{v})\delta(\omega+\omega') \tag{37}$$

By obtaining the last expression it was assumed that in all terms of (35) the particle distribution is thermal (satisfied with necessary accuracy). A small deviation from the thermal distribution is necessary to take into account only in terms not containing the time-derivatives on average distributions. By using the approximate expressions (36) (37) for correlation functions we find that the first term of, (35), which contains a full frequency derivative, is equal to zero and that

$$I_i^{(t)} = -\frac{Z_i^2 e^2}{2\pi^3}\frac{\partial}{\partial \mathbf{p}} \cdot \int \frac{\mathbf{k}}{k^2}\Phi_i d\omega d\mathbf{k}$$

$$\left\{ -\pi\delta(\omega - \mathbf{k}\cdot\mathbf{v}) \left(\frac{\partial \epsilon_{\mathbf{k},\omega}^{(0)}}{\partial t}\right) \frac{1}{\epsilon_{\mathbf{k},\omega}^{(0)}} \left(\frac{\partial}{\partial \omega}\frac{1}{\epsilon_{\mathbf{k},\omega}^{(0)}}\right) + \frac{(\mathbf{k}\cdot\mathbf{v})}{(\omega - \mathbf{k}\cdot\mathbf{v} + i0)} \times \right.$$

$$\left. \left\{ \frac{1}{\epsilon_{\mathbf{k},\omega}^{(0)}}\frac{\partial}{\partial\omega}\left[\frac{\partial \epsilon_{\mathbf{k},\omega}^{(0)}}{\partial t}\frac{1}{\omega} Im\left(\frac{1}{\epsilon_{\mathbf{k},\omega}^{(0)}}\right)\right] + \frac{1}{\omega} Im \left(\frac{1}{\epsilon_{\mathbf{k},\omega}^{(0)}}\right)\left(\frac{\partial \epsilon_{-\mathbf{k},-\omega}^{(0)}}{\partial t}\right)\frac{\partial}{\partial\omega}\left(\frac{1}{\epsilon_{-\mathbf{k},-\omega}^{(0)}}\right) \right\} \right\} \tag{38}$$

To calculate the change of the reaction rate due to the temporal evolution of the fluctuations, described by the expression (30) it is necessary to have in mind that we are interested only in effects of the first order in the nuclear reaction rates. In this limit we can obtain from expression (20) an approximate relation (compare with (27),(28)):

$$\int \langle \delta f_{i,\mathbf{k},\omega}^{(0)}(\mathbf{p}) \delta f_{j,\mathbf{k}',\omega'}^{(0)}(\mathbf{p}')\rangle e^{-(\omega+\omega')t} d\mathbf{k}' d\mathbf{p}'$$

$$= \delta_{i,j}\left[\Phi_i - \frac{i}{2}\frac{\partial \Phi_i}{\partial t}\left(\frac{\partial}{\partial\omega} + \frac{\partial}{\partial\omega'}\right)\right]\delta(\omega+\omega')\delta(\omega - \mathbf{k}\cdot\mathbf{v}). \tag{39}$$

The first term of the right hand side of (39) is zero since it leads to an conventional expression for Coulomb collision integral which makes all distributions to be a thermal distributions. For the thermal distribution the first term of (39) is exactly zero, and for non-thermal distributions the first term of (39) has a relative smallness of the order $1/N_d \ll 1$ and also can be neglected. In the corrections proportional to the derivative of the average distribution with respect of time one can assume that the distribution is thermal in the first place. We find

$$I_i^{(0)} = -\frac{Z_i^2 e^2}{4\pi^3}\frac{\partial}{\partial \mathbf{p}} \cdot \int \frac{\mathbf{k}}{k^2} d\mathbf{k} d\omega \left\{ \pi\delta(\omega - \mathbf{k}\cdot\mathbf{v})\frac{\partial \Phi_i}{\partial t}\left(\frac{\partial}{\partial\omega}\frac{1}{\epsilon_{\mathbf{k},\omega}^{(0)}}\right) \right.$$

$$\left. + \frac{(\mathbf{k}\cdot\mathbf{v})\Phi_i}{(\omega - \mathbf{k}\cdot\mathbf{v} + i0)\epsilon_{\mathbf{k},\omega}^{(0)}}\left[\left(\frac{\partial}{\partial\omega}\frac{1}{\omega\epsilon_{-\mathbf{k},-\omega}^{(0)}}\frac{\partial}{\partial t} Im\epsilon_{\mathbf{k},\omega}^{(0)}\right) + \frac{1}{\omega}\left(\frac{\partial}{\partial t} Im \epsilon_{\mathbf{k},\omega}^{(0)}\right)\left(\frac{\partial}{\partial\omega}\frac{1}{\epsilon_{-\mathbf{k},-\omega}^{(0)}}\right)\right]\right\} \tag{40}$$

One can perform the following simplification in the last expression: one can take the integration with respect to frequency by parts in the first term of (40) and not take into account (for a time) the time-derivative of the factor $1/(\omega - \mathbf{k}\cdot\mathbf{v} + i0)$. Then the term obtained in this way together with the second term of (40) contains the combinations

$$-\frac{1}{\epsilon^{(0)}_{-\mathbf{k},-\omega}}\frac{\partial}{\partial\omega}\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}+\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\frac{\partial}{\partial\omega}\frac{1}{\epsilon^{(0)}_{-\mathbf{k},-\omega}} \tag{41}$$

which does not change a sign for $\omega \to -\omega, \mathbf{k} \to -\mathbf{k}$, while the other factors $\mathbf{k}(\mathbf{k}\cdot\mathbf{v})/(\omega(\partial/\partial t)Im\epsilon_{\mathbf{k},\omega})$ are also not changing their sign and the only left is the part of the expression $1/(\omega-\mathbf{k}\cdot\mathbf{v}+i0)$ that also does not change in sign, i.e. $-i\pi\delta(\omega-\mathbf{k}\cdot\mathbf{v})$. This means that only the imaginary part of (41) is left. In the expression left with the derivative with respect to the frequency of the factor $1/(\omega-\mathbf{k}\cdot\mathbf{v}+i0)$ it is possible again to perform an integration by parts with respect to frequency. As a result of these calculations we get

$$I_i^{(0)}=-\frac{Z_i^2e^2}{4\pi^3}\frac{\partial}{\partial\mathbf{p}}\cdot\int\frac{\mathbf{k}}{k^2}d\mathbf{k}d\omega\left\{\pi\delta(\omega-\mathbf{k}\cdot\mathbf{v})\left[\frac{\partial\Phi_i}{\partial t}\left(\frac{\partial}{\partial\omega}\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\right)+\left(\frac{\partial}{\partial t}Im\epsilon^{(0)}_{\mathbf{k},\omega}\right)\times\right.\right.$$
$$\left.2\left[Im\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\right)\frac{\partial}{\partial\omega}Re\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\right)-Re\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\right)\frac{\partial}{\partial\omega}Im\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\right)\right]\right]+$$
$$\left.\frac{\omega\Phi_i}{(\omega-\mathbf{k}\cdot\mathbf{v}+i0)}\frac{\partial}{\partial\omega}\left(\frac{1}{\omega|\epsilon^{(0)}_{\mathbf{k},\omega}|^2}\frac{\partial}{\partial t}Im\epsilon^{(0)}_{\mathbf{k},\omega}\right)\right\} \tag{42}$$

