ISSN 0915-6372

NATIONAL INSTITUTE FOR FUSION SCIENCE

# Discussion Record of the Workshop on Nonlocal Transport

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(Received - Apr. 23, 1997)

NIFS-MEMO-23 June 1997

**RESEARCH REPORT**
**NIFS-MEMO Series**



NAGOYA, JAPAN

# Discussion Record of the Workshop on Nonlocal Transport

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The discussion on the problem of the transient response and nonlocal transport is reported. Problem of the transient response is surveyed, and several approaches are reviewed. The formulation based on the nonlocal transport is discussed. Example of the analysis is presented. Future study is identified.

Keywords: nonlocal transport, transient response, integral form of flux, long mean free path

## 1. Introduction

On occasion of the visit of Dr. Stroth to NIFS, the second working workshop on the nonlocal transport was held at Nagoya[1]. This is the extension of the first workshop which was held at Garching last summer. The discussion record is reported here.

This report covers the following aspects:

*Back-ground of the problem*

*Approach based on the nonlocal transport*

*Example of the analysis*

*Experimental Method*

*Future task and next step*

## 2. Back-ground of the problem

First, the background of the problem is briefly explained.

The effective thermal diffusivity is defined by the ratio of the heat flux to the gradient of the temperature as

$$q = -\chi_{eff} n \nabla T \qquad (1)$$

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## 2.1 Experimental Observations

The problem of the nonlocal transport, in toroidal confinement, has been stimulated by the observation of the transient response of plasmas.

In the beginning of the tokamak research, the transport coefficient was evaluated by studying the heat pulse propagation after the sawtooth. Consider that the perturbation is initially ($t = t_0$) localized at $r = r_1$. The perturbation shortly after, $t = t_1$, is still localized, but it diffuses away as time passes, $t = t_2$ (Fig.2.1). The perturbation is expressed as

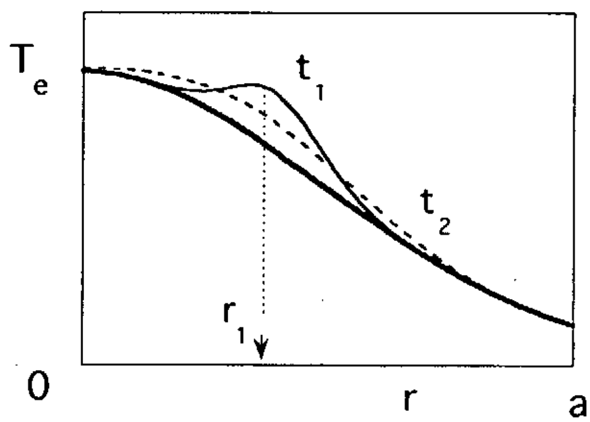



**Fig.2.1** Heat pulse propagation

$$\Delta T(r,t) \propto \frac{1}{\sqrt{\chi_{HP} t}} \, exp\left\{-\frac{(r-r_1)^2}{\chi_{HP} t}\right\} \tag{2}$$

From this fitting of the data, the thermal diffusivity $\chi_{HP}$, which is deduced from the heat pulse propagation, has been obtained experimentally. [Eq.(2) is given for the slab configuration for the simplicity. In the experimental analysis, the formula is obtained in cylindrical geometry.]

When the additional heating power become strong enough, the estimate of the heat deposition profile became more confident, in comparison with the Ohmic heating cases. Then the heat flux in a stationary state is experimentally estimated. By combining the heat flux and the measured temperature gradient, the thermal diffusivity is also evaluated. This diffusivity, being given by the power balance, is usually written as $\chi_{pb}$.

Usually, the relation

$$\frac{\chi_{HP}}{\chi_{pb}} > 1 \tag{3}$$

has been observed. Sometimes, however, the ratio is close to unity

$$\frac{\chi_{HP}}{\chi_{pb}} \simeq 1 \tag{4}$$

More recently, it has been widely recognized that the transport coefficient, i.e., the ratio between the flux and temperature gradient, at the observation point, $r = r_{obs}$, seems to change much faster than the local plasma parameter at $r = r_{obs}$. (That is, $\chi(r_{obs})$ changes faster than $T(r_{obs})$ and $\nabla T(r_{obs})$.) Such a fast change has been observed in the cases of

L/H transition

pellet injection

sawtooth crash

X-event

abrupt change of the heating power.

The understanding of the very fast change, together with the explanation of the difference between $\chi_{HP}$ and $\chi_{pb}$, is required.

### 2.2 Historical evolution of the models

If one looks back the evolution of the thermodynamics from the last century, there is a trial of J. C. Maxwell. Maxwell considered that the Fick's law (or Fourier's law), Eq. (1), must be corrected in a transient phase. The Maxwell's idea is to introduce the "inertia" of the heat flux, and write

$$q + \tau_{Maxwell}\frac{\partial}{\partial t}q = -\chi_{eff}n \nabla T \tag{5}$$

The motivation to keep the inertia term is that, according to the solution (2), the temperature perturbation exists at the distance for any time $t > 0$ (although the amplitude is exponentially small). This indicates that the change at distance occurs within the infinitesimally small time elapse. This "fast" (although exponentially small) propagation was thought *artificial* by him. In order to get rid of this fast propagation, the "inertial" term is introduced. The origin of this delay in the response could be formulated by many physics processes (e.g., the inertia of the energy carrier, growth time for the turbulence, etc.) Perhaps it was the first trial to think about the non-equilibrium nature of the transient heat transport problem.

This inertial effect is effective to slow down the response of the heat flux against the change of the gradient. Therefore this Maxwell's mechanism is not sufficient to explain the present problem in plasmas, although this process may work in the confined plasmas.

In the following, several attempts in the analysis of plasma confinement is illustrated.

*[A] Nonlinearity of* $q[\nabla T]$

The transient response could be expressed in terms of the perturbed temperature $\delta T$ and the perturbed heat flux $\delta q$ as

$$\frac{\partial}{\partial t}n\delta T = -\nabla \cdot \delta q \tag{6}$$

When the perturbation is small, the linearlized equation between $\delta T$ and $\delta q$ holds as

$$\delta q = -\chi_{HP}\nabla\delta T \tag{7}$$

If the thermal diffusivity is expressed in terms of the temperature (and its gradient) alone, the perturbed heat flux is given as

$$\delta q = -\left[\left(\frac{\partial \chi_{eff}}{\partial \nabla T}\right)_0 (\nabla T)_0 + (\chi_{eff})_0\right] \nabla \delta T - \left(\frac{\partial \chi_{eff}}{\partial T}\right)_0 (\nabla T)_0 \delta T \qquad (8)$$

where the suffix 0 indicates the unperturbed state. By definition, $\chi_{pb}$ is expressed as

$$\chi_{pb} = (\chi_{eff})_0 \qquad (9)$$

The relation (8) suggests that the heat-pulse diffusivity is composed of various ingredients. In case that the heat flux is a nonlinear function of the gradient, i.e., the power law

$$\chi_{eff} \propto |\nabla T|^{\alpha} \qquad (10)$$

holds (and $\chi_{eff}$ is independent of $T$), the relation

$$\chi_{HP} = (1 + \alpha)(\chi_{eff})_0 \qquad (11)$$

is derived. The power-degradation of the energy confinement time

$$\tau_E \propto P_{heat}^{-\alpha/(1+\alpha)} \qquad (12)$$

is related to the nonlinearity Eq.(10). $\chi_{pb}$ and $\chi_{HP}$ are illustrated in Fig.2.2.

