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M. Okamoto, A.A. Maluckov, S. Satake,
N. Nakajima and H. Sugama

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Transport and Radial Electric Field in Torus Plasmas

Masao Okamoto, Aleksandra A. Maluckov[†], Shinsuke Satake[‡], Noriyoshi Nakajima, and Hideo Sugama

National Institute for Fusion Science, Toki 509-5292 Japan
[†] *University of Nis, Prirodno-Matematicki Fakultet, FR Yugoslavia*
[‡] *Graduate University for Advanced Studies, Hayama 240-0193 Japan*

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Abstract

Transport phenomena in torus plasmas are discussed focusing on the genaration of the neoclassical radial electric field. A sophisticated δf Monte Carlo particle simulation code "FORTEC" is developed including the effect of finite orbit width (FOW), which is the non-local property of the plasma transport. It will be shown that the neoclassical radial electric field in the axisymmetric tokamak is generated due to this FOW effect. The Lagrangian approach is applied to construct a non-local transport theory in the region of near-axis. The reduction of the heat diffusivity toward the axis will be shown. From a statistical point of view, diffusion processes are studied in the presence of irregular magnetic fields. It is shown that the diffusion processes are non-local in almost all the cases if there are some irregularities in the magnetic field.

keywords: neoclassical transport, effect of finite orbit width, radial electric field, FORTEC, potato particle, Lagrangian formulation, heat diffusivity, diffusion process, irregular magnetic field, statistical point of view, non-local transport

§1 Introduction

It is well known that a large radial electric field can suppress various instabilities or turbulence in torus plasmas and thus improve the plasma confinement. Such a radial large electric field is generated in a plasma with a steep pressure gradient as seen in the ETB (External Transport Barrier or H-mode) and ITB (Internal Transport Barrier) discharges in tokamaks. The sheared radial electric field E_r' may be considered to destroy phase relations of $\tilde{p}\tilde{\vec{v}}, \tilde{n}\tilde{\phi}, \cdots$, resulting in suppressing the turbulence.

It is of great interest to investigate the generation mechanism of the radial electric field E_r. One possibility is the determination of E_r as a result from the neoclassical transport. Input of toroidal momentum or torque due to, e.g., the neutral beam injection for heating (NBI heating) can alter E_r. Another possibility is that the Reynolds stress $\tilde{\vec{v}}\tilde{\vec{v}}$ produced in anomalous transport may generate E_r through the inertia term $\nabla \cdot (\rho\tilde{\vec{v}}\tilde{\vec{v}})$.

Suppose that the radial particle flux Γ consists of ion and electron particle fluxes (assuming ionic charge number $Z_i = 1$)

$$\Gamma = -\Gamma_e^{nc} + \Gamma_i^{nc} - \Gamma_e^{an} + \Gamma_i^{an} + \Gamma_i^{cx} - \Gamma_e^{loss} + \Gamma_i^{loss} + \cdots\cdots \tag{1}$$

Here, Γ_e^{nc} and Γ_i^{nc} are electron and ion particle fluxes due to the neoclassical difusion, and Γ_e^{an} and Γ_i^{an} are electron and ion particle fluxes due to the anomalous diffusion, respectively. The charge exchange process and particle orbit loss remove momenta out of the plasma region to generate particle fluxes Γ_i^{cx}, Γ_e^{loss}, and Γ_i^{loss}. Γ given by Eq. (1) is a function of the radial electric field E_r. Owing to the plasma property of charge neutrality, the particle flux or radial electric current must vanish at the steady state to lead the ambipolarity condition $\Gamma(E_r) = 0$ at the steady state, which determines E_r.

In the present paper, we focus on the the neoclassical transport, which can be described only by particle guiding center motions and Coulomb collisions. The standard neoclassical transport theory [1,2] assumes regular flux surfaces (nested flux surfaces) and

$$\epsilon \equiv \frac{\rho_p}{L} \ll 1 \tag{2}$$

$$M_p \equiv \frac{B}{B_p}\frac{u_\parallel}{v_{th}} \ll 1 \tag{3}$$

where ρ_p is the poloidal Larmor radius, L is the plasma scale length, B is the magnetic field strength, B_p is the poloidal field, $u_\parallel$ is the parallel flow velocity, and v_{th} is the thermal velocity. The standard neoclassical theory is constructed in the limit of small orbit width (SOW). It can explain observed bootstrap current, electric conductivity, plasma rotation, and some others. On the other hand, the radial diffusion is governed by anomalous transport. The neoclassical transport gives the minimum radial diffusion.

However, the assummptions for the standard neoclassical theory in the SOW limit often break down. In the plasma with ITB, there exists a region where the pressure gradient is so steep that the condition $\epsilon \ll 1$ does not hold. Near and inside the ITB region, the diffusivities become drastically reduced to the level lower than those predicted by the standard neoclassical theory. It also breaks down near the magnetic axis in tokamaks because of complicated particle orbits, which are different from those of standard banana and passing particles [3,4]. The width of this exceptional region is very wide for the ITB plasma with negative magnetic shear, because it is roughly proportional to the central safety factor q_0 and the Larmor radius ρ or the ion temperature T_i, both of which are high in ITB plasmas. Magnetic islands and their overlappings generated by field irregularities or perturbations may alter the standard neoclassical transport drastically. For such plasmas to be understood, it is necessary to take into account the effect of finite orbit width (FOW), which has not been considered in the standard theory.

To take the FOW effects into account, a δf Monte Calro particle simulation code "FORTEC" (Finite ORbit Transport Extended Code) has been developed [5]. The code "FORTEC" clarified new physics which can not be obtained in the standard neoclassical theory. Further, the self-consistent radial electric field has been successfully calculated in a tokamak in both small and large ∇p plasmas [6]. Recently, the radial electric field in a rotating tokamak plasma has been calculated and compared with analytical estimations [7,8]. Plasma transport near the magnetic axis is analytically investigated by a Lagrangian method, in which the drift kinetic equation is solved using constants of motion. By averaging particle orbits, this formulation can include the FOW effect to correctly describe transport near the axis [9]. Diffusion processes in a partially destroyed magnetic field and in the fully destroyed field (stochastic field) are studied. In such plasmas, the condition for the standard neoclassical theory to hold breaks down, because it assumes regular magnetic flux surfaces. In this case, the non-local FOW effect is essential and is considered from a statistical point of view [10,11]. It should be noted that the transports including FOW effects have some characteristics of non-lolality.