Similar simplification is possible to use in the first term in the square brackets of (38). Integrating with respect to the frequency we get

$$I_i^{(t)}=-\frac{Z_i^2e^2}{2\pi^3}\frac{\partial}{\partial\mathbf{p}}\cdot\int\frac{\mathbf{k}}{k^2}\Phi_i d\omega d\mathbf{k}\left\{\pi\delta(\omega-\mathbf{k}\cdot\mathbf{v})\left[\frac{1}{|\epsilon^{(0)}_{\mathbf{k},\omega}|^2\epsilon^{(0)}_{\mathbf{k},\omega}}\left(\frac{\partial\epsilon^{(0)}_{\mathbf{k},\omega}}{\partial t}\right)\left(\frac{\partial\epsilon^{(0)}_{\mathbf{k},\omega}}{\partial\omega}\right)-\right.\right.$$
$$\left.2Im\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\right)\left[\left(\frac{\partial}{\partial\omega}Im\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k}\omega}}\right)\right)\frac{\partial}{\partial t}Re(\epsilon^{(0)}_{\mathbf{k},\omega})+\left(\frac{\partial}{\partial\omega}Re\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k}\omega}}\right)\right)\frac{\partial}{\partial t}Im(\epsilon^{(0)}_{\mathbf{k},\omega})\right]\right]$$
$$\left.+\frac{\omega}{(\omega-\mathbf{k}\cdot\mathbf{v}+i0)}\frac{\partial}{\partial\omega}\left[\frac{1}{\omega}Im\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\right)\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\frac{\partial}{\partial t}\epsilon^{(0)}_{\mathbf{k},\omega}\right)\right]\right\} \tag{43}$$

The sum of (42) and (43) allows to perform other simplifications. Collecting the terms which contain $\delta(\omega-\mathbf{k}\cdot\mathbf{v})$ we use the definition $I_i=I_i^{(t)}+I_i^{(0)}$ and have

$$I_i=\frac{Z_i^2e^2}{4\pi^2}\frac{\partial}{\partial\mathbf{p}}\cdot\int\frac{\mathbf{k}}{k^2}d\omega d\mathbf{k}\left\{\frac{\Phi_i}{\pi}\frac{\partial}{\partial t}\left[Re\left(\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\right)\left(\omega\frac{\partial}{\partial\omega}\frac{1}{\omega}\right)Im\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\right)\right.\right.$$
$$\left.Im\left(\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\right)\left(\omega\frac{\partial}{\partial\omega}\frac{1}{\omega}\right)Re\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\right)\right]-\delta(\omega-\mathbf{k}\cdot\mathbf{v})\times$$
$$\left.\left[\frac{\partial\Phi_i}{\partial t}\left(\frac{\partial}{\partial\omega}\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\right)+\frac{\Phi_i}{|\epsilon_{\mathbf{k},\omega}|^2}\left(\omega\frac{\partial}{\partial\omega}\frac{1}{\omega}\right)\frac{\partial}{\partial t}Re(\epsilon_{\mathbf{k},\omega})\right]\right\}. \tag{44}$$

In deriving these relations we used

$$Re\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\frac{\partial\epsilon^{(0)}_{\mathbf{k},\omega}}{\partial t}\right) = -\frac{1}{2}|\epsilon^{(0)}_{\mathbf{k},\omega}|^2\frac{\partial}{\partial t}\frac{1}{|\epsilon^{(0)}_{\mathbf{k},\omega}|^2} \tag{45}$$

$$Im\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\right) Im\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\frac{\partial\epsilon^{(0)}_{\mathbf{k},\omega}}{\partial t}\right) = \frac{1}{2|\epsilon^{(0)}_{\mathbf{k},\omega}|^2}\frac{\partial Re(\epsilon^{(0)}_{\mathbf{k},\omega})}{\partial t} + \frac{1}{2}\frac{\partial}{\partial t}Re\left(\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\right) \tag{46}$$

The expression in the first square brackets of (44) can be transformed to

$$Im\left[\frac{1}{\epsilon_{\mathbf{k},\omega}}\frac{\partial}{\partial\omega}\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\right] - Re\left(\frac{1}{\omega}\right) Im\left[\frac{1}{\epsilon_{\mathbf{k},\omega}}\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\right] \tag{47}$$

from which it becomes evident that after frequency integration the first term of (47)is zero. To find this result it is necessary to transfer the imaginary sign out of sign of time derivative and integration with respect to the frequency and to take into account that the derivative with respect to frequency of the factor $1/(\omega-\mathbf{k}\cdot\mathbf{v}+i0)$ has no poles in the upper part of the complex ω as well as the factor $1/\epsilon^{(0)}_{\mathbf{k},\omega}$(the latter due to analytical properties of dielectric permittivity). The second term also can be transferred to an expression not containing poles in the upper plane of the complex ω by adding and subtracting the correspondent expression with $Im(1/\omega+i0) = -\pi\delta(\omega)$. That procedure shows that the first square bracket of (44) gives

$$\int d\omega\pi\delta(\omega)Re\frac{1}{(\mathbf{k}\cdot\mathbf{v})\epsilon_{\mathbf{k},0}} = \int d\omega\frac{\pi}{\omega\epsilon_{\mathbf{k},0}}\delta(\omega-\mathbf{k}\cdot\mathbf{v}) \tag{48}$$

The last expression in (48) is used for obtaining an compact expression for the final result containing only $\delta(\omega-\mathbf{k}\cdot\mathbf{v})$. Thus

$$I_i = -\frac{Z_i^2e^2}{4\pi^2}\frac{\partial}{\partial\mathbf{p}}\cdot\int\frac{\mathbf{k}}{k^2}d\omega d\mathbf{k}\delta(\omega-\mathbf{k}\cdot\mathbf{v})\left\{\frac{\partial\Phi_i}{\partial t}\left(\frac{\partial}{\partial\omega}\frac{1}{\epsilon^{(0)}_{\mathbf{k},\omega}}\right) - \frac{\Phi_i}{\omega}\left(\frac{\partial}{\partial t}\frac{1}{\epsilon^{(0)}_{\mathbf{k},0}}\right)\right.$$
$$\left.+\frac{\Phi_i}{|\epsilon^{(0)}_{\mathbf{k},\omega}|^2}\left((\omega\frac{\partial}{\partial\omega}\frac{1}{\omega}\right)\frac{\partial}{\partial t}Re(\epsilon^{(0)}_{\mathbf{k},\omega})\right\} \tag{49}$$

