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The configuration dependence of ripple transport in LHD by the application of the GIOTA code

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Abstract: The GIOTA code is introduced with the emphasis on its logics and characteristics for the purpose of the evaluation of the ripple transport in helical systems. It is rather easy-to-use code with taking into account rigorously the effect of the finite rotational transform and the magnetic field topography. To demonstrate its capability, the effective helical ripple is calculated for LHD (Large Helical Device) configurations with a wide range scan of magnetic axis position, (R_{ax}) and plasma beta value (β).

Keywords: GIOTA code, ripple transport, effective ripple, helical systems, LHD

1. Introduction

In the helical systems, there exists the modulation of the magnetic field strength, B, along a magnetic field line due to the helical coil currents in addition to the toroidicity. This modulation of B gives the trapping of particles depending on the pitch angle. Since trapped particles move with the drift motion in addition to the bounce motion, large deviation from a flux surface occurs for the condition, $\omega_{\mathrm{b}} >> \nu_{\mathrm{eff}} >> \omega_{\mathrm{p}}$. Here, ω_{b} denotes the bounce frequency within a ripple of B, ν_{eff} the effective collision frequency, and ω_{p} the characteristic frequency of the poloidal drift motion. The diffusion coefficient for this range of collisionality, as a result, becomes inverse proportional to collisionality, which leads to the enhancement of diffusion coefficient for lower collisionality (*e.g.* higher temperature and/or lower density). This is so called ripple diffusion and the corresponding collisionality regime is called $1/\nu$ regime, which is of vital important to investigate in non-axisymmetric magnetic configurations.

In this report, GIOTA code, which is one of the numerical codes for evaluating ripple diffusion, is introduced with emphasis on the based-formulation and logics. It is noted that the GIOTA code has already been used as a part of the three dimensional equilibrium/one dimensional transport simulation code [1] for designing LHD device and analyzing LHD/CHS experimental data to estimate ripple transport. Although the applicable range of collisionality is limited to $1/\nu$ regime due to the employed ordering of the drift kinetic equation, GIOTA code can take into account rigorously the effect of the finite rotational transform, complicated three dimensionality of the magnetic field. The contribution to the diffusion from locally trapped particles is summed up for each toroidal field period as you will recognize in the following description. This is different methodology from that utilized in the NEO code [2], in which all magnetic field ripples even beyond the toroidal field period are taken into account. It is also rather light and easy-to-use code, which is especially appropriate to grasp the comprehensive dependence of ripple diffusion property on a wide range of magnetic configurations.

2. Description of the GIOTA code

The ripple diffusion in $1/\nu$ regime can be formulated based on the steady state drift kinetic equation considering the condition, $\nu_{\mathrm{eff}} >> \omega_{\mathrm{p}}$,

$$\mathrm{v}_{\|} \frac{\vec{B}}{B} \cdot \nabla f_1 + \vec{\mathrm{v}}_{\mathrm{D}} \cdot \nabla \psi \frac{\partial f_0}{\partial \psi} = C(f_1)\,, \tag{1}$$

where $\mathrm{v}_{\|}$ and $\vec{\mathrm{v}}_{\mathrm{D}}$ are the parallel velocity and the drift velocity of a particle, respectively. The following expression is obtained,

$$\vec{\mathrm{v}}_{\mathrm{D}} \cdot \nabla \psi = \frac{m\mathrm{v}_{\|}}{qB} \vec{B} \times \nabla \psi \cdot \nabla \left(\frac{\mathrm{v}_{\|}}{B} \right), \tag{2}$$

with m and q being the mass and charge of a particle. The f_0 and f_1 are the zero-th and first order distribution function in the ordering of Larmor radius, and f_0 is assumed to be the local Maxwellian. The ψ denotes a flux function and $C(f_1)$ is the linearlized collision term. It is noted that the stationality is assumed in Eq. (1) and the induced electric field is neglected. Furthermore, according to the condition that $\omega_{\mathrm{b}} \gg \nu_{\mathrm{eff}}$, the first order distribution function f_1 is expanded with the parameter $\nu_{\mathrm{eff}}/\omega_{\mathrm{b}} \ll 1$ as follows,

$$f_1 = \frac{\omega_{\mathrm{b}}}{\nu_{\mathrm{eff}}} f_1^{(-1)} + f_1^{(0)} + \frac{\nu_{\mathrm{eff}}}{\omega_{\mathrm{b}}} f_1^{(1)} + \cdots \quad . \tag{3}$$