The ratio is given as

$$\frac{\chi_{HP}}{\chi_{pb}} = 1 + \alpha > 1 \qquad (13)$$

The ratio is larger than unity, if the power degradation takes place $(\alpha > 0)$, as is shown in Fig.2.2.

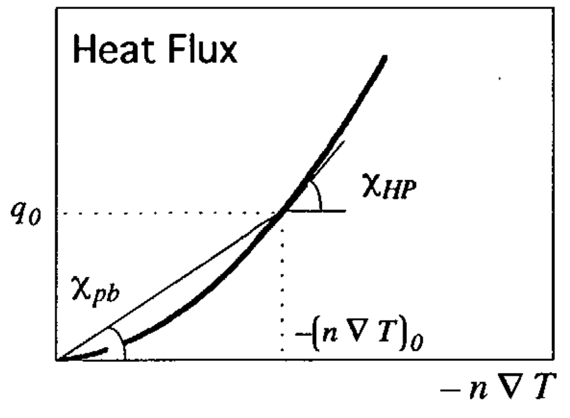



**Fig.2.2** Nonlinear relation $q[\nabla T]$ and the difference between $\chi_{pb}$ and $\chi_{HP}$.

The *difficulty* in this model is that (1) the ratio $\chi_{HP}/\chi_{pb}$ is constrained by the one parameter $\alpha$, and it does not include an enough freedom, and that (2) the perturbation propagates as diffusive process (i.e., the local flux change only if the local gradient changes). It means that the "transport time" is required for the propagation of perturbations. The very fast transmission of change could not be reproduced by this model.

*[B] Interference of Transport*

The nonlinearity of $q[\nabla T]$, as is discussed in Eq.(8), provides the difference between $\chi_{pb}$ and $\chi_{HP}$. The ratio is, however, solely determined by the functional relation $q[\nabla T]$. This, however, contradicts to the experimental observation. For instance, the dependence of the ratio $\chi_{HP} / \chi_{pb}$ on the frequency of the perturbation was found to be different from what was derived from Eq.(8).

The flux-gradient relation, in principle, includes the interference between different gradients. It should be expressed in terms of the transport matrix, e.g.,

$$\begin{pmatrix} \Gamma \\ q \end{pmatrix} = -\begin{pmatrix} D & M_{12} \\ M_{21} & \chi \end{pmatrix} \begin{pmatrix} \nabla n \\ \nabla T \end{pmatrix} \tag{14}$$

Due to the presence of the off-diagonal terms, the new time difference is introduced, and the response becomes more complex. As an example, let us consider the case that gradient changes in time sinusoidary, $sin\,(\omega t)$. It could be possible that there is a phase delay in density and temperature as,

$$\nabla\,\delta T = a_T sin\,(\omega t)\ , \quad \nabla\,\delta n = a_n sin\,(\omega t + \varphi)$$

In such a case, the phase difference appears between the temperature gradient and heat flux as

$$\delta q = \sqrt{\chi^2 a_T^2 + M_{21}^2 a_n^2 + 2\chi M_{21} a_T a_n}\ sin\,(\omega t + \hat{\varphi})$$

$$sin\,\hat{\varphi} = \frac{M_{21} a_n}{\sqrt{\chi^2 a_T^2 + M_{21}^2 a_n^2 + 2\chi M_{21} a_T a_n}}\ sin\,\varphi$$

An example is illustrated in Fig.2.3.

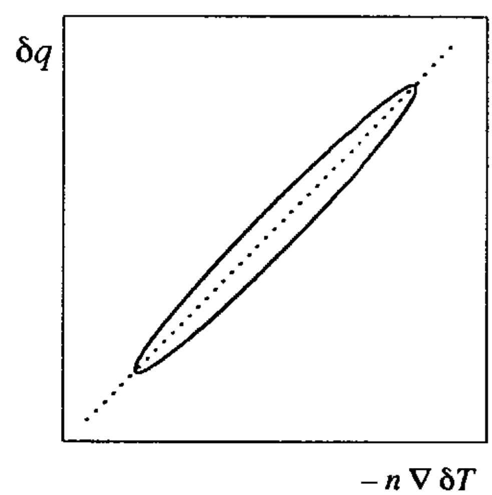



**Fig.2.3** Phase difference between the perturbed temperature gradient and the perturbed heat flux. Dotted line indicates the case without the phase difference.

The *difficulty* in this model [B] is similar to that in model [A]. The freedom by the off-diagonal term is, however, still insufficient.

(i) The sign of the off-diagonal term $M_{12}$ is different in many reports. The coefficient seems to depend on various discharge conditions. There should be still unidentified important internal freedom.

(ii) The perturbation still propagates with the transport process, and the "transport time" is necessary for the change. The very rapid transmission of the change cannot be explained within this framework.

*[C] Explicit heating-power dependence of $\chi_{eff}$*

There appeared an attempt to introduce the explicit dependence of the thermal diffusivity on the heating power as

$$\chi_{eff} \propto \chi_0 \left( \frac{P_{heat}}{P_0} \right)^{\gamma} \tag{15}$$

where $\chi_0$ and $P_0$ are normalizing constants which have the dimension of thermal diffusivity and power, respectively. The exponent $\gamma$ must be consistent with the transport analysis of the stationary profile. WVII-AS group has suggested

$$\gamma \simeq 0.6 \tag{16}$$

If this relation (15) holds, then the rapid change of the thermal conductivity, which is caused by the change of heating at the different location, is naturally deduced. Consider the case that the heating power is increased during a short interval between $(t_1, t_2)$ as is shown in Fig.2.4. During the rise phase $\left(t_1 < t < t_2, t_2 \to t_1\right)$, the plasma parameter (gradient) does not change. Nevertheless, the heat flux, Eq.(15), can change owing to the fast increase of the heating power. As a result of this, the gradient-flux relation has a hysteresis, as is illustrated in Fig.2.5.

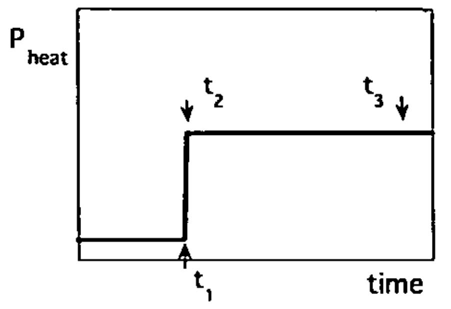



**Fig.2.4** Rapid increase of the heating power

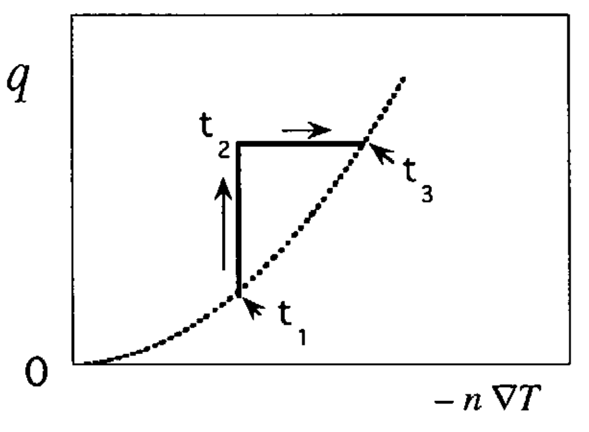



**Fig.2.5** Gradient-flux relation under the circumstance of Fig.3 (in case that $\chi_0$ and $P_0$ are constant). The dotted line indicates the relation in case of the slow increase.