6 Renormalization of the particle distributions

As the correlation corrections the corrections due to time evolution of the system can be written as some effective change of the probability of nuclear reaction rate. Notice that the correspondent equation (33) which takes into account the effects of the time evolution has a form different from that which is usually used when the plasma corrections are neglected. Namely, in the case the plasma corrections are taken into account the equation has a form

$$\frac{\partial\Phi_i}{\partial t} = I_i^{(0)} + I_i^{(t)} - \int w_{ij}\Phi_i\Phi_j\frac{d\mathbf{p}'}{(2\pi)^3}, \tag{50}$$

while in the case where the plasma corrections are not taken into account this equation has a form

$$\frac{\partial \Phi_i}{\partial t} = -\int w_{ij} \Phi_i \Phi_j \frac{d\mathbf{p}'}{(2\pi)^3}. \tag{51}$$

It is necessary to remind that here we restrict ourselves only to first order plasma corrections. With this accuracy the equation (50) can be converted to the standard form (51) by renormalization of the particle distribution and by introducing effective probability. Let us introduce the renormalized distribution $\Phi_i^{(R)}$ as a solution of the equation

$$\frac{\partial \Phi_i}{\partial t} - I_i^{(0)} - I_i^{(t)} = \frac{\partial \Phi_i^R}{\partial t} \tag{52}$$

and assume that the solution of it has a form

$$\Phi_i = (1 + \Lambda_{i,\mathbf{p}}^R) \Phi_i^{(R)} \tag{53}$$

Than by taking into account that the corrections are small and also the both of the distribution functions should be renormalized we find finally (by putting back the correlation corrections) a final equation in a usually used standard form

$$\frac{\partial \Phi_i^{(R)}}{\partial t} = -\int w_{ij}^{eff} \Phi_i^{(R)} \Phi_j^{(R)} \frac{d\mathbf{p}'}{(2\pi)^3} \tag{54}$$

where

$$w_{ij}^{eff} = w_{ij}(1 + \Lambda_{ij}^{(S)} + \Lambda_{i,\mathbf{p}}^{(R)} + \Lambda_{j,\mathbf{p}'}^{(R)}) \tag{55}$$

In solving the equation for Φ_i^R one should take into account that the redistribution of particles in momenta is occurring much faster than the inverse rate of nuclear reactions and therefore on time is depending only the particle density. The distribution function of the reacting nuclei is then a product of thermal distribution in momenta and the time-dependent density. The time derivative of dielectric permittivity entering in I^i is determined by all reacting nuclei and by the rate of change in time of their distribution function, i.e. by the time-derivatives of their concentration. This point is important in momenta integrations with respect to the particle distributions in the time derivatives of dielectric permittivities.

Further step can be made in the case we consider the nuclear cycle (in applications to the solar interior it could be the hydrogen cycle) and in the case we consider the asymptotic (for large times) state where the rate of all reactions become equal and are determined by the most slowest reaction. At the moment where the rates of all reaction started to be be equalized the renormalized distribution will describe an energy shift in the distribution of each sort of nuclei caused by all reacting species of the cycle. In this sense this shift is a collective energy shift. Since the rates of all reactions coincide asymptotically the relative derivatives on time in all terms of (50) are equal. That fact allows easily find the coefficient of renormalization for each nuclei species of the cycle. In this derivation it is necessary to take into account that the corrections are considered to be small and in the renormalization terms one should not distinguish between the renormalized and non-renotmalized distributions. The expression for $\Lambda_i^{(R)}$ obtained by this procedure is

$$\Lambda_{i,\mathbf{p}}^{(R)} = -\frac{Z_i^2 e^2}{4\pi^2} \int \frac{\mathbf{k} d\mathbf{k} d\omega}{k^2} \left(\frac{\partial}{\partial \mathbf{p}} - \frac{\mathbf{v}}{T} \right) \delta(\omega - \mathbf{k}\cdot\mathbf{v}) Re \left\{ \left(\frac{\partial}{\partial \omega} \frac{1}{\epsilon_{\mathbf{k},\omega}^{(0)}} \right) + \frac{1}{\omega (\epsilon_{\mathbf{k},0}^{(0)})^2} (\tilde{\epsilon}_{\mathbf{k},0} - 1) + \frac{1}{|\epsilon_{\mathbf{k},\omega}^{(0)}|^2} \left(\omega \frac{\partial}{\partial \omega} \frac{1}{\omega} \right) \left(\tilde{\epsilon}_{\mathbf{k},\omega}^{(0)} - 1 \right) \right\} \tag{56}$$

$$\tilde{\epsilon}_{\mathbf{k}\omega} = 1 + \frac{4\pi}{k^2} \sum_{\tilde{j}} \int \frac{Z_{\tilde{j}}^2 e^2}{(\omega - \mathbf{k}\cdot\mathbf{v}' + i0)} \left(\mathbf{k} \cdot \frac{\partial \Phi_{\tilde{j}}}{\partial \mathbf{p}'} \right) \frac{d\mathbf{p}'}{(2\pi)^3} \tag{57}$$

and the summation on $\tilde{j}$ is performed here only with respect to reacting nuclei of the cycle and also on electrons if they are taking part in nuclear reactions (as it is the case for hydrogen cycle).

It should be emphasized that the probability w_{ij} depends on the relative nuclear energy E_r only and in the limit $E_r \gg T$ has a sharp maximum close to the Gamov's energy E^G but the effective probability (55) depends also on the momentum of each reacting nuclei, i.e it depends on velocities of nuclei relative to the medium (plasma) which is natural for collective effects.

7 Change in collective electrostatic energy of nuclei and the energy shift

Each nuclei i in plasma is surrounded by a polarization screening charge and has an additional self-energy $E_i^{(s)}$ which, strictly speaking, has a meaning and can be calculated only in non-dissipation media (plasmas). The calculations can be performed only if one can neglect the imaginary part of dielectric plasma permittivity. This does not correspond to the problem we are dealing since the dissipation processes related with nuclear reactions play an important role. But asymptotically at large time intervals where the rates of all reactions become equalized it is possible to find the rate of change of the total electrostatic energy per single nuclei which plays a role of effective collective self-energy proportional to the square of the charge of the nuclei. In a large system the polarization charges of interacting nuclei are created by fluctuations of all remained plasma particles and their nature is collective. In presence of interacting and reacting nuclei the dissipation processes related with their reactions do not allow to introduce and define the self-energy. But the rate of change of the total electrostatic energy can be found and in the conditions where the rate of all reactions become equalized these changes can be attributed to each nuclei and describe some energy shift in their distribution. This energy shift can be considered as self-energy produced collectively by all reacting nuclei and, as we show, is directly related with nuclei distribution renormalization. Before the equalization of the rates such physical interpretation is not possible and it cannot be performed mathematically and some effective energy shift cannot be found.