Substituting Eq. (3) into Eq. (1) yields

$$\frac{\partial f_1^{(-1)}}{dl} = 0 \tag{4}$$

from the lowest order, where dl is the line element along the magnetic field line. The next order gives

$$\mathrm{v}_{\|} \frac{\vec{B}}{B} \cdot \nabla f_1^{(0)} + \vec{\mathrm{v}}_{\mathrm{D}} \cdot \nabla \psi \frac{\partial f_0}{\partial \psi} = C(f_1^{(-1)})\,. \tag{5}$$

By applying the bounce averaging operator

$$\langle A\rangle_b = \oint \frac{dl}{\mathrm{v}_\parallel} A\,, \tag{6}$$

to Eq. (5) leads to

$$\left\langle \vec{\mathrm{v}}_\mathrm{D}\cdot\nabla\psi\frac{\partial f_0}{\partial\psi}\right\rangle_b = \left\langle C(f_1^{(-1)})\right\rangle_b \ . \tag{7}$$

Once $f_1^{(-1)}$ is obtained from Eq. (7) utilizing Eq. (4), the particle and heat fluxes across a flux surface and also their diffusion coefficients can be evaluated through

$$\left\langle \vec{\Gamma}\cdot\nabla\psi\right\rangle = \left\langle \int d\mathrm{v}^3\vec{\mathrm{v}}_\mathrm{D}\cdot\nabla\psi f_1^{(-1)}\right\rangle, \tag{8}$$

$$\left\langle \vec{Q}\cdot\nabla\psi\right\rangle = \left\langle \int d\mathrm{v}^3\vec{\mathrm{v}}_\mathrm{D}\cdot\nabla\psi\left(\frac{1}{2}m\mathrm{v}^2\right)f_1^{(-1)}\right\rangle, \tag{9}$$

where $\langle\cdots\rangle$ is the flux surface average.

In the following, the Boozer coordinates (ψ, χ, ζ) is employed for the concrete evaluation. The magnetic field can be described in the Boozer coordinates as

$$\vec{B} = \nabla\psi\times\nabla\chi + (\iota/2\pi)\,\nabla\zeta\times\nabla\psi = \nabla\psi\times\nabla[\chi-(\iota/2\pi)\,\zeta], \tag{10}$$

where ψ is the toroidal flux divided by 2π, χ and ζ is poloidal and toroidal angle, respectively. The $(\iota/2\pi)$ is the rotational transform. Defining

$$\alpha = \chi-(\iota/2\pi)\,\zeta \tag{11}$$

as the alternative angle variable to form coordinates (ψ, α, ζ), leads to the fact that a magnetic field line can be described by ψ=const. and α=const. Only the pitch angle scattering is considered as the linearized collision operator as follows,

$$C(f_1) = 2\nu(K)\frac{\lambda^2}{B}\frac{\mathrm{v}_\parallel}{\mathrm{v}}\frac{\partial}{\partial\lambda}\left\{\lambda\frac{\mathrm{v}_\parallel}{\mathrm{v}}\frac{\partial f_1}{\partial\lambda}\right\}, \tag{12}$$

where $\lambda\equiv K/\mu$ with being K and μ the kinetic energy and magnetic momentum of a particle, respectively. Moreover,

$$\nu(K) = \nu_{ee}+\nu_{ei}\ \ \text{(for electron)}, \nu_{ii}\ \ \text{(for ion, since}\ \nu_{ie}\langle\langle\nu_{ii}) ,$$

$$\nu_{jk} \equiv \frac{4\pi n_k Z_j^2 Z_k^2 e^4 \ln\Lambda}{m_j^2\mathrm{v}_\mathrm{j}^3}\hat{K}_j^{-\frac{3}{2}}\left\{\left(1-\frac{1}{2\hat{K}_k}\right)\Phi\left(\sqrt{\hat{K}_k}\right)+\frac{1}{\sqrt{\pi\hat{K}_k}}e^{-\hat{K}_k}\right\},$$

$$\hat{K}_j = \frac{m_j \mathrm{v}^2}{2T_j}, \qquad \hat{K}_k = \frac{m_k \mathrm{v}^2}{2T_k}, \qquad \Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy. \tag{13}$$

