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Statistical theory for transition and long-time sustainment of improved confinement state*

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Abstract

The occurrence of stochastic transition is investigated in the presence of triggers by turbulence noise and external events. The probability of observing the transition is calculated under the circumstance that the global controlling parameters change in time. This is another important prediction of statistical theory in addition to the long time average. This clarifies the feature of transient response of the system with stochastic transitions. The interpretation of the experimental threshold database is discussed.

keywords: statistical theory, stochastic transition, transition rate, phase boundary, L-H transition, threshold database,

* Dedication:

This paper is dedicated to Prof. F. Wagner on occasion of the 60th anniversary of his birth.

I. Introduction

In understanding the improved confinement state, the exploration of statistical theory is an urgent issue. We have developed a statistical theory to analyze the transition phenomena in far-nonequilibrium systems[1, 2]. Improved confinement states are associated with transitions, and the empirical database must be compared to the results of statistical theory. Transitions include those to establish improved states (e.g., L-H transition) [3-6] and those to deteriorate barriers like neoclassical tearing mode (NTM) [7-13] which is an obstacle to sustain the improved confinement state for a long period. The condition of the transition has been studied by use of the threshold database, but the method to correlate the threshold database with nonlinear dynamics has not been well established.

A statistical model for the bifurcation of the radial electric field E_r has been analyzed in describing L-H transitions [14, 15]. The L-to-H and the H-to-L transition probabilities were calculated in the presence of micro fluctuations, and the effective phase limit is derived. We also applied this theoretical method to the problem of the stochastic trigger of NTM by microscopic turbulence. Suppression of NTM is necessary for the stationary operation of high temperature plasmas. The rate of excitation, the average of amplitude, and the boundary of phase (say, β_{p*}) have been derived [16, 17]. In this article, the occurrence of stochastic transition is investigated. The influence of turbulent trigger is investigated. The trigger by a large scale global event (like sawtooth) is also analyzed. The possibility of observing the transition is calculated under the circumstance that the global controlling parameters are changed in time. We obtained the distribution of global parameters where onsets (L-H transition or excitation of NTM) are observed. This is another important prediction of statistical theory in addition to the long time average. The interpretation of the experimental threshold database is discussed.

II. Statistical equation and transition rate

In many transition phenomena, the mechanisms of the sudden onset are attributed to subcritical excitations. Examples include the electric field bifurcation at edge in the case of the L-H transition and the excitation of the neoclassical tearing mode (NTM). The transition is triggered either by the turbulent noise or by the external large scale perturbations (e.g., sawtooth).

Theoretical procedure has been explained in literature [1, 2]. The dynamical equation of the quantity of interest X is given as a Langevin equation as

$$\frac{\partial}{\partial \tau} X + \Lambda X = w(\tau)\, g, \tag{1}$$

Excitations by large events are discussed later. In this expression, X is the radial electric field at edge when one studies the L-H transition and is taken as a perturbed helical vector potential if we study the onset of NTM. Λ represents the nonlinear damping rate of the mode. If $\Lambda < 0$ holds, X increases in time. The term $w(\tau)\, g$ denotes the noise term, and $w(\tau)$ is the white noise term. The magnitude of noise g is given by the amplitude of the background fluctuations. The nonlinear marginal stability condition is given by

$$\Lambda = 0 \tag{2}$$

which can have multiple solutions. Multiple solutions (X_A and X_B being separated by X_m) are illustrated in Fig.1. The parameter C is a schematic expression of the global parameter that controls the bifurcation. In the case of the L-H transition, C is chosen as the global pressure gradient (for fixed collisionality) [14]. In the case of the NTM excitation, C is expressed by the poloidal beta value [14]. Explicit forms of Λ and g are given in [14, 15] and [16, 17] for L-H transition and NTM, respectively.

The statistical theory has been developed for the system of Eq.(1), and statistical properties (statistical average, transition rate, phase boundary, etc.) have been obtained. Owing to the stochastic excitation, statistical average $\langle X \rangle$ becomes a smooth function of the controlling parameter C as is shown in Fig.1.