The effective self-energy found in the way described above should be different, and is indeed different, from the sum of self-energies of particles in a non-dissipative system. Although the collective self-energy per particle is similar to the self-energy of single particles, its value and sign are different, although both of them are proportional to the square of the nuclei charges. The main difference of the collective energy shift from the self-energy is that the total integral with respect to all particle momenta of a certain nuclei type is zero for the collective energy shift while it is not zero for the self-energy shift. Therefore the sign and the value of the correspondent corrections for nuclear reactions are determined by the values of nuclei momenta which contribute the most for corrections calculated with the weight of the probability of the nuclear reaction.

We will show that the rate of the change of the total electrostatic energy of the system is indeed related with the correction due to particle distribution renormalization. For the rate of the change of the total electrostatic energy of the system we have

$$\frac{dW^s}{dt} = \frac{1}{4\pi}\langle \mathbf{E}\cdot\frac{d\mathbf{D}}{dt}\rangle = \int \sum_i Z_i e \langle \delta f_{i,\mathbf{k},\omega}(-i\omega')\phi_{\mathbf{k}',\omega'}\rangle e^{-i(\omega+\omega')t} d\mathbf{k}\mathbf{k}' d\omega d\omega' \qquad (58)$$

This relation should be compared with the first term of the right hand side of (33). By taking into account that the change of the particle energy can be calculated by multiplying of the left hand side of (33) on the energy of a single particle with subsequent integration with respect on particle distributions, that after integration by parts the derivative with respect to momenta is substituted by the factor $\mathbf{k}\cdot\mathbf{v}$ and that due to (40(41) $\mathbf{k}\cdot\mathbf{v}$ this factor can be substituted by ω it is easy to find that (42) (43) correspond to such changes in particle distributions which describe the changes of total electrostatic energy of the system. This is the reason why the collective corrections to each particular nuclear reaction depend on the rates of the reaction of the whole cycle.

We can then show that the renormalization terms found in the previous section is possible to interpret as appearance of energy shift $\delta\epsilon_{i,\mathbf{p}}$ or, in other words, as appearance of additional energy, which should be added to the nuclear kinetic energy $\epsilon_{i,\mathbf{p}} = p^2/2m_i$ $\Phi_i^R(\mathbf{p}) \propto \exp(-\epsilon_{i,\mathbf{p}}/T - \delta\epsilon_{i,\mathbf{p}}/T) \approx \exp(-\epsilon_{i,\mathbf{p}}/T)(1 - \delta\epsilon_{i,\mathbf{p}}/T)$, where $\epsilon_{i,\mathbf{p}}$ is the particle energy i,and $\delta\epsilon_{i\mathbf{p}}$is the collective energy shift of particle i energy. By taking into account that $\Phi_i^{(R)} = \Phi_i(1 - \Lambda_{i,\mathbf{p}}^{(R)})$we find

$$\Lambda_{i,\mathbf{p}}^{(R)} = \frac{\delta\epsilon_{i,\mathbf{p}} - \delta\epsilon_i^{(0)}}{T} \qquad \delta\epsilon_i^{(0)} = \frac{\int \frac{\delta\epsilon_{i,\mathbf{p}}}{T}\exp\left(-\frac{\epsilon_{\mathbf{p}}}{T}\right)d\mathbf{p}}{\int \exp\left(-\frac{\epsilon_{\mathbf{p}}}{T}\right)d\mathbf{p}} \qquad (59)$$

where $\delta\epsilon_i^{(0)}$ is appearing due to renormalization. The relation (59) assumes only that the distribution of the reacting nuclei is still thermal and this does not allow to have any flexibility for other physical interpretation of the renormalization correction but the one related with appearance of energy shift.

The expression (56) has special property that the integral of (56) with thermal distribution is zero. But this does mean that

$$\frac{\delta\epsilon_{i\mathbf{p}}}{T} = \Lambda^{(R)}_{i,\mathbf{p}} \tag{60}$$

The expression (60) is different from that used in [10,12] where the time evolution of the average distribution was taken into account but the time evolution of fluctuations was not taken into account. The latter lead to the final expression for the corrections (49),(56) which are more general than that of [10,12]. In non-dissipative system we have [10-12]

$$\frac{E^s_{i,\mathbf{p}}}{T} = \frac{Z_i^2 e^2}{4T\pi^2}\int d\mathbf{k}d\omega\delta(\omega - \mathbf{k}\cdot\mathbf{v})\omega^2\frac{\partial}{\partial\omega}\left(\frac{1}{\omega Re(\epsilon^{(0)}_{\mathbf{k},\omega})}\right) \tag{61}$$

Contrary to (57) the integral of (61) with respect to the momenta with a thermal distribution is not zero. The collective energy shift (60) depends obviously only on the absolute value of the particle velocity, i.e. on particle energy. Therefore it should change the sign at certain energy due to zeros of the integral with respect to all momenta. The integration with a weight w_{ij} will depend both on the peak in relative energy at the Gamov energy and on the translational distribution which is thermal. For determination of the sign of the correction it is also important the angular averaging.

Due to assumption that the particle distribution is thermal the interpretation of the corrections as an energy shift is the only possible. The described energy shift is a collective one and in a certain sense is an analog of the Lamb shift. The shift itself is manifested as effective renormalized nuclei self-energy.