The distribution function, $f_1^{(-1)}$, for particles trapped in a helical ripple is now evaluated utilizing $dl = B\sqrt{g}d\zeta$ with $\sqrt{g} \propto 1/B^2$ being the Jacobian of the Boozer coordinates. The M helical ripple exists in the full range of $0 \le \zeta < 2\pi$ for a magnetic configuration with the toroidal field period of M. When one specifies a number of the helical ripple with i (=1, … , M), the left hand side of Eq. (7) is described by

$$\left\langle \vec{\mathrm{v}}_{\mathrm{D}} \cdot \nabla \psi \right\rangle_b = -\frac{2mB^2\sqrt{g}}{3q} \mathrm{v}\lambda^2 \frac{\partial}{\partial \lambda} H_1(\lambda, \alpha, i), \tag{14}$$

$$H_1(\lambda, \alpha, i) \equiv \int_{\zeta_A(\lambda,\alpha,i)}^{\zeta_B(\lambda,\alpha,i)} d\zeta \frac{1}{B^3}\sqrt{1-\frac{B}{\lambda}}\left(4-\frac{B}{\lambda}\right)\frac{\partial B}{\partial \alpha}, \tag{15}$$

where $\zeta_A(\lambda,\alpha,i)$ and $\zeta_B(\lambda,\alpha,i)$ describe the turning points for particles trapped in a i-th helical ripple, both depending on λ and α. Likewise, the right hand side of Eq. (7) becomes

$$\left\langle C(f_1^{(-1)}) \right\rangle_b = \frac{4\nu(K)B^2\sqrt{g}}{\mathrm{v}} \lambda^2 \frac{\partial}{\partial \lambda}\left\{\lambda \frac{\partial f_1^{(-1)}}{\partial \lambda} H_2(\lambda,\alpha,i)\right\}, \tag{16}$$

$$H_2(\lambda, \alpha, i) \equiv \int_{\zeta_A(\lambda,\alpha,i)}^{\zeta_B(\lambda,\alpha,i)} d\zeta \frac{1}{B^2}\sqrt{1-\frac{B}{\lambda}}. \tag{17}$$

It is recognized that $H_1(\lambda,\alpha,i)$ [$H_2(\lambda,\alpha,i)$] indicates the dependence of the bounce-averaged drift velocity across a magnetic surface [the bounce-averaged effective collision frequency] on the magnetic field. From Eqs. (7), (14) and (16), the $f_1^{(-1)}$ is obtained as

$$f_1^{(-1)}(K,\lambda,\alpha,i) \equiv \frac{K}{3q\nu(K)}\frac{\partial f_0}{\partial \psi}\int_{\lambda}^{B_{\max}(\alpha,i)} d\lambda' \frac{H_1(\lambda',\alpha,i)}{\lambda' H_2(\lambda',\alpha,i)}, \tag{18}$$

where $B_{\max}(\alpha,i)$ denotes the maximum reachable magnetic field strength for particles trapped in the i-th helical ripple. The domain for computing $f_1^{(-1)}$ is summarized with the notations in Fig.1.

The two important relations are now explained before coming to the evaluation of the particle and heat fluxes. One is the flux surface average and the other is the variable transformation for the integral in the velocity space. The flux surface average is defined by

$$\left\langle A \right\rangle = \oint A\frac{dl}{B} / \oint \frac{dl}{B}, \tag{19}$$

which can be written for the flux coordinates (ψ, α, ζ) as

$$\left\langle A \right\rangle = \frac{\int_0^{2\pi} d\alpha \int_0^{2\pi} A \frac{d\zeta}{B^2}}{\int_0^{2\pi} d\alpha \int_0^{2\pi} \frac{d\zeta}{B^2}} \quad . \tag{20}$$

Since the distribution function, $f_1^{(-1)}$, is calculated only in a helical ripple, Eq. (20) can be more modified as

$$\left\langle A \right\rangle = \frac{\int_0^{2\pi} d\alpha \sum_{i=1}^{M} \int_{\zeta_A(\lambda,\alpha,i)}^{\zeta_B(\lambda,\alpha,i)} A \frac{d\zeta}{B^2}}{\int_0^{2\pi} d\alpha \int_0^{2\pi} \frac{d\zeta}{B^2}} . \tag{21}$$

The integral in the velocity space is expressed by the variable transformation as

$$\int d^3\mathrm{v} = \int_0^{\infty} dK \int_{B_{\min}(\alpha,i)}^{B_{\max}(\alpha,i)} d\lambda \sum_{\sigma} \frac{2\pi BK}{m^2 \lambda^2 \left| \mathrm{v}_{\parallel} \right|} , \tag{22}$$

with $B_{\min}(\alpha,i)$ is the minimum of the magnetic field strength as a function of α in an i-th helical ripple and σ the sign of parallel velocity of a particle.