Future task in this approach is (1) to determine $\chi_0$ and $P_0$ which fulfill all the feature of the parameter-dependence of $\tau_E$. The dependence of $\chi_0$ and $P_0$ on the heating profile is not clear yet. Present *difficulty* in this approach is (2) the derivation of such a formula from the dynamical equation of the plasma. The particle flux

$$\Gamma = \frac{\langle \tilde{n} \cdot \tilde{E}_\theta \rangle}{B} \tag{17}$$

is obtained when the prediction of the fluctuation field is derived. How the field perturbation could be expressed in terms of the heating power at the different location?

Although the agreement of the result from Eq.(16) with the experimental observation is often obtained, theoretical foundation is difficult to be established.

*[D] Nonlocal relation of heat flux vs gradient*

Alternative approach is to employ the nonlocal effect in the heat flux. The local heat flux at $r$ can be dependent on the local plasma parameter at $r$ as well as on those at $r'$. Such an influence is effective within a finite distance

$$|r' - r| < \ell \tag{18}$$

The heat flux at $r$ is symbolically written as

$$q(r) = \int dr' K(r - r') Q(r', r) \tag{19}$$

where $Q(r, r')$ is the quantity which has the dimension of the heat flux,

$$\begin{aligned} Q(r, r') = &- n\,\chi_{NL,1}[T(r'),\ \nabla T(r') \cdots] \nabla T(r) \\ &- n\,\chi_{NL,2}[T(r'),\ \nabla T(r') \cdots] \nabla T(r') \end{aligned} \tag{20}$$

and $K(r - r')$ denotes the interaction between different locations. (See Fig.2.6)

For instance, the functional form like

$$K(r - r') = \frac{1}{\sqrt{2\pi}\ell} exp\left\{-\frac{(r - r')^2}{\ell^2}\right\} \tag{21}$$

is employed. The first term in Eq.(20) is interpreted that the thermal diffusivity at $r$ is influenced by the parameters within a distance $|r' - r| < \ell$. The second term in Eq.(20) is not directly proportional to the local gradient $\nabla T(r)$. If this term exists, it could be interpreted in experiments as "pinch" term.

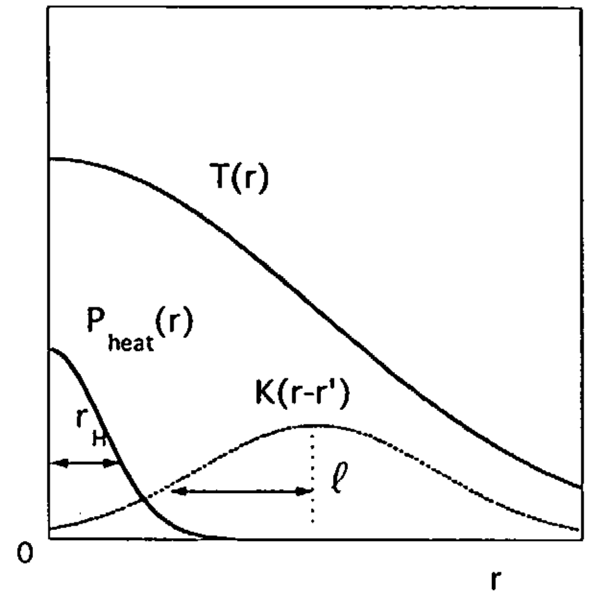



**Fig.2.6** Broad kernel $K(r-r')$ and peaked heating profile

If the relation like Eq.(19) holds, then the very fast change of heat flux is possible. The heat flux at $r$ changes when the plasma parameter at $r'$ is modified, without the change of the parameters at $r$. As is illustrated in Fig.2.4, the hysteresis could appear in the gradient-flux relation.

The underlying physics and some examples are explained in the next section.

## 3. Approach based on the Nonlocal Transport

### 3.1 Underlying Physics

*[A] Long Mean Free Path*

In various physics problems, there appear the nonlocal transport phenomena. For instance, if the mean free path of energy carriers (e.g., electrons in un-magnetized plasmas and photons in solid state) is much longer than the scale length of the gradient, then the heat flux is expressed not only by the local gradient, but also in terms of the parameters at the distance.

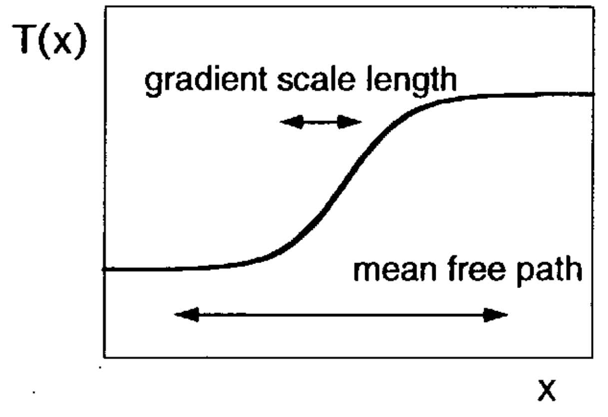



**Fig. 3.1** The case where mean free path is longer than the gradient scale length.

*[B] Large Scale Vortex*

The carrier of the energy can be waves. When the correlation length of the wave is longer than the scale length, the nonlocal transport becomes noticeable.

If one takes the example from the Benard cell problem, the effective thermal conductivity is given as

$$\chi_{eff} \simeq \left(\frac{\Re_a}{\Re_a}\right)^{1/3} \chi_c \tag{22}$$

in the large $\Re_a$-limit. In Eq.(22), $\Re_a$ is the Rayleigh number and is in proportion to the temperature difference between two plates,

$$\Re_a \propto T_1 - T_2 \tag{23}$$

$\chi_c$ is the thermal diffusivity owing to the molecular viscosity, which is a local quantity. Substituting Eqs.(22) and (23) into Eq.(1), the heat flux is expressed as

$$q \propto (T_1 - T_2)^{1/3} \nabla T \tag{24}$$

The heat flux depends on the temperature difference between two boundaries. This result clearly demonstrates that the local heat flux depends not only the local temperature gradient, but also on the parameter at far distance. In the Benard cell problem, this nonlocal dependence of the heat is caused by the fact that the turbulent transport is caused by the global waves, the wave length of which is of the order of the distance between two boundaries. (Fig.3.2)

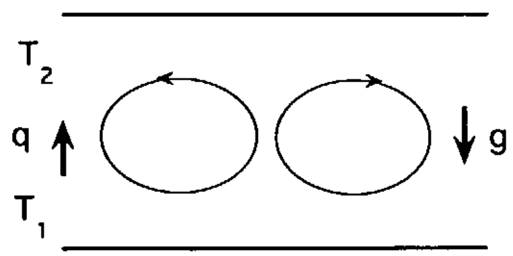



**Fig.3.2** Heat exchange in fluid

### 3.2 Some Models

In this workshop three types of models are discussed as typical examples.

*[A] Nonlocal conductivity*

In this approach, the heat flux is in proportion to the local gradient, but the coefficient ($\chi_{eff}$) depends on the global parameters. It is modelled as

$$q(r) = -n\chi_{NL} \nabla T(r) \tag{25}$$

$$q(r) = -\left\{\int dr' K(r-r') n \chi_{NL,\, l}[T(r'),\ \nabla T(r') \cdots]\right\} \nabla T(r) \tag{26}$$

This model is limited in a sense that only the diffusive term is taken (i.e., $q(r)$ vanishes if $\nabla T(r) \to 0$). At the same time, however, has an advantage: It could be demonstrated that the very rapid change could be reproduced even in the model of the diffusive transport process.