8 Simplification of corrections

The ratio of the nuclear reactions rate which takes into account the collective plasma corrections, R_{ij}, to its value in absence of plasma, $R^{(0)}_{ij}$ is given by

$$\frac{R_{ij}}{R^{(0)}_{ij}} = \frac{\int w^{eff}_{ij}\Phi_i(\mathbf{p})\Phi_j(\mathbf{p}')d\mathbf{p}d\mathbf{p}'}{\int w_{ij}\Phi_i(\mathbf{p})\Phi_j(\mathbf{p}')d\mathbf{p}d\mathbf{p}'} = 1 + \Lambda^{(C)} + \Lambda^{(T)} \tag{62}$$

where $\Lambda^{(C)}$ describes the correlation corrections and $\Lambda^{(T)}$ describes the corrections due to the time evolution (superscript T is used from the word "time"). The value $\Lambda^{(C)}$ being independent on particle momenta coincides with the earlier obtained expression and for $\Lambda^{(T)}$ we have

$$\Lambda^{(T)} = \frac{\int w_{ij}(\Lambda^{(R)}_{i,\mathbf{p}} + \Lambda^{(R,)}_{j,\mathbf{p}'})\Phi_i(\mathbf{p})\Phi_j(\mathbf{p}')d\mathbf{p}d\mathbf{p}'}{\int w_{ij}\Phi_i(\mathbf{p})\Phi_j(\mathbf{p}')d\mathbf{p}d\mathbf{p}'} \tag{63}$$

By introducing the relative velocity $\mathbf{v}_r = \mathbf{v} - \mathbf{v}'$ and the center of mass velocity $\mathbf{V} = (m_i\mathbf{v} + m_j\mathbf{v}'/(m_i + m_j)$ of two reacting nuclei it is possible to consider the expression (63) as an averaging with respect to the relative motion and to the center of mass motion

$$\Lambda^{(T)} = \frac{\int w_{ij}(\Lambda^{(R)}_{i,\mathbf{p}} + \Lambda^{(R)}_{j,\mathbf{p}'})e^{-\frac{E_r}{T} - \frac{V^2}{2(m_i+m_j)T}} d\mathbf{v}_r d\mathbf{V}}{\int w_{ij} e^{-\frac{E_r}{T} - \frac{V^2}{2(m_i+m_j)T}} d\mathbf{v}_r d\mathbf{V}} \tag{64}$$

The expression under the integral as a function of the relative energy $E_r = \mu_{ij} v_r^2/2$ has a sharp maximum at the Gamov's energy $E^G_{ij} = \mu_{ij}(v^G_{ij})^2/2$ where μ_{ij} is the reduced mass$\mu_{ij} = m_i m_j/(m_i + m_j)$. Therefore the integration with respect to the relative energy results in a substitution of Gamov's energy for the relative energy in all factors in front of the probability of the nuclear reaction. Then the integrals with respect of the relative energy is the same as in absence of plasma corrections. Of importance is that $\Lambda^{(R}_{i,\mathbf{p}} + \Lambda^{(R}_{j,\mathbf{p}'}$ depend on the momenta of nuclei relative to plasma and therefore on the absolute values of nuclear momenta. Apart of relative and translational energy they depend also on the angles. The expression (64) contains the correspondent angular averaging. After one extracts the angular dependence from the expression under the integral (64) it is possible to perform analytically the integration with respect to the absolute value of k by taking into account that the ratio $\omega/kv_{T\alpha}$(where $v_{T\alpha}$ is the thermal velocity of particles α) is independent on the absolute value of k due to $\omega = \mathbf{k}\cdot\mathbf{v}$. We will give later the explicit result of these calculations.

For real application to nuclear reaction rates in plasmas apart of such integration it is necessary to take into account that in the case the plasma contains a mixture of different reacting nuclei (as it is for hydrogen cycle) it is necessary to transfer the corrections to the form containing the relative mass abundances of different types of nuclei. We assume here that the ions are completely ionized, i.e. they are bared nuclei the charge of which is compensated in volume by free plasma electrons

$$n_e = \sum_i Z_i n_i$$

and we use the following known analytic expression for the dielectric permittivity

$$\epsilon^{(0)}_{\mathbf{k},\omega} = 1 + \frac{1}{k^2 d_e^2} + \sum_i \frac{1}{k^2 d_i^2} W(s_i) = 1 + \frac{1}{k^2 d^2 (1 + Z_{eff})}\left(1 + \sum_i Z_i^2 n_i W(s_i) / \sum_i Z_i n_i\right) \tag{65}$$

where the summation is performed only on nuclei. It is also assumed in (65) that the nuclei velocities are much less that the thermal electron velocity and therefore for the electrons the Debye screening approximation is used (the second term in the first equality (65)), d is the total Debye radius and $W(s)$ is the known plasma dispersion function

$$\frac{1}{d^2} = \frac{1}{d_e^2} + \sum_i \frac{1}{d_i^2}; \quad W(s) = 1 + s\exp(-s^2)\left(i\sqrt{\pi} - 2\int_0^s \exp(t^2)dt\right); \tag{66}$$

and

$$s_i = \frac{\omega}{\sqrt{2}kv_{Ti}}; \quad 1 + Z_{eff} = \frac{d_e^2}{d^2}; \quad Z_{eff} = \sum_i Z_i^2 n_i / \sum_i Z_i n_i \tag{67}$$

Mention that according to the consideration given above both s_i and $W(s_i)$ depend only on angular variables. In the case we use the notation $X_i = m_i n_i / \sum_j m_j n_i$ for relative mass density of nuclei, we can rewrite the dielectric permittivity in the form

$$\epsilon_{\mathbf{k},\omega}^{(0)} = 1 + \frac{1}{k^2 d^2 \left(1 + \sum \frac{Z_i^2 X_i}{m_i} / \sum \frac{Z_i X_i}{m_i}\right)} \left(1 + \sum_i \frac{Z_i^2 X_i}{m_i} W(s_i) / \sum_i \frac{Z_i X_i}{m_i}\right) \tag{68}$$

Notice that the expression $\Lambda_{i,\mathbf{p}}^{(R)}$ contains a sum with respect to the types of ions j' of the expressions, depending on k^2 and the angular variables though the $s_{j,i,j'}$ where

$$s_{i,j,j'} = \frac{\mathbf{k}\cdot\mathbf{v}}{k\sqrt{2}v_{Tj'}} = \sqrt{\frac{m_{j'}}{m_i + m_j}} \left(yx + \lambda_{ij} z \frac{m_j}{m_i + m_j}\right) \tag{69}$$

and

$$s_{j,i,j'} = \frac{\mathbf{k}\cdot\mathbf{v}'}{k\sqrt{2}v_{Tj'}} = \sqrt{\frac{m_{j'}}{m_i + m_j}} \left(yx - \lambda_{ij} z \frac{m_i}{m_i + m_j}\right). \tag{70}$$

Here x is the cosine of the vector $\mathbf{k}$ and the vector of the relative velocity $\mathbf{v_r}$ and z is the cosine of the vector $\mathbf{k}$ and the translational velocity $\mathbf{V}$, y is the normalized velocity of translational motion and $\lambda_{ij}$is the normalized velocity corresponding to the Gamov's energy

$$y = \frac{V}{\sqrt{2T/(m_i + m_j)}}; \quad \lambda_{ij} = \frac{v_{ij}^G}{\sqrt{2T/(m_i + m_j)}} \tag{71}$$