The paper presents three topics. First, in section 3, the δf simulation code "FORTEC" is described emphasizing the importance of FOW effect in determining radial electric field. Secondly, non-local transport near the axis is formulated by the Lagrangian method in section 4. Thirdly, the diffusion processes in destroyed magnetic fields are studied in section 5 from a statistical point of view.

§2 Neoclassical Radial Electric Field

In helical systems or stellarators such as LHD (The Large Helical Device), the electron and ion particle fluxes Γ_e and Γ_i are functions of the radial electric field E_r even on the lowest order of $\epsilon \equiv \rho_p/L$. The ambilorality condition has multiple solutions for E_r and thus a bifurcation of the solution may occur under some condition. The solution with negative E_r is called the ion root and positive one is called the electron root.

However, in axisymmetric tokamaks, $\Gamma_e = \Gamma_i$ is independent of E_r on the lowest order of ϵ. This is called the intrinsic ambipolarity. What determines the radial electric field in axisymmetric tokamaks is the subject in this section.

Consider the momentum balance equation at the steady state which is given, neglecting the inertia term, by

$$n_a e_a(\vec{E} + \vec{u}_a \times \vec{B}) + \vec{F}_a - \nabla P_a - \nabla \cdot \Pi_a = 0 \tag{4}$$

where n_a, e_a are the density and the charge for species a, respectively, $\vec{E}, \vec{B}$ are the electric and magnetic fields, $\vec{u}_a, \vec{F}_a, P_a$ and Π_a are the plasma flow velocity, friction force, scalar pressure, and stress tensor ($\nabla \cdot \Pi_a$ is the viscosity force), respectively, for species a. We define the coordinates (ψ, θ, ϕ) with ψ the flux as a radial coordinate, θ the poloidal angle, and ϕ the toroidal angle. Taking toroidal component $(R^2\nabla\phi\cdot)$ of Eq. (4) gives the particle flux

$$\Gamma_{a\psi} \equiv \langle n_a \vec{u}_a \cdot \nabla\psi \rangle = \langle R^2\nabla\phi \cdot n_a \vec{u}_a \times \vec{B} \rangle \tag{5}$$

$$\Gamma_{a\psi} = -\langle R\nabla\phi \cdot (\frac{1}{e_a}\vec{F}_a + n_a\vec{E}) \rangle + \frac{1}{e_a}\langle \nabla \cdot (R^2\nabla\phi \cdot \Pi_a) \rangle \tag{6}$$

In Eq. (6), $\sum F_a = 0$ (the momentum conservation during the collision), $\sum e_a n_a = 0$ (charge neutrality), and $\sum \Pi_a = 0$ (viscosity force balance). Then we obtain the ambipolarity condition

$$\sum e_a \Gamma_{a\psi} = 0 \tag{7}$$

at the steady state. This relation determines the radial electric field E_r since it depends on E_r.

The particle fluxes coming from the friction force $\vec{F}_a$ are obviously independent of E_r. These fluxes, which are called axisymmetric fluxes, can be calculated by the parallel force balance $\langle \vec{B} \cdot \vec{F}_a \rangle = \langle \vec{B} \cdot \nabla \cdot \Pi_a \rangle$. It should be noted that the axisymmetric fluxes are independent of E_r regardless of the order of ϵ. For tokamaks, if ϵ is a small parameter, the toroidal viscosity for ions, the last term in Eq. (6), is small due to the axisymmetry and on the order of ϵ^4 [12], whereas the axisymmetric fluxes for electrons and ions are on the order of ϵ^2. On the lowest order, the axisymmetric fluxes dominate and the relation Eq. (7) is independent of E_r. This is called the intrinsic ambipolarity. On the other hand, in non-axisymmetric tori (stellarators or helical systems) such as LHD (The Large Helical Device), the toroidal viscosity $\langle \vec{B}_t \cdot \nabla \cdot \Pi_a \rangle$ dominates the particle diffusion.

It is the ion particle flux generated by the toroidal viscosity that determines the radial electric field, since the intrinsic ambipolarity condition can not determine the radial electric field. In the case of small ϵ, the toroidal viscosity has been calculated by Rosenbluth et al., assuming $\nabla T_i = 0$ [12], where T_i is the ion temparature. The toroidal viscosity is small, but finite due to FOW effect even if ϵ is very small, because the drift surfaces of particle orbits deviate from the magnetic surfaces due to the toroidicity. For any value of ϵ, the intrinsic ambilorarity condition holds for axisymmetric fluxes, and the toroidal viscosity $\Gamma_i = \langle \nabla \cdot (R^2 \nabla \phi \cdot \Pi_a) \rangle$ depends on E_r and it is this flux to determine E_r. In the case of arbitrary ϵ, we need a numerical calculation to obtain E_r taking into account accurately the FOW effect. For this purpose, we develop a δf Monte Carlo particle simulation code "FORTEC" to solve the drift kinetic equation.

§3 δf Simulation – FORTEC –

In this section, we solve the drift kinetic equation for ions by a simulation code. Particle orbits are followed and Coulomb collisions are modelled by the Monte Carlo method. To reduce the statistical noises in the simulation, we employ the δf scheme. The code is called "FORTEC" (Finite ORbit Transport Extended Code), in which orbits of many test particles are accurately calculated to include the effect of finite orbit width (FOW). The code "FORTEC" is applied to the calculation of the radial electric field E_r with and without the plasma parallel flow.