The rates of transition (from A to B) and back-transition (B to A) are

$$r_{\mathrm{A}\to\mathrm{B}} = \frac{\sqrt{\Lambda_{\mathrm{A}}\Lambda_{\mathrm{m}}}}{2\pi} \exp\left(S(X_{\mathrm{A}}) - S(X_{\mathrm{m}})\right), \tag{3a}$$

$$r_{\mathrm{B}\to\mathrm{A}} = \frac{\sqrt{\Lambda_{\mathrm{B}}\Lambda_{\mathrm{m}}}}{2\pi} \exp\left(S(X_{\mathrm{B}}) - S(X_{\mathrm{m}})\right), \tag{3b}$$

respectively, where the time rates $\Lambda_{\mathrm{A,\,m,\,B}}$ are given as $\Lambda_{\mathrm{A,\,m,\,B}} = 2X\left|\partial\Lambda/\partial X\right|$ at $X = X_{\mathrm{A,\,m,\,B}}$ [1, 2]. (For the case of L-H transition, we read $\mathrm{A} \to \mathrm{L}$ and $\mathrm{B} \to \mathrm{H}$. In

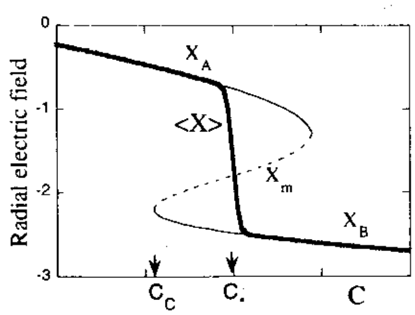

Fig.1 Bifurcation and statistical average as a function of the global controlling parameter C. The case of radial electric field in the L-H transition is shown. (X_A and X_B denote the L-mode and H-mode, respectively.) Thin line is for deterministic model, and thick line shows the statistical average.

the problem of NTM, B is for excited state.) The renormalized dissipation function is defined as $S(X) = \int^{X} 4\Lambda(X')g(X')^{-2}X'\,\mathrm{d}X'$. The phase limit between the states A and B is given by

$$S(X_{\rm B}) = S(X_{\rm A}) + \frac{1}{2}\ln(\Lambda_{\rm A}/\Lambda_{\rm B}) . \tag{4}$$

Apart from a weak logarithmic term, it is approximated as $S(X_{\rm B}) = S(X_{\rm A})$. Equation (4) determines the boundary $C = C_*$ for the statistical average. This is different from the ridge of the cusp ($C = C_c$ in Fig.1). The life-time of the state $X = X_A$ is given by $r_{A \to B}^{-1}$.

III Occurrence of transition and implication for the database

3.1 Trigger by turbulent noise

The statistical theory gives a new insight for the database of the transition condition. When database for the onset of L-H transition is collected, one measures the value of C when the transition occurs, under the circumstance that the relevant parameter C is gradually increased. This is a kind of transient problem and a new statistical evaluation is necessary, in addition to the long time average. This is obtained in the following.

The situation that the first transition is observed is a 'one-through' problem. We consider the situation that the state is in $X = X_A$ at $t = 0$ and the controlling parameter C is slowly increasing. The number of ensemble that is in the state $X = X_A$, N_A , satisfies the equation

$$\frac{\partial}{\partial t} N_A = - r_{A \to B} N_A . \tag{5}$$

The source by the back-transition from the state $X = X_B$ is not kept, because we study when the first transition from A to B occurs. Solving Eq.(5), we have

$$N_A = N_{A,0} \exp\left(- \int_0^t r_{A \to B}\,\mathrm{d}t \right) \tag{6}$$

where $N_{A,0}$ is the initial number of ensemble. The slow variation of the controlling parameter is prescribed as $C(t)$, and we here consider a linear dependence as

$$C(t) = C(0) + (\mathrm{d}C/\mathrm{d}t)t . \tag{7}$$

The relative abundance of occurrence of transitions is given by

$$P = -\left(\mathrm{d}C/\mathrm{d}t\right)^{-1} N_{A,0}^{-1} \frac{\partial}{\partial t} N_A \,. \tag{8}$$

We obtain

$$P(C) = \left(\mathrm{d}C/\mathrm{d}t\right)^{-1} r_{A \to B} \exp\left(-\left(\mathrm{d}C/\mathrm{d}t\right)^{-1} \int_{C(0)}^{C} r_{A \to B}\, \mathrm{d}C \right). \tag{9}$$

As C increases in the domain of multiple solutions, the transition from the A-state to the B-state can happen. When C approaches to C_*, transition becomes more frequent. When C far exceeds C_*, the state has already changed to the B-state, and only little A-to-B transition can occur. $P(C)$ has a broad peak around $C = C_*$.