Using the identity

$$\omega \frac{\partial W(s)}{\partial \omega} = s \frac{dW(s)}{ds} = (1 - 2s^2) W(s) - 1 \tag{72}$$

and

$$\int d\omega \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}} \delta(\omega - \mathbf{k}\cdot\mathbf{v}) L_{\mathbf{k},\omega} = \int d\omega \delta(\omega - \mathbf{k}\cdot\mathbf{v}) \frac{k^2 v_{Ti}^2}{T} \frac{\partial L_{\mathbf{k},\omega}}{\partial \omega} \tag{73}$$

it is useful to write the relation (56) in the form containing only the operators $\omega\partial/\partial\omega$

$$\Lambda_{i,\mathbf{p}}^{(R)} = \Lambda_{i,\mathbf{p}}^{(T)} + \tilde{\Lambda}_{i,\mathbf{p}}^{(T)} \tag{74}$$

$$\Lambda_{i\mathbf{p}}^{(T)} = \frac{Z_i^2 e^2}{4\pi^2 T} \int \frac{d\mathbf{k}}{k^2} Re \left\{ \left[1 + \frac{k^2 v_{Ti}^2}{\omega^2} \left(1 - \omega \frac{\partial}{\partial \omega}\right)\right] \omega \frac{\partial}{\partial \omega} \left(\frac{1}{\epsilon_{\mathbf{k},\omega}^{(0)}}\right) \right\}_{\omega = \mathbf{k}\,\mathbf{v}} \tag{75}$$

$$\tilde{\Lambda}_{i,\mathbf{p}}^{(T)} = \frac{Z_i^2 e^2}{4\pi^2 T} \int \frac{d\mathbf{k}}{k^2} Re \left\{ \left(1 + \frac{k^2 v_{Ti}^2}{\omega^2}\right) \left[\frac{(\tilde{\epsilon}_{\mathbf{k},0}^{(0)} - 1)}{|\epsilon_{\mathbf{k},0}^{(0)}|^2} - \frac{Re(\tilde{\epsilon}_{\mathbf{k},\omega}^{(0)} - 1)}{|\epsilon_{\mathbf{k},\omega}^{(0)}|^2} \right] + \right.$$

$$\left. \frac{1}{|\epsilon_{\mathbf{k},\omega}^{(0)}|^2} \omega \frac{\partial}{\partial \omega} Re(\tilde{\epsilon}_{\mathbf{k},\omega}^{(0)} - 1) + \frac{k^2 v_{Ti}^2}{\omega^2} \left(\omega \frac{\partial}{\partial \omega}\right) \left[2 \frac{Re(\tilde{\epsilon}_{\mathbf{k},\omega}^{(0)} - 1)}{|\epsilon_{\mathbf{k},\omega}^{(0)}|^2} - \frac{1}{|\epsilon_{\mathbf{k},\omega}^{(0)}|^2} \omega \frac{\partial}{\partial \omega} Re(\tilde{\epsilon}_{\mathbf{k},\omega}^{(0)} - 1) \right] \right\}_{\omega = \mathbf{k}\cdot\mathbf{v}} \quad (76)$$

The integration with respect to k in (76) can be performed analytically and the problem of explicit calculation of the corrections is reduced to an averaging with respect to 1) y with an form-factor $(4/\sqrt{\pi}) \int_0^\infty ... y^2 \exp(-y^2) dy$ 2) averaging on x with $(1/2) \int_{-1}^{1} ... dx$ and 3) averaging on z with $(1/2) \int_{-1}^{1} ... dz$. By using (66) we find the final expression which can be used in numerical calculations

$$\Lambda^{(T)}(y,x,z) = -\frac{Z_i^2 e^2}{4\sqrt{\pi} T d \sqrt{1 + Z_{eff}}} \int_0^\infty y^2 e^{-y^2} dy \int_{-1}^{1} dx \int_{-1}^{1} dz Re\{\Lambda_{ij}^{(T)}(y,x,z)\} + \tilde{\Lambda}_{ij}^{(T)}\}$$

$$+ i \leftrightarrow j \quad (77)$$

where the $\Lambda_{ij}^{(T)}(y,x,z)$ is determined by averaging of(75) and the $\tilde{\Lambda}_{ij}^{(T)}(y,x,z)$ is determined by averaging of (76),while the explicit expressions for $\Lambda^{(T)}(x,y,z)$ and $\Lambda_{ij}^{(T)}(y,x,z)$ are given in the Appendix . For $i \leftrightarrow j$ expressions(A1)(A2)will contain (70) instead of (69) and correspondingly $s_{j,i,j}$ instead of $s_{i,j,i}$ and Z_j^2 instead of Z_i^2. The mass of the nuclei can be calculated in the units of the proton mass since the masses enter in the same power in the numerators and denominators of (A1)(A2), i.e. m in (69),(70) and (A1)(A2) is the atom number of the correspondent nuclei.

The expressions(A1)(A2) allow to calculate the rates of nuclear reactions between the nuclei i and j for arbitrary mixture of nuclei in plasma. The summation with respect of j' includes both reacting and non-reacting nuclei , while the summation with respect to $\tilde{j}'$ includes only the reacting nuclei.

9 Numerical results

Let us give some numerical results of calculations of corrections inn the center part of the Sun using the presently accepted values [3]: $X_H = 0.3411, X_{He} = 0.6387, X_C = 0.00003, X_N = 0.0063, X_O = 0.0085$. Then $Z_{eff} = 2.551, \sum_j Z_j X_J / m_j = 0.661$. The parameter $e^2 dT$ depending on the temperature and density and abundance is a known parameter which we will exclude by nurmalizaing all corrections with respect to it. For temperature $T = 1.5 KeV$ and density $n = 5 \times 10^{25} cm^{-3}$ accepted in the present solar models we find $e^2/Td = 0.05$ which gives for the Salpeter's corrections for p,p reactions(the beginning of the hydrogen cycle) 5% and gives for the reactions with ^7Be (the end of the hydrogen cycle)$4e^2/Td \approx 20\%$. To exclude the trivial dependence on the temperature and density through the parameter e^2/Td we will present the relative value of the corrections in units of e^2/Td, namely for $\Lambda_{ij,N} \equiv \Lambda_{ij} Td/e^2$. The Table 1 presents the results of numerical computation for the collective plasma corrections obtained by using the expressions (77)(A1)(A2). The table contains the accepted value for the relative Gamov's velocities λ_{ij}, contains the value of for correlation corrections $\Lambda_{ij,N}^{(C)}$ and the value of the corrections due to the collective energy shift $\Lambda_{ij,N}^{(T)}$, as well as the Salpeter'e corrections $\Lambda_{ij,N}^{(S)}$ and the expression for the total corrections $\Lambda_{ij,N}^{(C)} + \Lambda_{ij,N}^{(T)}$ found in the present investigation.