3.1 Formulation of δf method

We consider only ions. The density n and temperature T are assumed to be functions of position $\vec{x}$ and constant with time t ; $n = n(\vec{x})$ and $T = T(\vec{x})$、whereas the electric potential depends both on position and time ; $\Phi = \Phi(\vec{x}, t)$. In $(\mathcal{E}, \mu, \vec{x})$ space, where $\mathcal{E} = mv^2/2 + e\Phi$ is the energy of a particle and $\mu = v_\perp^2/2B$ is the magnetic moment, the drift kinetic is given by [1]

$$\frac{\partial f}{\partial t} + \frac{e}{m}\frac{\partial \Phi}{\partial t}\frac{\partial f}{\partial \varepsilon} + (\vec{v_\parallel} + \vec{v_d}) \cdot \nabla f = C(f, f) \tag{8}$$

where f is the distribution function, $\vec{v_\parallel}$ is the velocity parallel to the magnetic field, and $\vec{v_d}$ is the guiding center drift velocity, which includes the $\vec{E} \times \vec{B}$ drift moltion [13]. $C(f, f)$ is the Coulomb collision operator.

We decompose f into two parts : $f = f_M + \delta f$, where f_M is a Maxwellian distribution function given by

$$f_M = e^{e\Phi/T} \frac{n}{(\pi v_{th}^2)^{3/2}} e^{-m\mathcal{E}/T} \tag{9}$$

with v_{th} is the thermal velocity. δf is the distribution function perturbed due to particle drift motions. The decomposed equations are

$$\frac{\partial f_M}{\partial t} + \frac{e}{m}\frac{\partial \Phi}{\partial t}\frac{\partial f_M}{\partial \varepsilon} + \vec{v_\parallel} \cdot \nabla f_M = 0 \tag{10}$$

$$\frac{\partial \delta f}{\partial t} + \frac{e}{m}\frac{\partial \Phi}{\partial t}\frac{\partial \delta f}{\partial \varepsilon} + \vec{v_d} \cdot \nabla f_M + (\vec{v_\parallel} + \vec{v_d}) \cdot \nabla \delta f = C(\delta f, f_M) + C(f_M, \delta f) \tag{11}$$

Here, we have neglected the collision term of $C(\delta f, \delta f)$ assuming $|\delta f| \ll f_M$. In Eq. (11), the term $\vec{v}_d \cdot \nabla \delta f$ represents the effect of FOW.

We denote the linearized collision operators in Eq. (11) by $C_{TP} \equiv C(\delta f, f_M)$ and $Pf_M \equiv C(f_M, \delta f)$. C_{TP} is the test particle collision operator with a background Maxwellian ions including pitch angle scattering, drag or slow down, and enegy diffusion. We employ the collision model for C_{TP} by Lin et al.[14] in the present paper. On the other hand, Pf_M is the collision term to garantee the particle number, momentum, and energy during the collisions. This operator compensates the excess momentum and energy produced in the process of test particle collisions C_{TP}. The most accurate collision model for Pf_M has been developed by Wang et al. [4], which is used in the present simulations.

We introduce a function $g(\vec{x}, \vec{v}, t)$ which satisfies

$$\frac{Dg}{Dt} = 0 \tag{12}$$

where the operator D/Dt is defined, for arbitrary function A, by

$$\frac{DA}{Dt} \equiv \frac{\partial A}{\partial t} + \frac{e}{m}\frac{\partial \Phi}{\partial t}\frac{\partial A}{\partial \varepsilon} + (\vec{v_\parallel} + \vec{v_d}) \cdot \nabla A - C_{TP} \tag{13}$$

with $C_{TP} = C(A, f_M)$. If we introduce weight fields $W(\vec{x}, \vec{v}, t), P(\vec{x}, \vec{v}, t)$

$$f_M = Pg \tag{14}$$

$$\delta f = Wg \tag{15}$$

Then we can derive the equations for P and W to satisfy,

$$\frac{DP}{Dt} = \frac{P}{f_M}\vec{v}_d \cdot \nabla f_M \tag{16}$$

$$\frac{DW}{Dt} = \frac{P}{f_M}[-\vec{v}_d \cdot \nabla f_M + Pf_M] \tag{17}$$

By solving Eqs. (16) and (17), we finally obtain the objective distribution functions f_M and δf from Eqs. (14) and (15).

3.2 δf Monte Carlo particle simulation

We solve the set of equations (12) to (17), by an orbit followong and Monte Carlo method. The function g corresponds to the test particle distribution function, which can be expressed by

$$g(\vec{x}, \vec{v}, t) = \sum_i \delta(\vec{x} - \vec{x}_i(t))\delta(\vec{v} - \vec{v}_i(t)). \tag{18}$$

where $\vec{x}_i(t)$ and $\vec{v}_i(t)$ are the position and the velocity of the i-th test particle at time t and obtained by solving the guiding center equations and collisions C_{TP}. We assign two weights to each test particle

$$w_i(t) = W(\vec{x}_i(t), \vec{v}_i(t), t) \tag{19}$$

$$p_i(t) = P(\vec{x}_i(t), \vec{v}_i(t), t) \tag{20}$$

The distribution functions δf and f_M can be obtained once w_i, p_i are determined

$$f_M(\vec{x},\vec{v},t) = \sum_i p_i(t)\delta(\vec{x}-\vec{x}_i(t))\delta(\vec{v}-\vec{v}_i(t)) \tag{21}$$

$$\delta f(\vec{x},\vec{v},t) = \sum_i w_i(t)\delta(\vec{x}-\vec{x}_i(t))\delta(\vec{v}-\vec{v}_i(t)) \tag{22}$$

We can derive the equations for weights w_i and p_i from Eqs. (16) and (17), paying attention to the problem of weight spreading [15]. The equations are given by [5,15]

$$\frac{dw_i}{dt} = \frac{p_i}{f_{Mi}}\left[-\vec{v_d}\cdot\nabla f_M + C(f_M, f_1)\right]_i - \eta\left[w_i - \bar{W}(\vec{x}_i,\vec{v}_i,t)\right] \tag{23}$$

$$\frac{dp_i}{dt} = \frac{p_i}{f_{Mi}}\left[\vec{v_d}\cdot\nabla f_M\right]_i - \eta\left[p_i - \bar{P}(\vec{x}_i,\vec{v}_i,t)\right] \tag{24}$$