*[B] General dependence*

In general, the heat flux at $r$, $q(r)$, could be dictated not by the local gradient, $\nabla T(r)$, by more strongly by the gradient at the distance, $\nabla T(r')$. The second term of Eq.(20)

$$q(r) = -\left\{\int dr' K(r-r') n \chi_{NL,2}[T(r'), \nabla T(r') \cdots] \nabla T(r')\right\} \tag{27}$$

is one of the examples of this kind. If the term $\chi_{NL,2}$ is expressed in terms of the power of temperature and temperature gradient,

$$\chi_{NL,2} \propto \{\nabla T(r')\}^{\alpha} T(r')^{\beta} \cdots\cdots \tag{28}$$

one may have the heat flux like

$$q(r) = -\int dr' K(r-r') n \left[|\nabla T(r')|^{\alpha} T(r')^{\beta} \cdots\cdots\right] \nabla T(r') \tag{29}$$

It would be useful to consider the underlying physics which suggests the forms like Eqs.(26) and (27). The details of the derivation of the formula is left elsewhere, and the idea is briefly discussed. Let us consider the case that the flows (26) or (27) are caused by fluctuations.

The flux is expressed in terms of the fluctuating quantity as Eq.(17). The formula (27) suggests that the flux at $r$, that is governed by the field perturbation at $r$, is governed by the parameters at distance. In other words, the field perturbation at $r$ is influenced by the gradient far from the location $r$. Such a situation is expected that the flow is caused by a large scale convections, the scale length of which is comparable to the gradient scale length. (Fig.3.3) The large-scale convection is predicted to be more stable, compared to the microscopic one, within the framework of the self-sustained turbulence. Therefore the large scale convection could be induced by the nonlinear interactions of the small scale fluctuations. If the large-scale one is excited, then the heat flux includes the terms like Eq.(27).

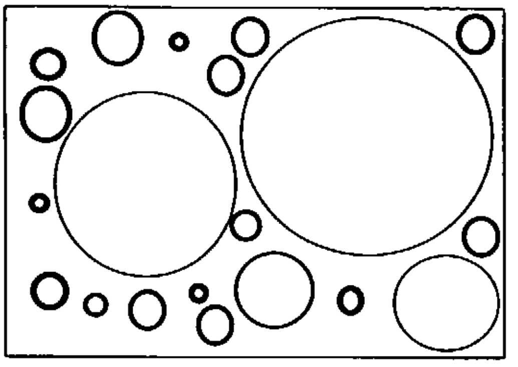


**Fig.3.3** Strong and small scale vortex and weak-and large scale vortex.

The heat flux like Eq.(26) has the nature of the diffusive process, because it is in proportion to the local gradient. It implies that the flux is caused by the small scale turbulence, which induces the diffusive flux. At the same time, the magnitude of the diffusive flow is dependent on the plasma parameters at the distance. It indicates that the amplitudes of the local and small-scale fluctuations are influenced by the plasma parameters at the distance. Such an influence on the small scale fluctuations could happen if the small scale fluctuations have interactions with large scale modes. The large scale one could be induced by the microscopic turbulence through the nonlinearity, leading the flux like Eq.(27). The driven-large-scale fluctuations then influence on the microscopic turbulence, through the nonlinearity too.

The schematic correspondences between these interactions and nonlocal fluxes are drawn in Fig.3.4.

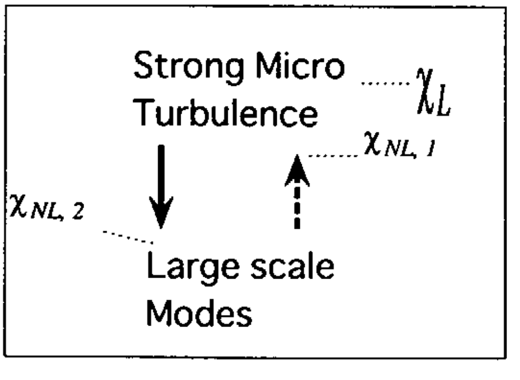



**Fig.3.4** Various fluctuation and nonlinear interactions, and their relations to the local or nonlocal fluxes.

*[C] Constrained Evolution*

Independent approach is to introduce the idea of the memory effect. The consideration like "favourable profile" (some has called "canonical profile", --- but what is the meaning of "canonical"----) has been studied long.

Assume that there is a relation between the stationary temperature profile and the stationary heating profile. In other words, if one writes as

$$T_0(r) = T(0)U(r) \tag{30}$$

$$P(r) = P(0)u(r) \tag{31}$$

it is assumed that the constraint between the profiles $U(r)$ and $u(r)$ is strongly preserved during the transient response. If the shape-relation is solid (unchanged), it is the extension of the idea of "profile consistency" to the dynamical problems. When the relation between $U(r)$ and $u(r)$, during the transient phase, is not fixed but changes only little, then the profile change during the transient phase could be different from the case where the profile change due to the slow variation of the heating power. This strong relation between $U(r)$ and $u(r)$ is introduced *apriori*, and the physics basis is not discussed.

First, argument is developed by taking one heating profile $u(r)$. Temporal variation of the heating is summarized as $P(0; t)$.

The thermal diffusivity is given by a sum of the local term and nonlocal term as

$$\chi = \chi_L + \chi_{NL} \tag{32}$$

The local part, $\chi_L$, connects the stationary profiles $T_0(r)$ and $P(r)$. The profiles $u(r)$ and $U(r)$ are controlled by $\chi_L$. The nonlocal term is related to the deviation of the (transient) profile from the stationary profile. The idea of this model implies (1) that the deviation of the profile from $U(r)$ causes the larger heat flux (2) and that it accelerates the change of heat flux during the transient phase. The nonlocality of the transport appears that the heat flux at $r$ is dictated by the difference of the shape form the target profile. (It is evident, that the concept of the dependence of the heat flux $q(r)$ on the "profile" $U(r)$ requires the information form the different radial locations $r'$.) The nonlocal term may be expressed in a simplest case like

$$\chi_{NL} \propto \left(\frac{\nabla T(r)}{\nabla T_d(r)}\right)^x - 1 \tag{33}$$

The form can be easily extended to more complex situations.

When the relation such as Eq.(33) is employed, then the gradient-flux relation behaves as is illustrated in Fig.2.5. The fast "recovery" to the preferable profile $U(r)$ in a stationary state is deduced.

There is several analysis based on this way of thinking.

If we think about the foundation of the idea, it seems that this model includes the ansatz about the time integration. Consider the extension of the model. Let us take the situation that the heating profile $u(r)$ changes simultaneously at $t_2$ in Fig.2.4 ($u_1(r) \rightarrow u_3(r)$). Under this situation, one may simply use the form like

$$\chi_{NL} \propto \left(\frac{\nabla T(r)}{T_0 \nabla U_3(r)}\right)^x - 1$$

This functional form, however, introduces some difficulty related with the time. This form implies that the transport nature just after the change ($t = t_2 + 0$) is controlled by the profile that will be realized as an asymptotic form. The consideration on the causality is required in order to generalize the model Eq.(33).

This is related to the consideration of the non-equilibrium nature of the system. This must be studied more fundamentally in the future.