Based on these two relations, Eqs. (21) and (22), performing the same procedures to derive Eq. (14) leads to the expression for the particle and heat fluxes,

$$\left\langle \vec{\Gamma} \cdot \nabla \psi \right\rangle = -\int_0^{\infty} dK \frac{2\mathrm{v}K^2}{q^2 m \nu(K)} \frac{\partial f_0}{\partial \psi} G_D \ , \tag{23}$$

$$\left\langle \vec{Q} \cdot \nabla \psi \right\rangle = -\int_0^{\infty} dK \frac{2\mathrm{v}K^3}{q^2 m \nu(K)} \frac{\partial f_0}{\partial \psi} G_D \ . \tag{24}$$

Here G_D is the geometrical factor, which does not depend on K. The G_D is expressed as

$$G_D = \frac{2\pi}{9} \frac{\int_0^{2\pi} d\alpha \sum_{i=1}^{M} \int_{B_{\min}(\alpha,i)}^{B_{\max}(\alpha,i)} d\lambda \frac{\left[H_1(\lambda,\alpha,i) \right]^2}{\lambda H_2(\lambda,\alpha,i)}}{\int_0^{2\pi} d\alpha \int_0^{2\pi} \frac{d\zeta}{B^2}} \quad . \tag{25}$$

Assuming that f_0 is the local Maxwellian,

$$f_0(\psi, E) = \left(\frac{m}{2\pi T} \right)^{3/2} n e^{-\frac{E - q\phi}{T}} \quad , \tag{26}$$

with E, ϕ and T are the energy, electrostatic potential and temperature, respectively. Substituting Eq. (26) into Eqs. (23) and (24) leads to the following expressions for particle and heat fluxes (a=*ion, electron*).

$$\left\langle \vec{\Gamma} \cdot \nabla \psi \right\rangle = -\frac{G_D}{\nu_a^*} \left(\frac{T_a}{q_a} \right)^2 \left\{ L_{11a} \frac{dn_a}{d\psi} + L_{12a} \frac{n_a}{T_a} \frac{dT_a}{d\psi} + L_{13a} \frac{n_a q_a}{T_a} \frac{d\phi}{d\psi} \right\}, \quad (27)$$

$$\left\langle \vec{Q} \cdot \nabla \psi \right\rangle = -\frac{G_D}{\nu_a^*} \left(\frac{T_a}{q_a} \right)^2 \left\{ L_{21a} T_a \frac{dn_a}{d\psi} + L_{22a} n_a \frac{dT_a}{d\psi} + L_{23a} n_a q_a \frac{d\phi}{d\psi} \right\}. \quad (28)$$

Here,

$$L_{j1a} = E_{ja}, \quad (29)$$

$$L_{j2a} = (E_{j+1a} - \tfrac{3}{2} E_{ja}), \quad (30)$$

$$L_{j3a} = L_{j1a} \quad (31)$$

$$E_{ja} = \frac{2}{\sqrt{\pi}} \int_0^\infty d\hat{K} \frac{\hat{K}^{j+3/2} e^{-\hat{K}}}{2\pi \nu(\hat{K}) / \nu_a^*}, \quad (32)$$

$$\nu_a^* \equiv \frac{4\pi n_a Z_a^4 e^4 \ln \Lambda}{m_a^2 \mathrm{v}_a^3}. \quad (33)$$

3. Application of the GIOTA code for LHD configurations

So called “effective ripple (ε_{eff})”, has been frequently considered to estimate the level of the ripple transport in helical systems as the comparative parameter among different configurations [3]. This parameter reflects the effect of the multiple helicity of the magnetic configuration on the ripple transport. Here, the configuration dependence of the effective ripple in LHD is examined by applying the GIOTA code. The definition of the effective ripple [3] is

$$\varepsilon_{\text{eff}} = \left[\frac{9\sqrt{2}\pi}{16} \frac{\nu}{\mathrm{v}_{\text{d}}^2} D \right]^{2/3}, \quad (34)$$

where ν, v_{d} and D are the collision frequency, drift velocity and particle diffusion coefficient, respectively.