Here, the sufixes i means that any quantity must be evaluated in the phase space of $(\vec{x}_i(t), \vec{v}_i(t))$ and η is the damping rate to be specified. After calculating Eqs. (23) and (24), we average the weights w_i and p_i of test particles in a small phase space element to obtain $\bar{W}(\vec{x}_i,\vec{v}_i,t)$ and $\bar{P}(\vec{x}_i,\vec{v}_i,t)$. Then, new weights for i-th test particle are chosen from the averaged weights $\bar{W}$ and $\bar{P}$; $w_i(t) = \bar{W}(\vec{x}_i,\vec{v}_i,t)$, $p_i(t) = \bar{P}(\vec{x}_i,\vec{v}_i,t)$.

3.3 Equation for the radial electric field

From Poisson equation and the continuity equation, we obtain the time development equation for the radial electric field E_r,

$$\frac{\partial E_r}{\partial t} = -4\pi J_r \tag{25}$$

J_r is the current density in the radial direction consisting of

$$J_r = J_{rp}^{C} + J_{rp}^{NC} + J_{rd}^{NC} \tag{26}$$

We neglect the electron current, since it is much samaller than ion current by mass ratio. J_{rp}^{NC} and J_{rp}^{NC} are neoclassical polarization and drift current which are calculated in the simulation, while the classical polarization current J_{rp}^{C} is not calculated in the simulation and is given by the analytical expression. In general coordinates (cgs-units), the radial electric field is given by

$$\left[\langle|\nabla\psi_p|^2\rangle + 4\pi nmc^2\langle\frac{|\nabla\psi_p|^2}{B^2}\rangle\right]\frac{\partial^2\Phi}{\partial t\partial\psi_p} = 4\pi e\Gamma_i \tag{27}$$

The ion particle flux Γ_i is defined by

$$\Gamma_i = \langle\int d\vec{v}(\vec{v}_d\cdot\nabla\psi_p)f\rangle \tag{28}$$

At every time step in the simulation, the distribution function f is calculated and thus Γ_i can be obtained. Eq. (27) is solved by a predictor-corrector scheme.

3.4 Analytical estimation

We estimate analytically the time behaviour of the radial electric field and its value in the long time limit assuming a distribution function. We restrict ourselves only on the case of $\nabla T = 0$. In such a case, the drift kinetic equation has a solution of a shifted Maxwellian distribution function in the standard neoclassical theory in the SOW limit [1]. We assume for f as

$$f_{SM}(\vec{v},\vec{r}) = \frac{n}{(\pi v_{th}^2)^{3/2}} exp[-(v_\perp^2 + (v_\parallel - u_\parallel)^2)/v_{th}^2] \tag{29}$$

Taking the time derivative we have a coupled equation of E_r and f_{SM}

$$(1+\frac{c^2}{v_A^2})\frac{\partial^2 E_r}{\partial^2 t} = -4\pi e \langle \int d^3v \frac{\partial f}{\partial t} v_{dr} \rangle \tag{30}$$

where $v_A = B/\sqrt{4\pi nm}$ is the Alfven velocity. From the drift kinetic equation, we calculate $\partial f/\partial t$

$$\frac{\partial f}{\partial t} = -\frac{e}{m_i}\frac{\partial \Phi}{\partial t}\frac{\partial f_{SM}}{\partial \mathcal{E}} + (\vec{v}_\parallel + \vec{v}_d)\cdot \nabla f_{SM} \tag{31}$$

The wave equation (30) becomes

$$\frac{\partial^2 E_r}{\partial t^2} + \omega_{GAM}^2 E_r = S \tag{32}$$

ω_{GAM} is the frequency of Geodesic Acoustic Mode [16]. Generally, E_r has a following form [17]

$$E_r(t) = \bar{E}_r + Ae^{-i\omega_{GAM}t}e^{-\gamma_{GAM}t} + Be^{-\gamma_{MP}t} \tag{33}$$

γ_{GAM} depends strongly on q and γ_{MP} is the damping term due to the magetic pumping term with a small damping rate. In the limit of $t = \infty$

$$\bar{E}_r = \frac{T}{e}\{\frac{1}{n}\frac{dn}{dr} + 2(\frac{I_1}{I_0} - \bar{u}_\parallel)\frac{d\bar{u}_\parallel}{dr}\} + \frac{T}{e}\alpha\frac{I_2}{I_0}\bar{u}_\parallel \tag{34}$$

with $I_1/I_0 \simeq 3\bar{u}_\parallel$, $I_2/I_0 \simeq 0.43$, and $\alpha = r/(qR\rho)$. It will be shown that this analytical estimation agrees well with simulation results [7,8]. In case of no toroidal momentum, Boltzman relation holds from Eq. (34)

$$\bar{E}_r = \frac{T}{e}\frac{\partial n}{\partial r} \tag{35}$$

3.5 Calculation of E_r by FORTEC

Using the δf Monte Carlo particle simulation code "FORTEC"-version S, the simulations are carried out for a simple magnetic field geometry in the large aspect ratio limit. Many particles are initially loaded in the simulation domain to form a local shifted-Maxwellian distribution with T in each cell devided in the r and θ-directions. In the present simulation, the effect of the parallel flow is treated as a perturbation in δf according to the procedure described in section 2. The flow is initially given by

$$\bar{u}_{\parallel}(x, t=0) = \bar{u}_0 \exp\left(-\alpha_u (x - x_0)^{\beta_u}\right) \quad (36)$$

Here, $x = r/a_p, \bar{u}_0 = u_{\parallel}(x = x_0)/v_{th}$, α_u, β_u, and x_0 are input parameters. In the present paper, three cases of $\bar{u}_0 = 0$, and ± 0.1 are examined. The parameters throughout the calculations are $B_0 = 3\mathrm{T}$ (magnetic field strength), $R_0 = 3\mathrm{m}$ (plasma major radius), $a_p = 0.5\mathrm{m}$ (plasma minor radius), $n_0 = 5.0 \times 10^{19}\mathrm{m}^{-3}$ (central density), $T = 1.5\mathrm{keV}$, $\alpha_u = 2$, $\beta_u = 2$, $x_0 = 0$, and $N_{test} = 10^6$ (number of test particles).