**4. Example of the analysis**

An example of the transport analysis is reported by Iwasaki. As a first step, the general form as Eq.(29) is chosen. The gradient-flux relation is written as

$$q(r) = -C_L n\chi_0 \nabla T(r) - C_{NL} n\chi_0 \int dr' K(r-r') \nabla T(r') \tag{34}$$

where $\chi_0$, $C_L$ and $C_{NL}$ are kept constant. When $K(r-r')$ is a delta function, or $C_{NL} = 0$ holds, the flux reduces to that in the local transport model. The local term is also superposed in Eq.(34). This is for the test of code, and it sometimes help numerical stability of the code. The kernel of the integral is given as

$$K(r-r') = \frac{1}{\sqrt{2\pi}\ell} exp\left\{-\frac{(r-r')^2}{\ell^2}\right\} \frac{r}{r'} \tag{35}$$

The weighting function $r / r'$ in Eq.(35) is introduced to impose the boundary condition

$$q(0) = 0 \tag{36}$$

In this model, the coefficient $\chi_0$ is constant, and does not depends on the plasma parameter. This eliminates the mechanism of the section 2.2[A], i.e., the nonlinearity in the transport coefficient induces the difference between $\chi_{pb}$ and $\chi_{HP}$. In the local limit, $\ell \to 0$, Eq.(34) leads to the linear relation $q(r) \propto \nabla T(r)$. The details of the analysis is attached. Time evolution of the heating power is chosen as in Fig.4.1

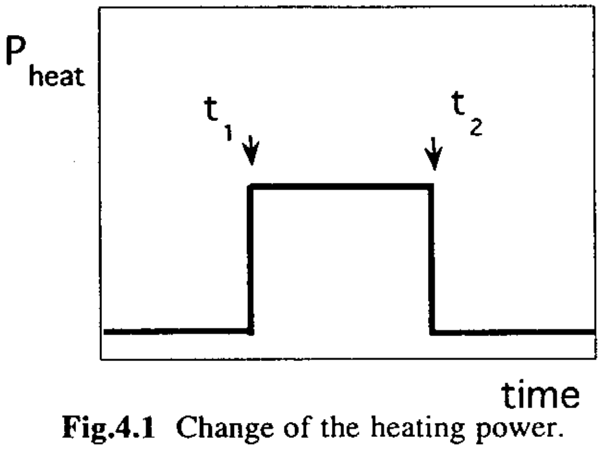



**Fig.4.1** Change of the heating power.

Three cases are tested for the radial profiles:

$$P_1(r) \propto const \tag{37-1}$$

$$P_2(r) \propto (1 - r^2/a^2) \tag{37-2}$$

$$P_3(r) \propto exp\left\{-\left(\frac{10r}{a}\right)^2\right\} \tag{37-3}$$

It is confirmed that the hysteresis in the flux-gradient relation appears in this model ($P_1$ and $P_2$). It is also noted that when the profile changes as

$$P_1 \rightarrow P_3 \rightarrow P_1 \tag{38}$$

very prominent deviation takes place in the relation of $q$ and $\nabla T$. Further test, where profile is fixed as $P_3$, is calculated in the future.

## 5. Experimental Method

Two cases are explained, i.e., change of heating power, and the heat pulse propagation.

### *[A] Standard Parameters*

As an example, the following plasma parameters are in mind.

The power degradation of the confinement time in the stationary state suggests that the relation

$$q \propto (\nabla T)^2 \tag{39}$$

holds. Within various combinations like Eq.(28), the cases

$$\alpha + \beta \sim 1 \tag{40}$$

seem to be consistent with Eq.(39). (The survey will be made putting an emphasis on this combination.)

Typical numbers are,

$$P_{heat} = 0.1 \sim 1\ MW, \quad P(r) \propto exp\left\{-\left(\frac{r}{r_H}\right)^2\right\}, \ r_H \simeq a/10 \tag{41a}$$

$$n \sim 4 \times 10^{19} m^{-3}, \tag{41b}$$

$$a \sim 0.2\ m \tag{41c}$$

$$\tau_E \sim \tau_L(scaling) \sim \tau_{ISS95}(scaling), \qquad \chi \sim 0.5 - 2\ m^2/s \tag{41d}$$

*[B] Fast change of heating power*

As is illustrated in Fig.4.1, the heating power is changed quickly while the heating profile is kept constant (41a).

*[C] Modulation of heating power and heat pulse propagation*

The response of the plasma temperature perturbation against the modulation of the heating power is commonly studied. The heating power, which is centrally localized (41.a), is modulated in time as is shown in Fig.5.1. The amplitude is small

$$\frac{\Delta P}{P} \simeq \pm 10\ \% \tag{42}$$

and frequency is chosen in the range

$$f_M = 23 \cdots 92 \cdots 368\ (Hz) \tag{43}$$

and $f_M = 92\ (Hz)$ is often employed. The heating profile is fixed.

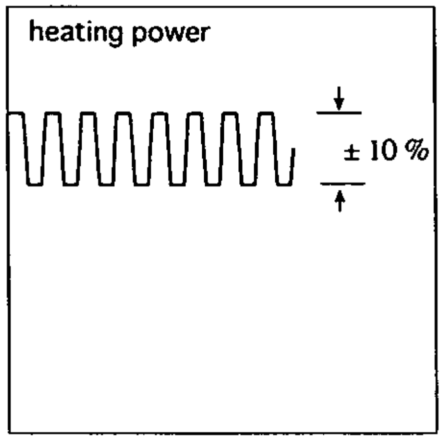



**Fig.5.1** Modulation of the heating power

Responding to the power modulation, there appears the modulation of the temperature. If one writes the perturbed temperature as $\delta T(r, t)$, it is Fourier-decomposed as

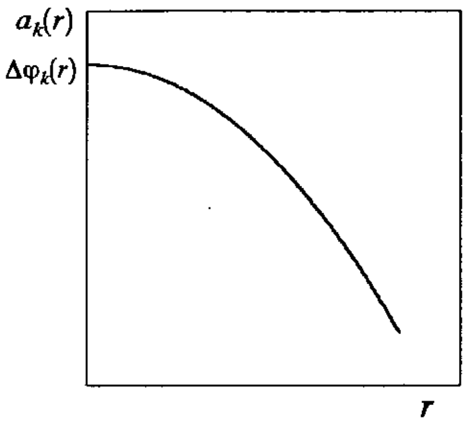
$$\delta T(r, t) = \sum_{k=1}^{\infty} a_k(r) \cos\left(2\pi k f_M t + \varphi_k(r)\right) \quad (44)$$

where $a_k(r)$ and $\varphi_k(r)$ are the amplitude and phase of the k-th harmonics at $r$. The phase difference is defined as

$$\Delta\varphi_k(r) = \varphi_k(r) - \varphi_k(0) \quad (45)$$

and the profiles of $a_k(r)$ and $\Delta\varphi_k(r)$ are discussed. In usual analyses, at present, up to second harmonics are kept. Examples are illustrated in Figs.5.2 and 5.3. The prediction from the model is compared to the experimental observations.

In some cases, $\Delta\varphi_k(r)$ turns out to be a non-monotone function of the radius. (Fig.5.3) If it is true, the signal seems to propagate from the edge, although the perturbation is excited at the center. In this case, the importance of the additional mechanisms (such as the radiation cooling) has been thought.



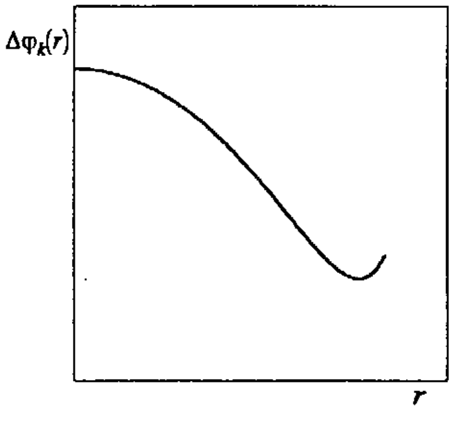




**Fig.5.2** (left) Radial profile of responses
**Fig.5.3** (right) Radial profile of the delay. Some times, the delay is not an increasing function of the radius. Superficially, perturbation seems to propagates form the edge.