Figure 2 shows the contour of $\log(\varepsilon_{\text{eff}}^{3/2})$ on the (R_{ax}, β) plane for the radial position of (a) ρ=0.2, (b) ρ=0.5 and (c) ρ=0.8, respectively. Here, R_{ax} denotes the magnetic axis position at vacuum configuration, β the volume averaged beta value. The MHD equilibria for these calculations are based on the fixed-boundary VMEC [4]. This condition corresponds to an operation with a feedback control of the vertical field to keep the plasma position same as that for the vacuum case. The pressure profile employed for these VMEC calculations is $P(\rho)= P(0)(1-\rho^2)(1-\rho^8)$.

This kind of parameter scan calculations in a wide range of configuration space can be relatively easily done by the GIOTA code. This is the significant advantage of the GIOTA code. The designated numbers denote the value of $\log(\varepsilon_{\mathrm{eff}}^{3/2})$ on each contour. The minimum of $\varepsilon_{\mathrm{eff}}^{3/2}$ appears around R_{ax} of about 3.53-3.55 m regardless of radial position, ρ, for vacuum cases. This feature well reproduces the previous finding of the "neoclassical-optimized configuration in LHD" by the DCOM code [5]. The $\varepsilon_{\mathrm{eff}}^{3/2}$ increases as β is increased for configurations with $R_{\mathrm{ax}}\geq 3.53$ m. It is not the case, however, for configurations with R_{ax} less than that value. For those configurations, $\varepsilon_{\mathrm{eff}}^{3/2}$ decreases as β is increased in this β range. It is also recognized that this property appears regardless of ρ from plasma core to edge region. Comprehensive understandings have not yet been reached on this characteristic, which will be examined in detail in the near future.

As for reference, the magnetic axis position of VMEC equilibria (from vacuum to finite β cases) are shown as the contour plot on (R_{ax}, β) plane in Fig.3. The monotonous increase of magnetic axis position (outward shift) is seen as β is increased if one sees in the vertical direction starting from vacuum axis position, R_{ax}. It is interesting to note here that the variation of $\varepsilon_{\mathrm{eff}}^{3/2}$ on (R_{ax}, β) plane is similar to that of the magnetic axis position. More concretely, the minimum region of $\varepsilon_{\mathrm{eff}}^{3/2}$ is well aligned to the region with the magnetic axis position of around 3.5 to 3.6 m. This fact implies that the magnetic topography in a wide range of magnetic configurations in LHD is strongly correlated to the position of plasma column. This feature will also be investigated elsewhere in detail.

4. Conclusion

The GIOTA code is introduced as the light-to-use code for evaluating ripple transport property in helical systems. The magnetic topography can be rigorously treated by the code including the effect of the finite rotational transform. To demonstrate its capability, the effective helical ripple has been evaluated on a wide range of LHD equilibria, reproducing the reported fact [5] that its minimum value appears around a configuration with Rax=3.53m. The physical reasons of this fact will be investigated in detail in a separate paper by utilizing the rigorous treatment of the configuration properties capable with the GIOTA code.