We show the calculation results for the case with a small density gradient and positive shear of $q(r)$. The potato region of ions is $\Delta_p/a_p = 0.1$ and ions lie in the banana regime in $x = r/a_p = 0.2$ to 1.0. In other region ions are in the plateau regime.

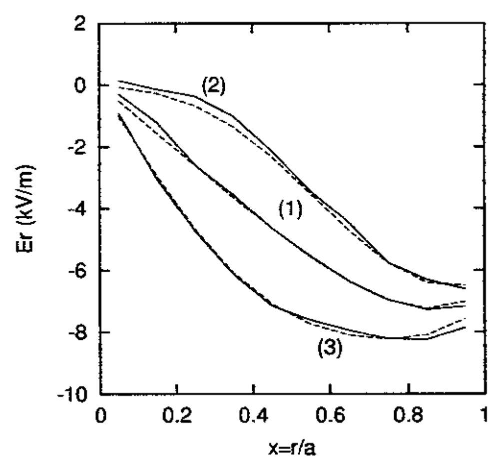

Figure 1: E_r profiles in a rotating plasma.

The particle flux Γ_i first oscilates with the GAM frequency and damps in a short time to become zero. The time behaviour of E_r synchronizes with that of Γ_i and finally it reaches the value of $\bar{E}_r$ gven by Eq. (34). The collision operator in "FORTEC-S" ensuring momentum and energy conservations keeps $\Gamma_i = 0$ and constant E_r.

Figure 1 demonstrates the radial profiles of E_r for three cases of (1) $\bar{u}_{\parallel} = 0$,(2) $\bar{u}_{\parallel} = -0.1$, and (3) $\bar{u}_{\parallel} = +0.1$. Dotted lines are simulation results and the dashed lines are analytical results by Eq. (34). Simulation results are obtained by time averaging from $t = 4.8\tau_i$ to $t = 5.0\tau_i$, where τ_i is the ion-ion collision time. Figure 1 shows clearly that the simulation results and the analytical results are in good agreement.

§4 Transport Near the Magnetic Axis

The orbits of particles moving near the magnetic axis are different from those of passing and banana particles apart from the axis. Those particles near the axis are genarally called potato particles. The typical orbit width of potato particles is $\Delta_p \sim (q^2\rho^2 R_0)^{1/3}$, and the fraction is $\sim (q\rho/R_0)^{1/3}$, where q, ρ, and R_0 are the safety factor, Larmor radius, and major radius, respectively. Potato particles, which move complicatedly with Δ_p comparable to the particle radius position r, may alter transport near the axis. Here, we apply first the Lagrangian formulation for transport to the near-axis region, which can include FOW effect [9].

4.1 Particle orbits near the magnetic-axis

Near the magnetic axis, $\dot{\theta}$ is given as

$$\dot{\theta} = (v_\parallel \vec{b} + \vec{v}_d) \cdot \nabla\theta \simeq \frac{1}{qR_0}\left[v_\parallel - \frac{q}{r\Omega_0}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\cos\theta\right] \quad (37)$$

Note that $\vec{v}_d \cdot \nabla\theta \propto r^{-1}$ and $r \simeq \Delta_p$ (potato width). Assumptions for the standard neoclassical theory $v_d \ll v_\parallel$, and $\Delta_b (\simeq \Delta_p) \ll r$ break down near the axis.

We identify particle orbits, which are different from those of standard banana and passing particles, by the number of the turning points of $v_\parallel$ and the turning points of $\dot{\theta}$. The classification of particle orbits is shown in Fig.1 with parameters of $q = 3, B_0 = 4$T, $T = 10$keV for hydrogen plasma [4] ; (A) standard banana particle, (B) fattest banana, (C) passing particle, (D) inner-circulating, (E) outer-circulating, (F) kidney particle, and (G) concave-kidney particle. In Fig.1, the mark bar means a turnibg point of $v_\parallel$ and the mark square indicates a turning point of $\dot{\theta}$.

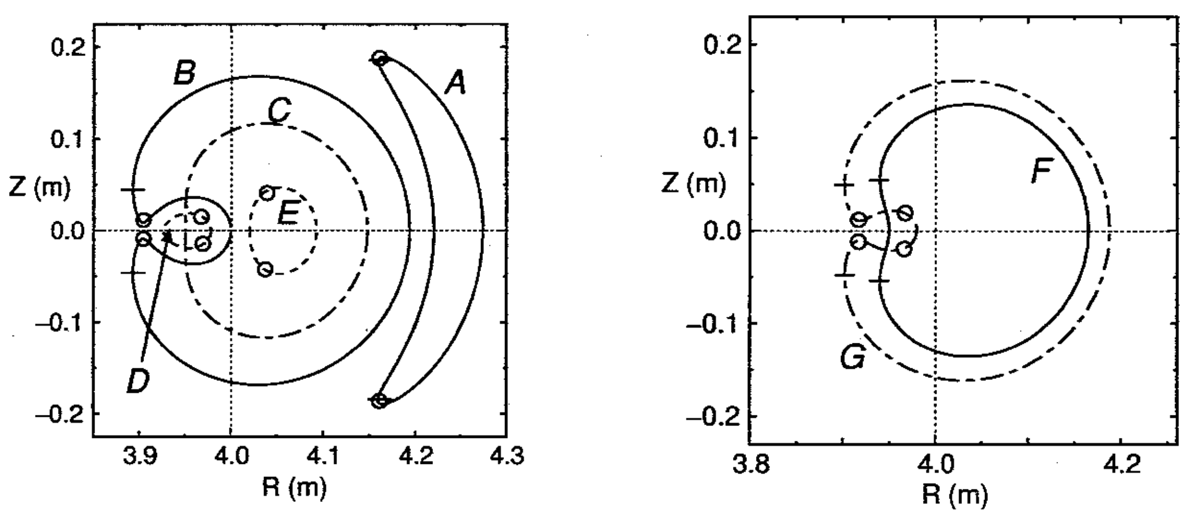

Figure 2: Types of orbits : (A) standard banana, (B) the fattest banana, (C) passing, (D) inner-circulating, (E) outer-circulating, (F) kidney, (G) concave-kidney orbits.