## 6. Future Task and Next Step

[A] Numerical simulation of Section 4.

The following study is necessary.

(1) Confirm hysteresis characteristics in the case of the heating profile $P_3(r)$.

(2) Dependence of the hysteresis on the distance $\ell$. It is expected that the hysteresis becomes prominent if

$$\ell > r_H \tag{42}$$

where $r_H$ characterizes the broadness of the heating profile. (For example, see Fig.2.6) Experimental profile is given in Eq.(41a). The dependence on $\ell$ needs to be studied.

(3) Dependence of the hysteresis on the abruptness the change of heating. If the change of the heating power takes place with the time period of $\Delta t$ (as is shown in Fig.6.1), then the hysteresis characteristics depends on $\Delta t$. The hysteresis would be prominent if the condition

$$\Delta t << \tau_E \tag{43}$$

is satisfied.

(4) Numerical accuracy is better to be confirmed.

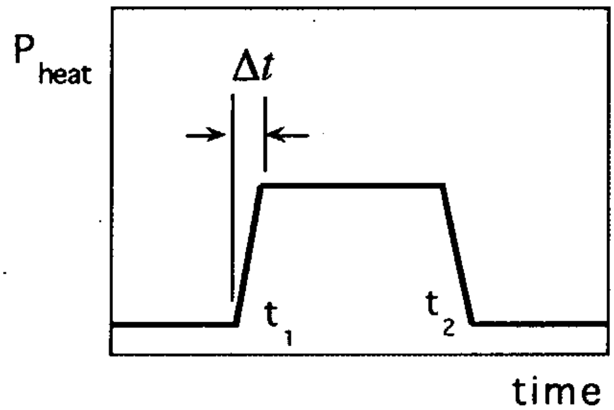



**Fig.6.1** Change of heating power with finite time interval.

*[B] Benchmark data*

For the standard parameters, the study of the stationary profile is necessary (in the presence of the nonlocal term). The model of Eq.(26) seems better for the initial analysis. ($\alpha = 1$, $\beta = 0$) Next, the power change (Fig.4.1 or Fig.6.1) is examined.

*[C] EPS paper*

Level of 6[A] plus some of 6[B] must be performed for EPS paper. For the abstract, we depend on SII.

*[D] Further studies*

For the future study, the following process is planned:

(1) Study of modulation will be performed.

(2) Survey with respect to ($\alpha$, $\beta$) would be necessary.

(3) Thinking of the model of Eq.(27) must be advanced.

(4) Fundamental theoretical study to derive the nonlocal formula will be continued.

(5) Power dependent model (like Eq.(15)) is studied by the Stroth's student.

**Appendix: Viewgraph Copy of**

# Comparison between the Non-local Transport Model and Experimental Results

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## <PURPOSE>

*To investigate the non-local effect of the transport on a phenomenological point of view.

*Comparison between the non-local transport model and experimental results.

## <BASIC EQUATION>

$$\frac{\partial}{\partial t}\left(\frac{3}{2} n_e T\right) = -\nabla \cdot q(r,t) + \Sigma Q$$

$\Sigma Q$ ; SOURCES AND SINKS

## <MODEL OF HEAT FLUX>

Generally , the heat flux can be written as follows:

$$q(r,t) = -\int_0^{\infty} K(r-r') \cdot \frac{r}{r'} \cdot n_e \cdot \chi(T,\nabla T) \cdot \nabla T(r',t)\,dr'$$

$K(r-r')$ ; kernel

$\chi(T,\nabla T)$ ; diffusivity

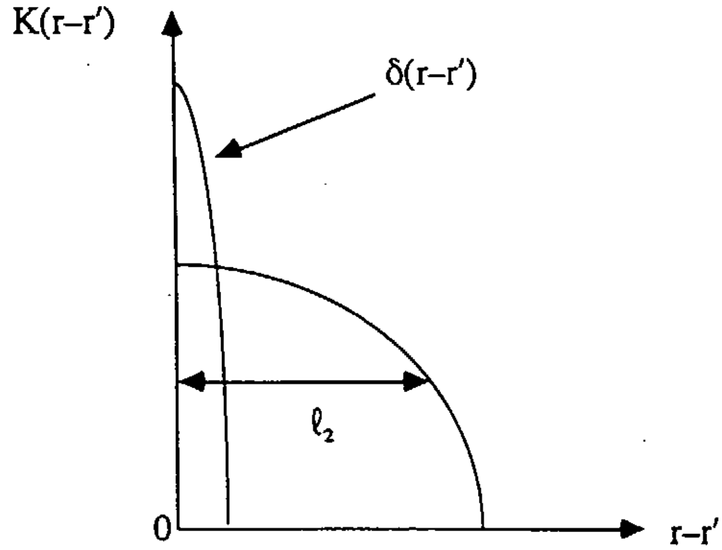
In the limit of $K(r-r') = \delta(r-r')$ , $q(r,t)$ is reduced to the local model.

## <DETERMINATION OF KERNEL>

$$K(r-r') = C_{local}\delta(r-r') + C_{global}\frac{1}{\sqrt{\pi}\ell_2}\exp\left\{-\left(\frac{r-r'}{\ell_2}\right)^2\right\}$$

local effect    non-local(global) effect

$C_{local}$ and $C_{global}$ are the coefficient to determine the magnitude of the local effect and non-local effect.




The second term becomes delta function as $\ell_2 \to 0$.

## <STEADY STATE、 $\chi = const$>

### Boundary Condition

$\frac{\partial T}{\partial r} = 0$   $(q = 0)$   at   $r=0$

$T = 0$   at   $r=a$

The data is shown in Fig.1

If $\ell_2 \to 0$ , temperature profile can be calculated analytically.

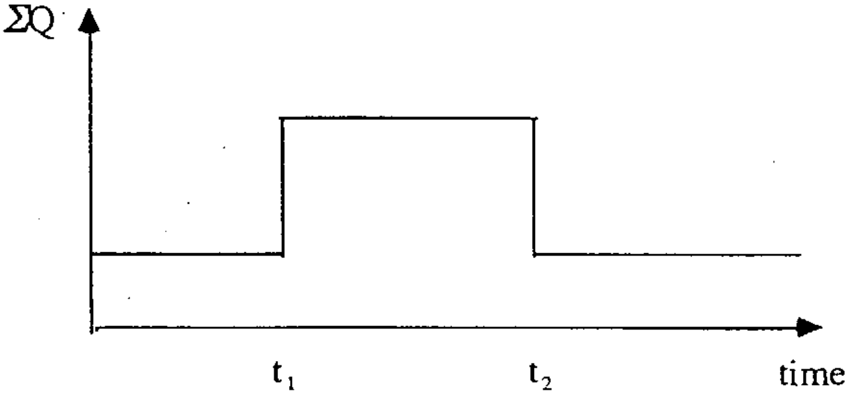
$T(r) = T_0(1 - r^2)$ , $T_0=1$ when $\Sigma Q=P=8$

In the following analysis , we use P=8 for the source term.