Equation (9) shows the competition between the transition rate and the rate of variation of the controlling parameter $\mathrm{d}C/\mathrm{d}t$. It has been shown that the transition rate changes as

$$r_{A \to B}(C) \propto \exp\left(\left(C - C_*\right)/\sigma \right) \tag{10}$$

in the vicinity of $C = C_*$. (The parameter of width σ is given by $\sigma^{-1} = -\int_{X_A}^{X_m} \mathrm{d}X' 4X' \, \partial\left\{ \Lambda(X') g(X')^{-2} \right\} / \partial C$.) Based on the results in [14-17], we employ a model

$$r_{A \to B}(C) \simeq f_\infty \left\{ 1 + \exp\left(-\left(C - C_*\right)/\sigma \right) \right\}^{-1}, \tag{11}$$

where f_∞ denotes the transition rate at $C >> C_*$. Substituting Eq.(11) into (9), we have

$$P(C) = \frac{\sigma^{-1} h \exp\left(\left(C - C_*\right)/\sigma \right)}{\left\{ 1 + \exp\left(\left(C - C_*\right)/\sigma \right) \right\}^{1+h}}, \tag{12}$$

where $h \equiv \sigma f_\infty \left(\mathrm{d}C/\mathrm{d}t\right)^{-1}$.

Figure 2(a) illustrates $P(C)$ for various values of h. When the change of the controlling parameter is slow, $h > 1$, the transition is peaked around $C \simeq C_*$, and the

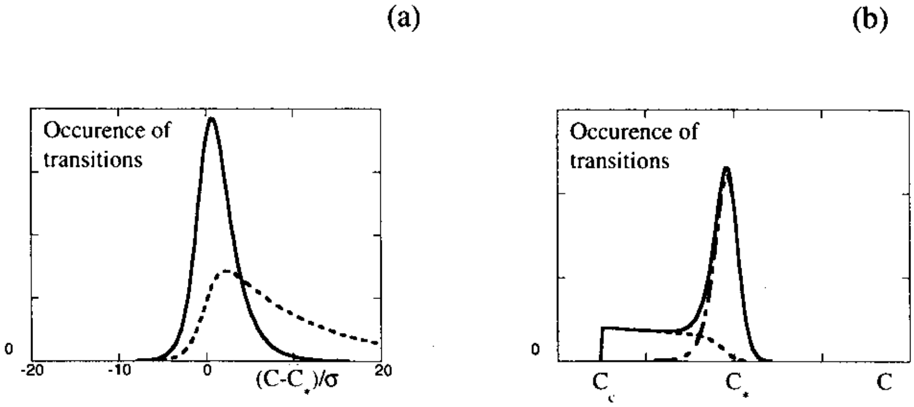

Fig.2 Occurrence of noise-trigger transition (e.g., the L-H transition) when the controlling parameter C is changing in time (a). The case where the change of C is slow ($h \equiv \sigma f_\infty (\mathrm{d}C/\mathrm{d}t)^{-1} = 1$, solid line) and that is fast ($h = 0.1$, dashed line) are shown. The contribution of the trigger by external events is included in (b). The contribution of the trigger of noise (broken line, $h = 1$) and that by the external events (dashed line) compose the total (slid line). (Parameters: $\sigma f_{\mathrm{event}} (\mathrm{d}C/\mathrm{d}t)^{-1} = 0.1$ and $C_c - C_* = 15\sigma$)

width of the peak is characterized by σ . When the change of controlling parameter is faster than the rate of transitions, $h << 1$, the transition is observed abundantly in the region of $C > C_*$ having an asymptotic dependence $P(C) \propto \exp\left(-h\left(C - C_*\right)/\sigma\right)$. The width becomes σ/h . The result shows that the rate of the change of global parameters, $\mathrm{d}C/\mathrm{d}t$, influences the distribution $P(C)$. This clarifies the feature of transient response of the system with stochastic transitions.