4.2 Lagrangian transport theory

In the Lagrangian formulation [18,19,20], the drift kinetic equation is described in terms of three constants of motion (COM) of $\mathcal{E}$ (particle energy), μ (magnetic moment), and $\langle\psi\rangle$

or $\langle r \rangle$ (averaged radial position of a particle). The average particle radial position is defined by

$$\langle \psi \rangle = \frac{1}{\tau_p} \oint \frac{d\theta}{\dot{\theta}} \psi \tag{38}$$

with $\tau_p = \oint \frac{d\theta}{\dot{\theta}}$ thr poloidal period. $\langle \cdots \rangle$ means the average over a particle orbit. Using these COM's $(\mathcal{E}, \mu, \langle \psi \rangle)$, the drift kinetic equation can be written as

$$\frac{\partial}{\partial t} f_a(\mathcal{E}, \mu, \langle \psi \rangle, \theta, t) + \dot{\theta} \frac{\partial f_a}{\partial \theta} = C_{ab} \tag{39}$$

We assume that the plasma is in the collisionless regime $\delta_c = \nu_c^{\rm eff} \tau_p \ll 1$, where $\nu_c^{\rm eff}$ is the effective collision frequency and τ_p is the poloidal period. In the zero-th order $O(\delta_c^0)$, we obtain

$$\dot{\theta} \frac{\partial f_0}{\partial \theta} = 0 \qquad \therefore f_0 = f_0(\mathcal{E}, \mu, \langle \psi \rangle), \tag{40}$$

The next order $O(\delta_c)$ gives

$$\frac{\partial}{\partial t} f_0(\mathbf{z}, t) + \frac{1}{J_z} \frac{\partial}{\partial \theta} \left(J_z \frac{d\theta}{dt} f_1 \right) = \frac{1}{J_z} \frac{\partial}{\partial \mathbf{z}} \cdot \left(J_z \frac{\partial \mathbf{z}}{\partial \mathbf{v}} \cdot \mathbf{\Gamma}(f_0) \right) \tag{41}$$

Taking the orbit average yields the reduced kinetic equation

$$\frac{\partial \bar{f}}{\partial t} = \frac{1}{J_c} \frac{\partial}{\partial \mathbf{z}} \cdot \left(J_c \left\langle \frac{\partial \mathbf{z}}{\partial \mathbf{v}} \cdot \mathbf{\Gamma}(\bar{f}) \right\rangle \right) = \bar{C} \tag{42}$$

where $\bar{f} = f_0$ and J_c is the Jacobian in the $(\mathcal{E}, \mu, \langle \psi \rangle)$-space

$$J_c = \frac{\tau_p}{m^2} |1 - \delta^*| \qquad \left(\delta_* \equiv \frac{\partial}{\partial \langle \psi \rangle} \left\langle \frac{I}{\Omega} v_\parallel \right\rangle \right) \tag{43}$$

δ_* is found to be negligible for almost all orbit-types of particles.

We employ a model collision operator [21] which conserves particle number, energy, and parallel momentum on a local point of the particle. The averaged operator is given by

$$\bar{C}_i(\bar{f}_i) = \frac{1}{J_c} \frac{\partial}{\partial \mathbf{z}} \cdot J_c \frac{\nu_i}{2} \left[\left\langle \frac{\partial \mathbf{z}}{\partial \mathbf{v}} \cdot \mathsf{V} \cdot \frac{\partial \mathbf{z}}{\partial \mathbf{v}} \right\rangle \frac{\partial \bar{f}_i}{\partial \mathbf{z}} - \left\langle \frac{m_i u_{i\parallel}}{T_i} \frac{\partial \mathbf{z}}{\partial \mathbf{v}} \cdot \mathbf{w} f_{iM} \right\rangle \right] \tag{44}$$

Here, $\mathsf{V}(\mathbf{v}) = v^2 \mathsf{I} - \mathbf{v}\mathbf{v}$ and $\mathbf{w} = \mathsf{V} \cdot \mathbf{b}$. $u_{i\parallel}$ is determined to satisfy the relation $\int d^3 v v_\parallel C_i(f_i) = 0$;

$$u_{i\parallel} = \frac{\tau_i}{2 n_i \mathcal{K}_1} \int d^3 v \nu_i v_\parallel f_i, \tag{45}$$

where $\mathcal{K}_1 = 0.2664$ and τ_i is the ion-ion collision time.

4.3 Transport equation

The reduced kinetic equation (42) for ions is expanded by a parameter $\delta_b \equiv \Delta_b / L \ll 1$ where Δ_b is a typical orbit width ($\Delta_b \simeq \Delta_p$) and L is the typical gradient scale length. We assume the following orderings

$$\frac{\partial}{\partial \mathcal{E}} \sim \frac{\partial}{\partial \mu} \sim O(\delta_b^0) \ , \ \frac{\partial}{\partial \langle \psi \rangle} \sim O(\delta_b^1) \ , \ \frac{\partial \bar{f}}{\partial t} \sim O(\delta_b^2) \ , \ u_{i\parallel} \sim O(\delta_b^1)$$

We neglect C_{ei} here. By expanding $\bar{f} = \bar{f}_0 + \delta_b \bar{f}_1 + \delta_b^2 \bar{f}_2 + \cdots$ and $\bar{C} = \bar{C}^{(0)} + \bar{C}^{(1)} + \cdots$ in (42), the kinetic equation is solved order by order. In the order $O(\delta_b^0)$, $\bar{C}_i^{(0)}(\bar{f}_{i0}) = 0$, which yields

$$\bar{f}_{i0} = \bar{n}_i \left(\frac{m}{2\pi T}\right)^{3/2} \exp\left[-\frac{\mathcal{E} - e\Phi}{T}\right] \tag{46}$$