## <TIME EVOLUTION OF TEMPERATURE GRADIENT AND HEAT FLUX>

$$\frac{3}{2}n_e\frac{\partial T(r,t)}{\partial t}=C_{local}\frac{1}{r}\frac{\partial}{\partial r}\left(m_e\chi\frac{\partial T(r,t)}{\partial r}\right)$$

$$+C_{global}\frac{1}{r}\frac{\partial}{\partial r}\left(r\int_0^a\frac{1}{\sqrt{\pi}\ell_2}\exp\left\{-\left(\frac{r-r'}{\ell_2}\right)^2\right\}\cdot\frac{r}{r'}\cdot n_e\cdot\chi(T,\nabla T)\cdot\frac{\partial T(r',t)}{\partial r'}dr'\right)+\Sigma Q$$




case1 $\Sigma Q = \text{const} = P \longrightarrow 10P \longrightarrow P$

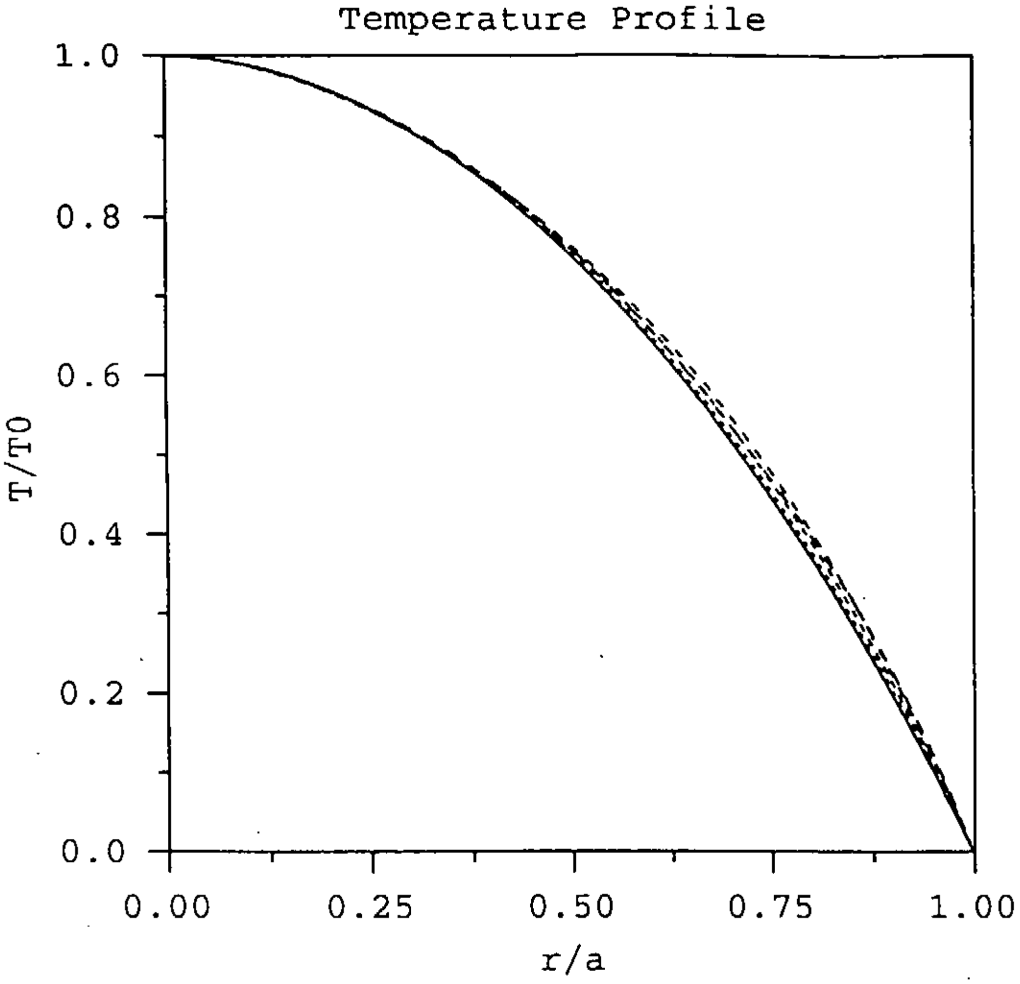
case2 $\Sigma Q = P(1-r^2) \longrightarrow 10P(1-r^2) \longrightarrow P(1-r^2)$

case3 $\Sigma Q = P \longrightarrow 10P\exp(-100r^2) \longrightarrow P$

In all the cases , we set $\ell/a = 0.5$.
These graphs are after $t_1$.

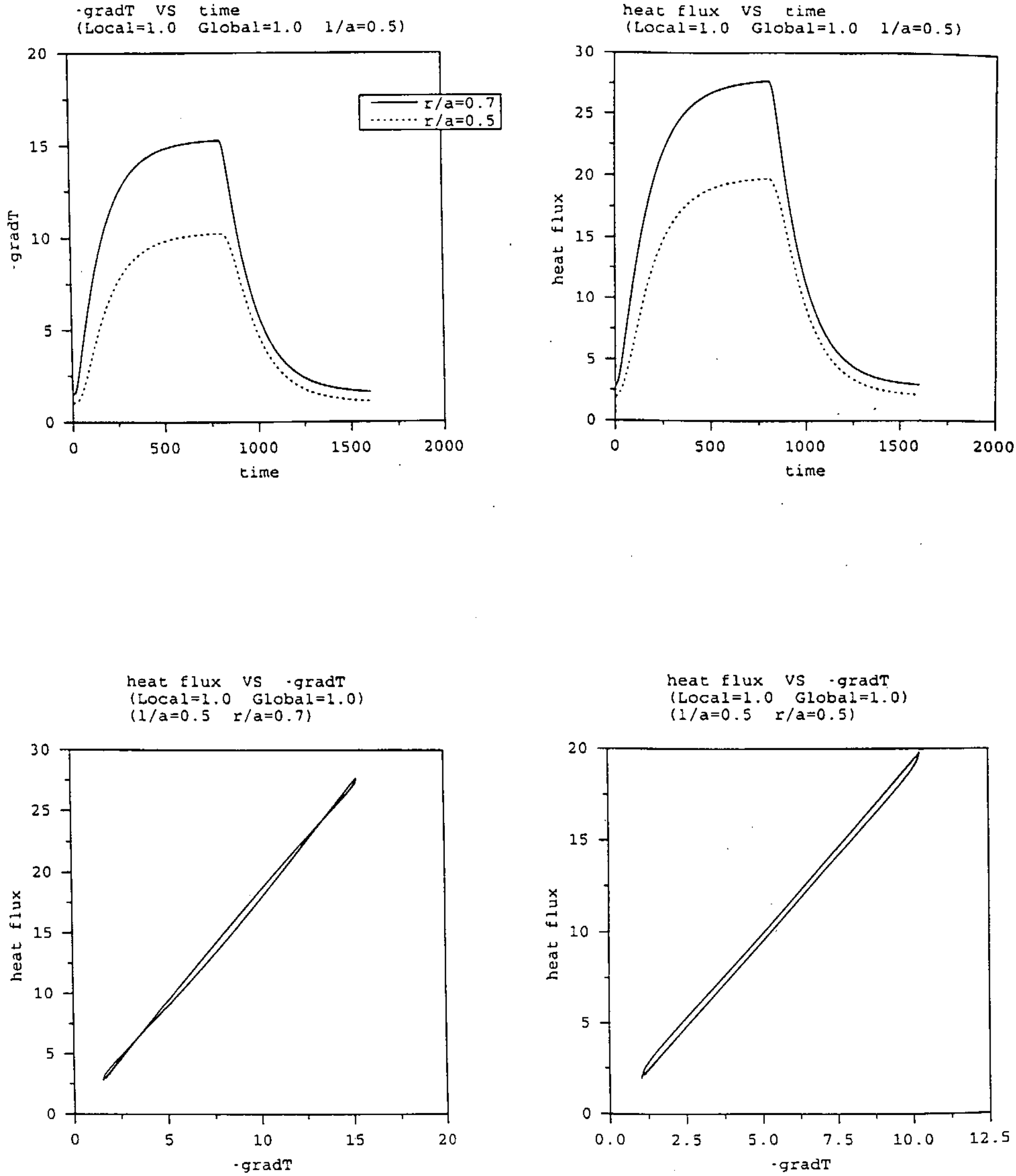
| $(C_{local}, C_{global})$ | (1.1) | (1.2) | (1.4) | (0.1) | (1.0) |
|---|---|---|---|---|---|
| case1 | Fig.2 | Fig.5 | Fig.8 | Fig.11 | Fig.14 |
| case2 | Fig.3 | Fig.6 | Fig.9 | Fig.12 | Fig.15 |
| case3 | Fig.4 | Fig.7 | Fig.10 | Fig.13 | Fig.16 |

