§16. Collisionless Kinetic-Fluid Model of Zonal Flows

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A novel kinetic-fluid model is presented, which describes collisionless time evolution of zonal flows in toroidal plasmas [1]. The new zonal-flow closure relations are derived from the gyrokinetic model and they can reproduce the gyrokinetic long-time zonal-flow responses to the initial condition and to the turbulence source.

The velocity moments of the gyrokinetic equation are taken to obtain the equations which govern time evolution of the fluid variables defined by $[\delta n^{(\mathrm{g})}_{\mathbf{k}_\perp}, n_0 u_{\|\mathbf{k}_\perp}, \delta p_{\|\mathbf{k}_\perp}, \delta p_{\perp\mathbf{k}_\perp}] = \int d^3v \ \delta f^{(\mathrm{g})}_{\mathbf{k}_\perp}[1, v_\|, mv_\|^2, \frac{1}{2}mv_\perp^2]$, where $\delta f^{(\mathrm{g})}_{\mathbf{k}_\perp}$ represents the perturbed gyrocenter distribution function. Consequently, for the zonal flow component with the wave number vector $\mathbf{k}_\perp = k_r \nabla r$ perpendicular to the magnetic flux surface, we obtain the perturbed gyrocenter density equation,

$$\frac{\partial \delta n^{(\mathrm{g})}_{\mathbf{k}_\perp}}{\partial t} + \mathbf{B}\cdot\nabla\left(\frac{n_0 u_{\|\mathbf{k}_\perp}}{B}\right) = i\frac{c}{eB^2}\mathbf{k}_\perp\cdot(\mathbf{b}\times\nabla B)\Big[\delta p_{\|\mathbf{k}_\perp} + \delta p_{\perp\mathbf{k}_\perp} + n_0 e\phi_{\mathbf{k}_\perp} e^{-b/2}(2 \quad b/2)\Big] + \mathcal{N}_{0\mathbf{k}_\perp}, \tag{1}$$

the parallel momentum balance equation,

$$mn_0\frac{\partial u_{\|\mathbf{k}_\perp}}{\partial t} + \mathbf{B}\cdot\nabla\left(\frac{\delta p_{\|\mathbf{k}_\perp}}{B}\right) + \frac{\delta p_{\perp\mathbf{k}_\perp}}{B}\mathbf{b}\cdot\nabla B = i\frac{mc}{eB^2}\mathbf{k}_\perp\cdot(\mathbf{b}\times\nabla B)\left(q_{\|\mathbf{k}_\perp} + q_{\perp\mathbf{k}_\perp} + 4p_0 u_{\|\mathbf{k}_\perp}\right) \quad n_0 e\mathbf{b}\cdot\nabla\left(\phi_{\mathbf{k}_\perp}e^{-b/2}\right) + \frac{n_0 eb}{2B}\phi_{\mathbf{k}_\perp}e^{-b/2}\mathbf{b}\cdot\nabla B + \mathcal{N}_{1\mathbf{k}_\perp}, \tag{2}$$

the perturbed parallel pressure equation,

$$\frac{\partial \delta p_{\|\mathbf{k}_\perp}}{\partial t} + \mathbf{B}\cdot\nabla\left[\left(q_{\|\mathbf{k}_\perp} + 3p_0 u_{\|\mathbf{k}_\perp}\right)/B\right] + \frac{2}{B}\left(q_{\perp\mathbf{k}_\perp} + p_0 u_{\|\mathbf{k}_\perp}\right)\mathbf{b}\cdot\nabla B = i\frac{c}{eB^2}\mathbf{k}_\perp\cdot(\mathbf{b}\times\nabla B)\Big[m\left(\delta r_{\|,\|\mathbf{k}_\perp} + \delta r_{\|,\perp\mathbf{k}_\perp}\right) + p_0 e\phi_{\mathbf{k}_\perp}e^{-b/2}(4 \quad b/2)\Big] + \mathcal{N}_{2\|\mathbf{k}_\perp}, \tag{3}$$

and the perturbed perpendicular pressure equation,

$$\frac{\partial \delta p_{\perp\mathbf{k}_\perp}}{\partial t} + \mathbf{B}\cdot\nabla\left[\left(q_{\perp\mathbf{k}_\perp} + p_0 u_{\|\mathbf{k}_\perp}\right)/B\right] \quad \frac{1}{B}\left(q_{\perp\mathbf{k}_\perp} + p_0 u_{\|\mathbf{k}_\perp}\right)\mathbf{b}\cdot\nabla B = i\frac{c}{eB^2}\mathbf{k}_\perp\cdot(\mathbf{b}\times\nabla B)\Big[m\left(\delta r_{\|,\perp\mathbf{k}_\perp} + \delta r_{\perp,\perp\mathbf{k}_\perp}\right) + p_0 e\phi_{\mathbf{k}_\perp}e^{-b/2}(3 \quad 3b/2 + b^2/8)\Big] + \mathcal{N}_{2\perp\mathbf{k}_\perp}, \tag{4}$$