3.2 Trigger by external global events

The onset of transition by external events is also discussed by stochastic model. In this case, the stochastic equation (1) takes the form

$$\frac{\partial}{\partial \tau} X + \Lambda X = F(t) \tag{13}$$

where $F(t)$ represents a random sequence of strong impulses. By one pulse, the barrier $X = X_m$ can be overcome. Let us consider an impulse, $F(t) = F_0$ for $0 < t < t_0$ and $F(t) = 0$ otherwise. The strength of impact is characterized by $G \equiv F_0 t_0$. Eq.(13) is solved as

$$\int_{X_A}^{X} \frac{\mathrm{d}X}{F_0 - \Lambda X} = t \tag{14}$$

The condition that the crossing over barrier $X = X_m$ occurs is expressed as

$$t_0 F_0 > G_c = \int_{X_A}^{X_m} \frac{dX}{1 - F_0^{-1} \Lambda X} \tag{15}$$

and $F_0 > \Lambda X$. The right hand side of Eq.(15) is estimated by employing a quadratic model $\Lambda X = \Lambda_m (X - X_A)(X_m - X)$ as

$$G_c = 2\xi F_0 \Lambda_m^{-1} \tan^{-1}\left(\xi (X_m - X_A)/2\right), \tag{16}$$

where $\xi^2 = \Lambda_m F_0^{-1} / \left(1 - 4\Lambda_m F_0^{-1} (X_m - X_A)^2\right)$ holds. In the limit of strong impact $F_0 >> |\Lambda X|$, it gives $G_c \simeq X_m - X_A$.

The rate of the excitation of transition is given by the rate of the occurrence of the impulse that satisfies the condition Eq.(15). The rate of transition by the external events is given by the rate that the impact of $G > G_c$ happens. We write the rate of such strong events f_{event}. If the noise and events coexist, $r_{\mathrm{A} \to \mathrm{B}}$ in Eq.(5) is replaced by $r_{\mathrm{A} \to \mathrm{B}} + f_{\mathrm{event}}$. The life time of the state A is given by $\left(r_{\mathrm{A} \to \mathrm{B}} + f_{\mathrm{event}}\right)^{-1}$. Consequently, $P(C)$ is given by the formula

$$P(C) = \left(\mathrm{d}C/\mathrm{d}t\right)^{-1}\left(r_{\mathrm{A} \to \mathrm{B}} + f_{\mathrm{event}}\right) \exp\left(-\left(\mathrm{d}C/\mathrm{d}t\right)^{-1} \int_{C(0)}^{C} \left(r_{\mathrm{A} \to \mathrm{B}} + f_{\mathrm{event}}\right) \mathrm{d}C\right). \tag{17}$$

When the infrequent external events occur independently with C, the PDF of the occurrence of transition by external events becomes nearly a stepwise function with respect to C. Figure 2(b) illustrates schematically the expected observation of the onset.

IV. Summary and implication to experiments

A statistical model for the bifurcation is analyzed in view of understanding of the threshold data base, taking examples of L-H transition and onset of NTM. The transition probabilities are calculated, and the effective phase limit is derived. The distribution in parameter space for observing transitions $P(C)$ is obtained under the circumstance of the temporal change of C. The method to compare the threshold database and transition physics is presented.

Implications to experimental threshold database are as follows: First, the ensemble averages of $\langle X \rangle$ and related quantities do not show hysteresis against global parameters when the change of parameters are slow. (This is in contrast to the deterministic model.) Second, the ensemble average of the transition condition in plasma parameters is different from the ridge of cusp in deterministic model. They may show the different parameter dependencies. The peak of the database must be compared to the ensemble averages of statistical models, and not to conditions of ridge in deterministic models. Third, each transition occurs being scattered around the ensemble average due to the noise as is illustrated by $P(C)$. The width of peak of distribution in database is influenced by both the noises and by the rate of change of global parameters. The observation of hysteresis in experiments critically depends on the speed of global parameter change. These must be noticed in the future comparison of experimental database with many theories. Relevant comparison between the theory and experimental threshold database is summarized in Table 1.

Threshold Database	**Theories**
Peak of distribution of observed transitions	Peak of PDF $P(C)$ in statistical theory
Width of the peak of observed transitions (adiabatic change of parameters)	width of PDF
Width of the peak (rapid change of parameters)	Competition between transition rate and rate of global parameter change
Boundaries of parameters where transitions are observed	Ridges of cusp in deterministic part of theory

Table 1: Approaches in comparison study of experiments and theories. Appropriate theoretical method must be employed to relevant experimental approaches.

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