Fig.1




STEADY STATE、 $\chi = const$

Fig.2



-gradT VS time
(Local=1.0 Global=1.0 1/a=0.5)
r/a=0.7
r/a=0.5
-gradT
time
heat flux VS time
(Local=1.0 Global=1.0 1/a=0.5)
heat flux
time
heat flux VS -gradT
(Local=1.0 Global=1.0)
(1/a=0.5 r/a=0.7)
heat flux
-gradT
heat flux VS -gradT
(Local=1.0 Global=1.0)
(1/a=0.5 r/a=0.5)
heat flux
-gradT

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T.Tsuzuki, K.Toi and K.Matsuura,"Feedback Control of Plasma Equilibrium with Control System Aided by Personal Computer on the JIPP T-II Tokamak"; Apr.1991 (In Japanese)

NIFS-MEMO-2 久保田雄輔、大林治夫、宮原 昭、大野和子、中村光一、堀井憲爾、「環境磁界の研究 (I) 三次元積分磁束計の開発」1991年 7月
Y.Kubota, H.Obayashi, A.Miyahara, K.Ohno, K.Nakamura and K.Horii, "Development of Three-Dimensional Compact Magnetic Dosimeter for Environmental Magnetic Field Monitoring", July,1991 (In Japanese)

NIFS-MEMO-3 「核融合科学研究所技術研究会; 1991年3月19､20日 土岐文化プラザ」1991年8月
"Proceedings of Symposium on Technology in Laboratories; Mar. 19, 20, 1991" Aug. 1991 (In Japanese)

NIFS-MEMO-4 「プラズマ中におけるカオス現象」1991年8月
"Chaotic Phenomena in Plasmas ", Aug. 1991 (In Japanese)

NIFS-MEMO-5 山崎耕造、「ファジー論理のトロイダルプラズマ制御への応用」1992年2月 "Fuzzy Logic Application to Troidal Plasma Contol ", Feb. 1992 (In Japanese)

NIFS-MEMO-6 平成３年度 核融合科学研究所 支援調査共同研究報告書 「大出力マイクロ波源の開発に関する調査研究」1992年4月
"Development of High Power Microwave Source ", Apr. 1992 (In Japanese)

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「プラズマにおけるカオス現象」1992年11月
"Chaotic Phenomena in Plasma, II", Nov. 1992 (In Japanese)

NIFS-MEMO-8 NIFSシンポジウム報告書「核融合炉材料照射用強力中性子源の設計と大学の役割」1993年1月
"Design of Intense Neutron Source for Fusion Material Study and the Role of Universities", Jan. 1993 (In Japanese)

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(Ed.) K. Yatsu and T. Hatori "Potential Formation and Transport in Open Field Line Systems", Apr. 1993 (In Japanese)

NIFS-MEMO-10 伊藤公孝、「人口太陽は燃えるか ー核融合研究最前線ー原子力オープン・スクール講演」１９９３年５月
K. Itoh, Reports of NIFS Collaboration Workshop from 1990 to 1992 at National Institute for Fusion SCience.
"At the Front of Fusion Research -Introductory Lecture Open to Public-", May 1993 (In Japanese)

NIFS-MEMO-11 (Ed.) T. Kato
"Atomic Physics and Radiation in Divertor and/or Disruption Plasmas",
Aug. 1993 (In English)

NIFS-MEMO-12 平成４年度　核融合科学研究所　共同研究（調査支援研究）研究成果報告書
「プラズマのカオス現象（まとめ）」1993年10月
"Chaotic Phenomena in Plasma, III", Oct. 1993
(In Japanese)

NIFS-MEMO-13 久保田雄輔、野田信明、相良明男、井上徳之、赤石憲也、本島修,
「プラズマ対向材料の開発・研究用高熱負荷試験装置ACT」1994年3月
"Electron Beam Irradiation Test Facility for Research and Development of Plasma Facing Components", Mar. 1994
(In Japanese)

NIFS-MEMO-14 「平成５年度核融合科学研究所技術研究会
日時：1994年3月23日-24日
場所：セラトピア土岐、土岐商工会議所」1994年7月
"Proceedings of Symposium on Technology in Laboratories by Department of Engineering and Technical Services", July 1994
(In Japanese)

NIFS-MEMO-15 S.-I. Itoh, K. Itoh, G. Soltwisch, K. H. Finken、
"Study on Sawtooth and Transport
-- A Report of Japan-TEXTOR Collaboration --", Jan. 1995
(In English)

NIFS-MEMO-16 久保田　雄輔、野田信明、相良明男、井上徳之、赤石憲也、山本純也、本島修
「LHD用ダイバータ板の開発・研究各種カーボン・銅接合材の熱負荷試験」
Y. Kubota, N. Noda, A. Sagara, N. Inoue, K. Akaishi, J. Yamamoto, O. Motojima,
"Research and Development of Divertor Plates for LHD (High Heat Flux Tests of Various Kinds of Materials)", Apr. 1995 (In Japanese)

NIFS-MEMO-17 江尻　晶,
「LHDにおける窓の熱設計」
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"Calculation of the Heat Transfer in the Windows of the Large Helical Device"
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核融合科学研究所共同研究
「プラズマと壁との相互作用に関する研究会」
核融合科学研究所、平成７年９月 12 日 - 9 月 13 日
(Eds.) T. Tanabe and K. Morita
"Proceedings of the Symposium on Plasma-Materials Interaction"
NIFS, Sep. 12-13, 1995,
Dec. 1995 (In Japanese)

NIFS-MEMO-19 K. Itoh (ed.),
Study on Sawtooth and Transport in Part of Japan-TEXTOR Collaboration 1995; Feb. 1996

NIFS-MEMO-20 編集責任、核融合科学研究所、太刀川恭治、山本純也、
「核融合科学研究所共同研究、核融合炉用先進超伝導導体、研究調査報告書　1
$Nb_3Sn$系超伝導導体」

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"Studies on Advanced Superconductors for Fusion Device"
Part 1 - Present Status of $Nb_3Sn$ Conductors-", Mar. 1996 (In Japanese)

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「LHDにおけるD-$^3$He核融合の検討」
A. Iiyoshi, M. Fujiwara, M. Okamoto, N. Ohyabu, O. Kaneko, M. Sasao, S. Sudo, R. Kanno, T. Mutoh, S. Murakami, Y. Takeiri, Y. Tomita and K. Yamazaki,
"Studies on D - $^3$He Fusion in LHD", Mar. 1996 (In Japanese)

NIFS-MEMO-22 編集責任、核融合科学研究所、太刀川恭治、山本純也、三戸利行
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"Studies on Advanced Superconductors for Fusion Device"
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"Discussion Record of the Workshop on Nonlocal Transport"
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