where $\mathcal{N}_{0\mathbf{k}_\perp}$, $\mathcal{N}_{1\mathbf{k}_\perp}$, $\mathcal{N}_{2\|\mathbf{k}_\perp}$, and $\mathcal{N}_{2\perp\mathbf{k}_\perp}$ are the nonlinear source terms. Here, the perturbed parallel and perpendicular temperatures $(\delta T_{\|\mathbf{k}_\|}, \delta T_{\perp\mathbf{k}_\perp})$ are defined by $\delta p_{\|\mathbf{k}_\perp} = n_0\delta T_{\|\mathbf{k}_\perp} + T\delta n^{(\mathrm{g})}_{\mathbf{k}_\perp}$ and $\delta p_{\perp\mathbf{k}_\perp} = n_0\delta T_{\perp\mathbf{k}_\perp} + T\delta n^{(\mathrm{g})}_{\mathbf{k}_\perp}$. The right-hand sides of Eqs. (2)–(4) contain the third-order fluid variables (or parallel heat fluxes), $[q_{\|\mathbf{k}_\perp}, q_{\perp\mathbf{k}_\perp}] = \int d^3v \ \delta f^{(\mathrm{g})}_{\mathbf{k}_\perp} v_\|[(mv_\|^2 \quad 3T), (\frac{1}{2}mv_\perp^2 \quad T)]$, and the fourth-order fluid variables, $[\delta r_{\|,\|\mathbf{k}_\perp}, \delta r_{\|,\perp\mathbf{k}_\perp}, \delta r_{\perp,\perp\mathbf{k}_\perp}] = \int d^3v \ \delta f^{(\mathrm{g})}_{\mathbf{k}_\perp} m[v_\|^4, \frac{1}{2}v_\|^2 v_\perp^2, \frac{1}{4}v_\perp^4]$.

We write the parallel heat fluxes as the sum of long- and short-time evolution parts,

$$q_{\|\mathbf{k}_\perp} = q^{(l)}_{\|\mathbf{k}_\perp} + q^{(s)}_{\|\mathbf{k}_\perp}, \qquad q_{\perp\mathbf{k}_\perp} = q^{(l)}_{\perp\mathbf{k}_\perp} + q^{(s)}_{\perp\mathbf{k}_\perp}. \tag{5}$$

Using the analytical solution of the gyrokinetic equation, which describes the long-time behavior of the perturbed gyrocenter distribution function, we obtain

$$q^{(l)}_{\|\mathbf{k}_\perp} = 2q^{(l)}_{\perp\mathbf{k}_\perp} = 2p_0 U_{\mathbf{k}_\perp}\left(B \quad \frac{{}_2}{{}_1}B^2\right), \tag{6}$$

where

$$U_{\mathbf{k}_\perp} \equiv {}_1\left({}_1 \quad \left\langle B^{-2}\right\rangle\right)^{-1} \times\left(\left\langle\frac{u_{\|\mathbf{k}_\perp}}{B}\right\rangle \quad \left\langle B^{-2}\right\rangle\left\langle Bu_{\|\mathbf{k}_\perp}(t=0)\right\rangle \quad \frac{\langle B^{-2}\rangle}{{}_1 n_0}\left\langle\int d^3v \ F_0 R_{\mathbf{k}_\perp}(t)\overline{\left(\frac{v_\|}{B}\right)}\right\rangle\right), \tag{7}$$

$${}_1 \equiv \frac{3}{2}\int_0^{1/B_M}\frac{d\lambda}{\langle B/(1 \quad \lambda B)^{1/2}\rangle}, \tag{8}$$

$${}_2 \equiv \frac{15}{4}\int_0^{1/B_M}\frac{\lambda d\lambda}{\langle B/(1 \quad \lambda B)^{1/2}\rangle}, \tag{9}$$

and B_M denotes the maximum field strength over the flux surface. For the short-time evolution, the parallel heat fluxes are given in the same dissipative form as in Hammett and Perkins [2],

$$q^{(s)}_{\|m\mathbf{k}_\perp} = 2\left(\frac{2}{\pi}\right)^{1/2} i\frac{m}{|m|}n_0 v_t \delta T_{\|m\mathbf{k}_\perp}, \qquad q^{(s)}_{\perp m\mathbf{k}_\perp} = \left(\frac{2}{\pi}\right)^{1/2} i\frac{m}{|m|}n_0 v_t \delta T_{\perp m\mathbf{k}_\perp}, \tag{10}$$

where m denotes the poloidal Fourier mode number. The fourth-order variables are approximated by

$$[\delta r_{\|\|\mathbf{k}_\perp}, \delta r_{\|\perp\mathbf{k}_\perp}, \delta r_{\perp\perp\mathbf{k}_\perp}] = v_t^2 T\delta n^{(\mathrm{g})}_{\mathbf{k}_\perp}[3, 1, 2]. \tag{11}$$

References
1) H. Sugama, T.-H. Watanabe, and W. Horton, Phys. Plasmas **14**, 022502 (2007).
2) G. W. Hammett and F. W. Perkins, Phys. Rev. Lett. **64**, 3019 (1990).