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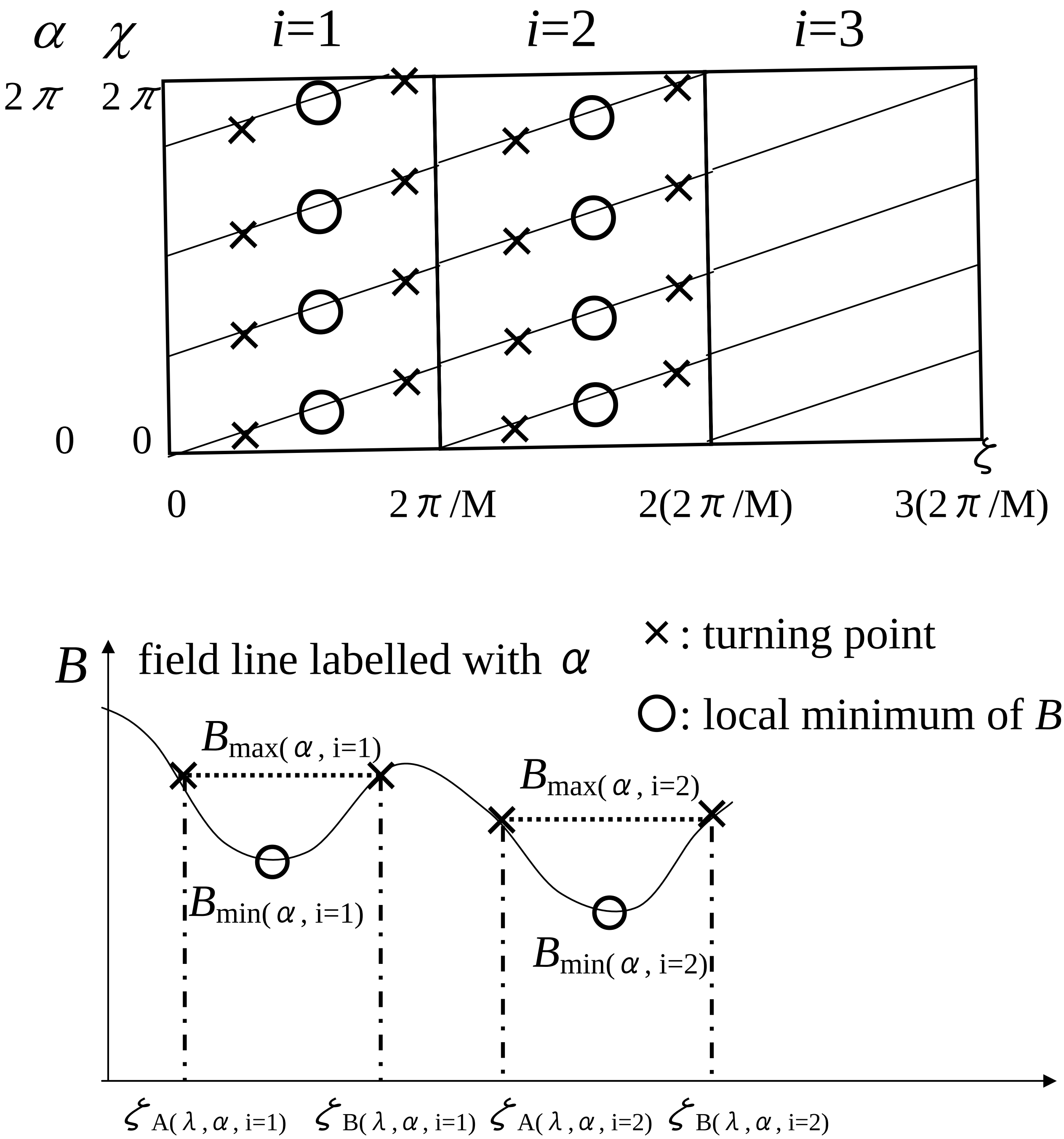
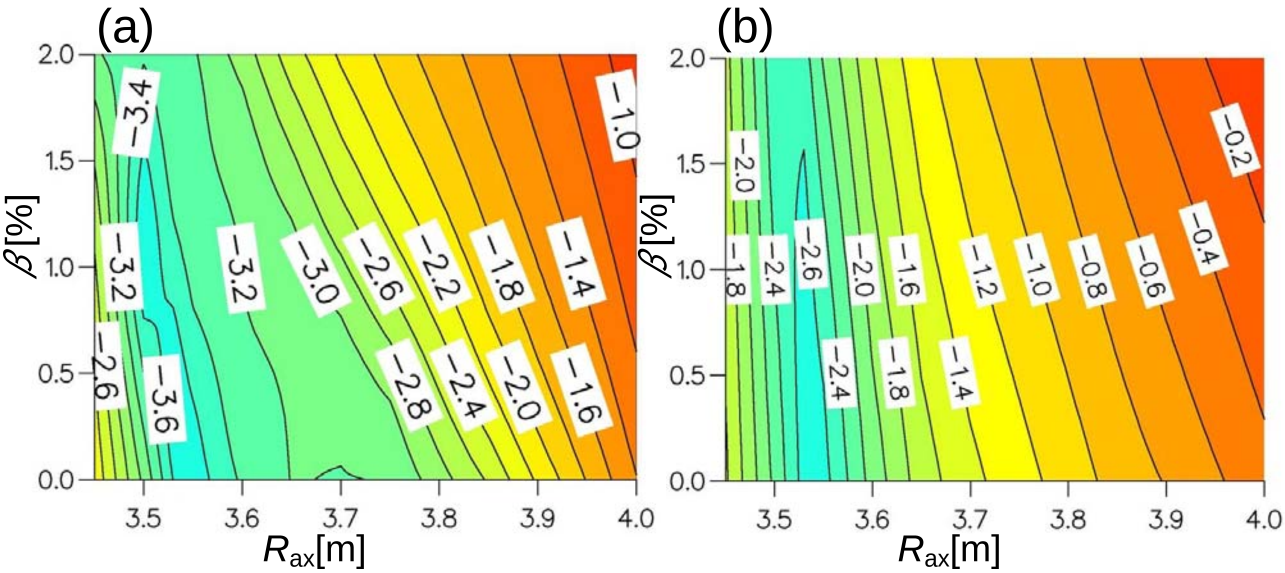