Table 1

N.	Reaction	$\lambda_{i,j}$	$\Lambda_{ij,N}^{(S)}$	$\Lambda_{ij,N}^{(C)}$	$\Lambda_{ij,N}^{(T)}$	$\Lambda_{ij,N}^{(C)} + \Lambda_{ij,N}^{(T)}$
1.	$p+p$	4.280	1	1	+0.357	1.357
2.	$p+^2H$	4.757	1	1	+0.313	1.313
3.	$^3He+^3He$	8.150	4	4	+0.98	4.98
4.	$^3He+^4He$	8.420	4	4	+0.943	4.943
5.	^7Li+p	10.234	3	3	+0.676	3.676
6.	^7Be+p	11.264	4	4	+1.056	4.707
7.	^7Be+e	0	0	−4	+0.788	−3.212

10 Discussion of the results

Previously it was assumed that all collective corrections are the Salpeter's corrections and they lead to an enhancement of the reaction rates. As can be seen from the Table 1 not all of the collective corrections give an enhancement of the reaction rate but some of them, for example the reactions with ^7Be capturing electrons, describe a decrease of the reaction rate. The latter effect is important since the reaction with ^7Bc has two branches and the effect found here affects the branching, the value of which which was for long time a hot point in discussions in solar nuclear physics. The decrease of the reaction of electron capture by ^7Be nuclei decreases the number of Boron nuclei which are creating neutrinos detected in first Homestake experiment and also in Super-Kamiokande experiment. For all other reactions of the hydrogen cycle the calculated corrections are larger than that previously accepted and give a larger enhancement of the reaction rates. The total correction due to the time evolution $\Lambda_{ij}^{(T)}$ are positive and are the sum of the corrections found in [10-12] (describing a decrease of the reaction rates) and the correction due to temporal evolution of fluctuation caused by reactions which create neutrinos freely leaving the system and increasing the nuclear reaction rate. The latter effect appears to be larger than that of [10-12] leading to total enhancement of the nuclear reaction rate. On the other hand, the proved absence of the Salpeter's screening removes the interference effects between the changes in probabilities and correlation effects. Left are only the correlation effects which lead to an enhancement of the reaction rates. The proved statement that the Salpeter's correction are in first approximation zero has an indirect confirmation from observations. Indeed, if the Salpeter's corrections will be not zero their interference with the correlation effect will lead to a result which contradicts the observations. By taking into account the correction of the sign of the interference term (mentioned in the end of [10-12]) we find that (without the temporal evolution effects) the corrections will be 4 times larger than Salpeter's corrections, i.e. totally (with temporal corrections) will be about 84% for reactions 3 4 of the Table 1. The latter contradicts the recent data on solar seismology. The correlation effects except the last line of the Table 1 "recover" the result often used in the present standard solar models. but the total correction given in Table 1 are larger than that usually used. The existing experience of numerical calculations of the solar models shows that even a small corrections in the probabilities of nuclear reactions can affect the final result of the parameters of an evolving in time nuclear system. Thus the difference of the final expressions of the Table 1 from the Salpeter's corrections could be of importance. The last line of the Table 1 clearly illustrates the quantitative difference of the correlation corrections from the Salpeter's corrections. The correlation corrections have even an opposite sign. Different is also the physical meaning of the correlation corrections and the Salpeter's corrections.

We should discuss validity for neglecting the terms which leads to the result of zero value of the Salpeter's corrections. It is determined by the possibility of neglecting of the second term in the Eg.(6). The latter describes the influence of the fluctuating electric field on the rates of nuclear reactions. The present theory of weak plasma fluctuations provides for this effect an expression which is divergent at large k. If for an estimate one uses $k_{max} \approx 1/r$ where r as a characteristic size of nuclear tunneling one finds that the effect related with fluctuating electric fields is r/R times smaller than that taken into account here (R is the characteristic size of fluctuations). If, on other hand, one uses $k_{max} \approx N_d/d$ (large angle scattering in collisions), then the relative contribution of the second term of (6)is even smaller $\approx r^2 N_d/R^2$.

The dense plasmas can be found in laboratory laser fusion experiments where the temperature are of the same order as in the solar interior but the density could be two orders of magnitude higher. Then the factor e^2/dT will be one order of magnitude larger and the correlation corrections to $D+T$ reaction will be 50% instead of 5% in solar interior. The total corrections calculated numerically by using the present theory will be larger than the correlation corrections by the factor 1.194 which corresponds to 51% increase of the reaction rate.

According to the present investigation the physical meaning of the collective corrections is related with a change of nuclear distribution (of correlation type and of temporal type) but not with change of the probabilities of nuclear reactions as was assumed earlier. In this sense the corrections found here are not specific for nuclear reactions and will appear for any other processes, for example, for the chemical reactions or for the reactions in the colloids or in the biological macro-molecules. One should always have in mind that the explicit expressions were obtained here assuming that the rate of the reactions should be less than the characteristic time of fluctuations. This emphasizes once more the physical difference of the corrections found in the present research from that related with screening of nuclear reactions. The present results also resolve the old paradox between the dynamic and static screening of nuclear reactions since the corrections are nothing to do with effects of influencing the reaction rates. The correlation corrections in the plasmas can be determined by and are determined by the static dielectric permittivity.

It was usually accepted in astrophysical applications that the approximation of weak Salpeter's screening can be applied only as first approximation and it is not sufficient for more precise description of the nuclear processes in Solar interior. It is much more insufficient for description of nuclear reactions in stars on other stages of their evolution, which are different than that of the present Sun. Usually the interpolation formulas between the weak and strong screening were used for this purpose. From the results of the present work it is clear that the interpolation formulas cannot be used, not speaking on the Salpeter's screening itself. To obtain the desired interpolation formulas it is necessary to describe the correlation corrections more precisely. In the present paper there was obtained only the result for weak correlations by expansion in a small parameter, the number of particle in the Debye sphere. In solar interior this parameter is $1/5 - 1/7$ and the better accuracy is desired to be available Also the small parameter used is different from that in Salpeter's corrections, namely the ratio of the thermal energy T to the Gamov's energy E_G. The correct interpolation formulas can be obtained in the case if strong correlations can be treated analytically. Contrary to the Salpeter's corrections, where the strong screening effect can be treated, the strong correlation effect was not yet properly considered in any plasma theory and in general it depends on many unsolved problems. Up to present only the problem of weak correlation has an explicit analytical solution. Although different approaches were used to find some strong correlation effects, the approaches used are based on several assumptions not proved yet. Concerning the correlation effects for nuclear reactions one can on basis of the present consideration propose a method and a procedure to find the correlation effects in the next order in small parameter $1/N_d$ (although a cumbersome calculations needed to be performed to find these corrections). But the effects of strong correlations in a general form cannot be found in nearest future. This means that there appears a doubt about the accuracy of the existing models for description of nuclear reactions during the stellar evolution where the Salpeter's interpolation results where used.