The next order gives $\bar{C}_i^{(0)}(\bar{f}_{i1}) = -\bar{C}_i^{(1)}(\bar{f}_{i0})$, or equivalently,

$$\frac{\partial}{\partial \mu}\left[J_c \mu \left\langle \frac{m_i v_\parallel^2}{B} \right\rangle \frac{\partial \bar{f}_{i1}}{\partial \mu}\right] = -\frac{\partial}{\partial \mu} J_c \mu \left[\left\langle \frac{I\gamma}{\Omega_i} v_\parallel \right\rangle \frac{\partial \bar{f}_{i0}}{\partial \langle \psi \rangle} + \frac{m_i \Omega_{i0} \overline{u_{i\parallel}}}{I_0 T_i} \left\langle \frac{I v_\parallel}{\Omega_i} \right\rangle \bar{f}_{i0}\right] \tag{47}$$

where $\bar{u}_{i\parallel}$ is the lowest order of $u_{i\parallel}(\psi, \theta)$. The second order $O(\delta_b^2)$ is

$$\frac{\partial \bar{f}_{i0}}{\partial t} = \bar{C}_i^{(2)}(\bar{f}_{i0}) + \bar{C}_i^{(1)}(\bar{f}_{i1}) + \bar{C}_i^{(0)}(\bar{f}_{i2}) \tag{48}$$

Taking moments of this equation, we have the equations for particle and heat fluxes.

By changing the radial coordinate from $\langle \psi \rangle$ to $\langle r \rangle$, we have

$$\frac{\partial \bar{n}_i}{\partial t} + \frac{\lambda_n}{V'} \frac{\partial}{\partial \langle r \rangle}(V' \Gamma_i) = 0 \tag{49}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}\bar{n}_i T_i\right) + \frac{\lambda_q}{V'} \frac{\partial}{\partial \langle r \rangle}\left[V'\left(q_i + \frac{3}{2}\Gamma_i T_i\right)\right] = -\lambda_q e_i \Gamma_i \frac{d\Phi}{d\langle r \rangle} \tag{50}$$

where

$$\lambda_n(\langle r \rangle) \equiv \bar{n}_i V' \mathcal{N}_i^{-1} \quad , \quad \lambda_q(\langle r \rangle) \equiv \frac{3\bar{n}_i T_i}{2} V' \mathcal{Q}_i^{-1} \tag{51}$$

$$\mathcal{N}_i = \sum_{\sigma_t} \int d\mathcal{E} d\mu J_c \bar{f}_{i0} \quad , \quad \mathcal{Q}_i \equiv \sum_{\sigma_t} \int d\mathcal{E} d\mu J_c W \bar{f}_{i0} \tag{52}$$

$\mathcal{N}_i$ is the total particle numbers which have the same $\langle \psi \rangle$ whereas $\mathcal{Q}_i$ is the kinetic energy of particles having the same $\langle \psi \rangle$. Numerical calculations show that $\lambda_n \simeq 1$ and $\lambda_q \simeq 1$. Then the direct comparison of the transport equations between the two representations (Eular and Lagrangian) is valid.

Particle and heat fluxes are given by

$$\begin{bmatrix} \Gamma_i \\ \frac{q_i}{T_i} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot \begin{bmatrix} \frac{d\ln \bar{n}_i}{d\langle \psi \rangle} + \frac{e_i}{T_i} \frac{d\Phi}{d\langle \psi \rangle} \\ \frac{d\ln T_i}{d\langle \psi \rangle} \end{bmatrix} \tag{53}$$

(54)

It is shown [9] that the transport coefficients A_{jk}'s are Onsagar-symmetric. Transport coefficients are in the following form

$$A_{jk}(\langle r \rangle) = \sum_{\sigma_t} \int dx d\lambda_0 \tau_p F_{jk}(x, \lambda_0, \langle r \rangle, \sigma_t) \tag{55}$$

where $x = \exp(-\mathcal{E}/T)$, $\lambda_0 = \mu B_0/\mathcal{E}$, and σ_t is the orbit type. Along the orbit of n-th particle $(x_n, \lambda_{0n}, \langle r \rangle; \sigma_{tn})$, we calculate τ_p and F_{jk}. (F_{jk} contains orbit-averaged functions such as $\langle B_0/B \rangle$, $\langle B_0 v_\parallel/B \rangle$, etc.) Particle orbits are numerically traced without collisions. It should be noted that the flux-force relation, Eq. (53) contains the property of nonlocal transport, although it has the same form as that of the standard neoclassical theory or the local transport theory. The nonlocal property comes from F_{jk} in Eq. (55), which contains information of all the particles having the same $\langle \psi \rangle$.

We calculate the ion thermal difusivity using Eqs. (53) and (55). The calculation parameters are $B_0 = 4$T, $q = 3$, $T = 20$keV, $n = 10^{20}$m^{-3}. The fattest banana width is $r_p = 2(q^2 \rho_{i0}^2 R_0)^{1/3} \sim 0.24m$. We have obtained the ion thermal diffusivity decreasing from $r = r_p$ toward the magnetic axis [9]. Outside the position of r_p it agrees with the prediction by the standard neoclassical theory

§5 Diffusion in Destroyed Magnetic Field

The standard neoclassical transport theory assumes nested magnetic surfaces. If there are some irregularities in the magnetic field, the neoclassical transport (determined only by particle orbits and Coulomb collisions) may drastically change. In this section, we investigate the particle radial diffusion in a magnetic field with irregularities as a realization of collisional (statistical) stochasticity and the magnetic field (deterministic) stochasticity. The diffusion process will be studied from the statistical view point.