Fig.1 The domain for computing $f_1^{(-1)}$ is shown with notations.



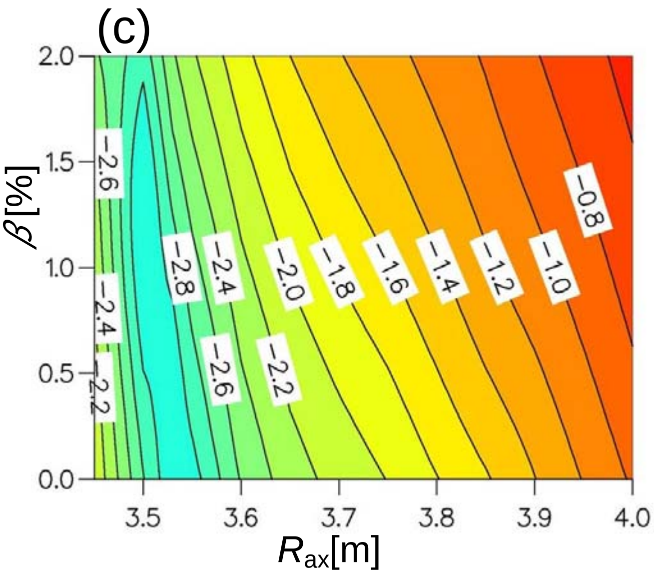


Fig.2 Contour of log($\varepsilon_{\mathrm{eff}}^{3/2}$) on the ($R_{ax}$, β) plane for the radial position of (a) ρ=0.2, (b) ρ=0.5 and (c) ρ=0.8, respectively on a wide range of LHD fixed-boundary equilibria.

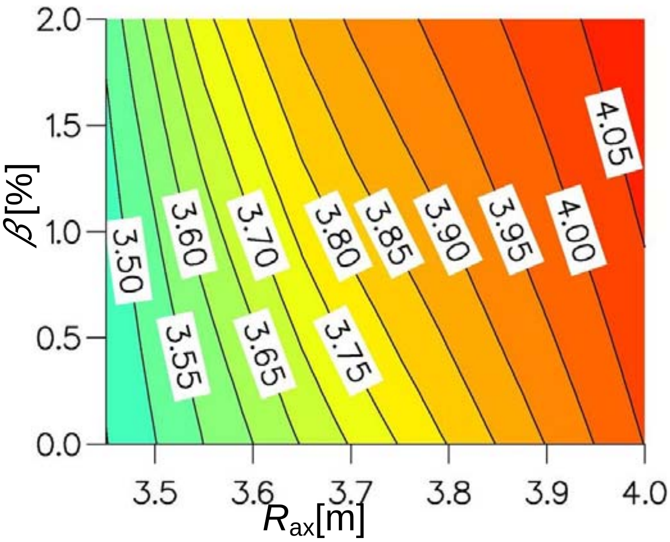


Fig.3 Contour of the magnetic axis position of VMEC equilibria on (R_{ax}, β) plane on a wide range of LHD fixed-boundary equilibria.