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APPENDIX

The calculations give the following expression for $\Lambda_{ij}^{(T)}(y.x,z)$

$$\Lambda_{ij}^{(T)}(y,x,z)=\frac{\sum_{j'}\frac{Z_{j'}^2X_{j'}}{m_{j'}}\left[\left(1+\frac{3s_{i,j,j'}^2}{s_{i,j,i}^2}-2s_{i,j,j'}^2\left(1+\frac{s_{i,j,j'}^2}{s_{i,j,i}^2}\right)\right)W(s_{i,j,j'})-\left(1+\frac{s_{i,j,j'}^2}{s_{i,j,i}^2}\right)\right]}{\sqrt{\sum_{j''}\frac{Z_{j''}X_{j''}}{m_{j''}}\left[\sum_{j'}\frac{Z_{j'}X_{j'}}{m_{j'}}+\sum_{j'}\frac{Z_{j'}^2X_{j'}}{m_{j'}}W(s_{i,j,j'})\right]}}+$$

$$\frac{1}{4s_{i,j,i}^2}\frac{\left[\sum_{j'}\frac{Z_{j'}^2X_{j'}}{m_j'}\left[(1-2s_{i,j,j'}^2)W(s_{i,j,j'})-1\right]\right]^2}{\sqrt{\sum_{j''}\frac{Z_{j''}X_{j''}}{m_{j''}}\left[\sum_{j'}\frac{Z_{j'}X_{j'}}{m_{j'}}+\sum_{j'}\frac{Z_{j'}^2X_{j'}}{m_{j'}}W(s_{i,j,j'})\right]^3}}\qquad (A1)$$

and for $\tilde{\Lambda}_{ij}^{(T)}(y,x,z)$

$$\tilde{\Lambda}_{ij}^{(T)}(y,x,z)=$$

$$-\frac{1}{\sqrt{\sum_{j''}\frac{Z_{j''}X_{j''}}{m_{j''}}}}\left\{\frac{Re\left\{\tilde{\sum}_{\tilde{j}'}\frac{Z_{\tilde{j}'}^2X_{\tilde{j}'}}{m_{\tilde{j}'}}\left[\left(1+\frac{3s_{i,j,\tilde{j}'}^2}{s_{i,j,i}^2}-2s_{i,j,\tilde{j}'}^2\left(1+\frac{s_{i,j,\tilde{j}'}^2}{s_{i,j,i}^2}\right)\right)W(s_{i,j,\tilde{j}'})-\left(1+\frac{s_{i,j,\tilde{j}'}^2}{s_{i,j,i}^2}\right)\right]\right\}}{Re\left\{\sqrt{\left[\sum_{j'}\frac{Z_{j'}X_{j'}}{m_{j'}}+\sum_{j'}\frac{Z_{j'}^2X_{j'}}{m_{j'}}W(s_{i,j,j'})\right]}\right\}}+\right.$$

$$\frac{1}{2s_{i,j,i}^2}Re\left\{\frac{Re\left[\tilde{\sum}_{\tilde{j}'}\frac{Z_{\tilde{j}'}^2X_{\tilde{j}'}}{m_j'}\left[(1-2s_{i,j,\tilde{j}'}^2)W(s_{i,j,\tilde{j}'})-1\right]\right]}{Re\left[\sqrt{\left[\sum_{j'}\frac{Z_{j'}X_{j'}}{m_{j'}}+\sum_{j'}\frac{Z_{j'}^2X_{j'}}{m_{j'}}W(s_{i,j,j'})\right]}\right]}+\frac{\left[\sum_{j'}\frac{Z_{j'}^2X_{j'}}{m_j'}\left[(1-2s_{i,j,j'}^2)W(s_{i,j,j'})-1\right]\right]}{\sqrt{\left[\sum_{j'}\frac{Z_{j'}X_{j'}}{m_{j'}}+\sum_{j'}\frac{Z_{j'}^2X_{j'}}{m_{j'}}W(s_{i,j,j'})\right]}}\times\right.$$

$$\left.\left.\frac{Re\left[\tilde{\sum}_{\tilde{j}'}\frac{Z_{\tilde{j}'}^2X_{\tilde{j}'}}{m_j'}\left[(1-2s_{i,j,\tilde{j}'}^2)W(s_{i,j,\tilde{j}'})-1\right]\right]-2Re\left[\tilde{\sum}_{\tilde{j}'}\frac{Z_{\tilde{j}'}X_{\tilde{j}'}}{m_{\tilde{j}'}}+\tilde{\sum}_{\tilde{j}'}\frac{Z_{\tilde{j}'}^2X_{\tilde{j}'}}{m_{\tilde{j}'}}W(s_{i,j,\tilde{j}'})\right]}{2\left[Re\left[\sqrt{\left[\sum_{j'}\frac{Z_{j'}X_{j'}}{m_{j'}}+\sum_{j'}\frac{Z_{j'}^2X_{j'}}{m_{j'}}W(s_{i,j,j'})\right]}\right]\right]^2}\right\}\right\}-$$

$$Re\left\{\left(1+\frac{1}{2s_{i,j,i}^2}\right)\left[\frac{1+\tilde{Z}_{eff}}{\sqrt{1+Z_{eff}}}-\frac{Re\left[\tilde{\sum}_{\tilde{j}'}\frac{Z_{\tilde{j}'}X_{j'}}{m_{\tilde{j}'}}+\tilde{\sum}_{\tilde{j}'}\frac{Z_{\tilde{j}'}^2X_{\tilde{j}'}}{m_{\tilde{j}'}}W(s_{i,j,\tilde{j}'})\right]}{\sqrt{\sum_{j''}\frac{Z_{j''}X_{j''}}{m_{j''}}\left[\sum_{j'}\frac{Z_{j'}X_{j'}}{m_{j'}}+\sum_{j'}\frac{Z_{j'}^2X_{j'}}{m_{j'}}W(s_{i,j,j'})\right]}}\right]\right\}\qquad (A2)$$

Here, as previously, the subscript $\tilde{j}$ corresponds the summation with respect to that nuclei which take part in nuclear reaction cycle and we used the following notation

$$\tilde{Z}_{eff}=\frac{\tilde{\sum}_{\tilde{j}'}\frac{Z_{\tilde{j}'}^2X_{\tilde{j}'}}{m_{\tilde{j}'}}}{\sum_j\frac{Z_jX_j}{m_j}}\qquad (A3)$$

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