5.1 Model

We solve the guiding center equations for electron orbits and give pitch angle scattering collisions to the electrons by random numbers. Specify the magnetic field with irregularities in a tokamak,

$$\vec{B} = \vec{B}_{eq} + \delta\vec{B} \tag{56}$$

with

$$\delta\vec{B} = \nabla \times (b\vec{B}_{eq}) \tag{57}$$

where $\vec{B}_{eq}$ is the regular nested magnetic field and b is

$$b(\psi, \theta, \zeta) = \sum_{m,n} b_{mn}(\psi) \cos(m\theta - n\zeta + \zeta_{m,n}) \tag{58}$$

where $\zeta_{m,n}$ is the phase and $\mid b_{mn} \mid /a \ll 1$. The form of b_{mn} is given by

$$\frac{b_{mn}(\psi)}{a} = s \cdot \exp\left(-\frac{(\psi - \psi_{mn})^2}{\Delta\psi^2}\right) \tag{59}$$

where s indicates the strength of perturbation, the stochasticity parameter. If $s = 0$, the magnetic field is regular and has nested magnetic surfaces. To make a magnetic field with irregularities, we specify the value of s, rational surface(s) with the mode number of m and n, and the width of the perturbation $\Delta\psi$.

5.2 Statistical approach

The statistical measure with respect to the radial displacement is

$$\delta r(t) = r(t) - r(0) \tag{60}$$

We define the first and second cumulants by $C_1(t) = \langle \delta r(t) \rangle$ and $C_2(t) = \langle (\delta r(t) - \langle \delta r(t) \rangle)^2 \rangle$, respectively. The diffusion coefficient is defined by

$$D(t) \equiv \frac{dC_2(t)}{2dt} \tag{61}$$

The power law equivalent diffusion coefficient is

$$D_{pw}(t) \equiv \frac{C_2(t)}{2t} \simeq t^{\alpha-1} \tag{62}$$

If $\alpha = 1$, the diffusion is called the normal diffusion, if $\alpha > 1$, it is the super diffusion, and if $\alpha < 1$, the diffusion is sub diffusive. The 3rd cumulant, which indicates the asymmetry of the distribution function, is defined by

$$\gamma_3(t) = \frac{C_3(t)}{C_2^{3/2}(t)} \tag{63}$$

$$C_3(t) = \langle (\delta r(t) - \langle \delta r(t) \rangle)^3 \rangle \tag{64}$$

The 4th cumulant represents the peakedness of the distribution function

$$\gamma_4(t) = \frac{C_4(t)}{C_2^2(t)} \tag{65}$$

$$C_4(t) = \langle (\delta r(t) - \langle \delta r(t) \rangle)^4 \rangle - 3C_2(t)^2 \tag{66}$$

It is noted that γ_n's are relative measures with respect to Gaussianity because $\gamma_{n\geq 3} = 0$ for the Gaussian process. The auto-correlation coefficient is defined as

$$A(t,t') = \frac{\langle (\delta r(t) - \langle \delta r(t) \rangle)(\delta r(t') - \langle \delta r(t') \rangle) \rangle}{\sqrt{\langle (\delta r(t) - \langle \delta r(t) \rangle)^2 (\delta r(t') - \langle \delta r(t') \rangle)^2 \rangle}} \tag{67}$$

5.3 Statistical results

The equilibrium magnetic field is a simple tokamak field $\vec{B}_{eq}$, in which magnetic surfaces are circular and concentric. The perturbed field is given by Eqs. (57), (58), and (59). Initially many test particles (electrons) having the same energy are loaded on a specified magnetic surface in the equilibrium field by using uniform random numbers. The guiding center equations are solved in a very short time step and pitch angle scatterings are given to each particle at every time step. The particles do not change their energy during the motions. We measure the position of $i-$th partilce $r_i(t)$ and calculate cumulants and auto-correlation function.

In the case of $\delta\vec{B}=0$, the stochasticity origin is only pitch angle collisions. The orbit-following and Monte Carlo calculation recovers the standard neoclassical diffusion. Statistical analysis shows that the diffusion without $\delta\vec{B}$ is the Wiener process or Gaussian Brownian motion [10]. Transport in a magnetic field with $\delta\vec{B}$ are studied in detail from the statistical point of view [11]. In this study the Coulomb collisional statistical stochasticity and the deterministic magnetic stochasticity coexist. Four cases are studied. The first is the field having only one island. The second is a disturbed field generated by overlapping of two islands. The third is the case of many island overlapping, but the field is still partially destroyed. The final case is a fully destroyed and stochastic field. In all the cases, perturbed (partially or fully) regions are bounded. This situation is different from that discussed by Rechester and Rosenbluth [22]. By analyzing the statistical quantities, the radial diffusions in almost cases are recognized as the strange process cheracterized by a power-law auto-correlation coefficient, subdiffusivity, neither Gaussianity nor uniformity, and statistically non-stationary. The strict justification of Markovianity remains open problems.

§6 Discussion and Summary

Neoclassical transports or collisional transports in toroidal systems have been considered beyond the framework of the standard neoclasssical theory including the non-local property of FOW effect. FOW effects have been emphasized in generating neoclassical radial electric fields, in transport near the magnetic axis, and diffusion in irregular magnetic fields. All the results in this paper have non-local property of transport attributed to FOW effects.

The δf Monte Carlo simulation code "FORTEC" has been developed to explore neoclassical transport including FOW effects, employing a new method for collisions and two weighting scheme. It has been cleared that the radial electric field in axisymmetric tokamaks is determined by the FOW effect.

The Lagrangian formulation has been firstly applied to the region near the magnetic axis, where particle orbits are completely different from ordinary banana and passing particles. The formulation was successful to include the non-local property of FOW effect.

Non-local transports in perturbed magnetic fields $\delta\vec{B}$ have been studied from a statistical point of view. Without $\delta\vec{B}$, standard neoclassical diffusion is recovered. The diffusion process is Wiener process or Gaussian Brownian motion. In the bounded region with $\delta\vec{B}$, the diffusion is non-Gaussian, non-Markovian, non-stationary, sub-diffusive. However, further studies are needed on this subject.

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