§18. Selection of a Radial Wavelength of Zonal Flows

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Turbulence of magnetized and high temperature plasmas is subject of intense studies, and understanding of a structure formation in plasmas has shown steady progress. In particular, the advancement was made on the nonlinear interplay between microscopic drift wave turbulence and mesoscopic zonal flows. The generation of zonal flows by microscopic turbulence is known to play an important role in determining turbulence and a turbulent transport. Various nonlinear processes have been studied and tested by direct nonlinear simulations. Although the noticeable achievement has been made, there are yet unresolved fundamental issues. For instance, it remains unanswered whether induced small scale zonal flows evolve into large-scale flows. In the previous article, a nonlinear dynamical equation for the zonal flow field has been derived, based upon the method of higher order kinetics. The coherent radial structure was obtained in a stationary state. However, the question whether mesoscale solutions evolve into the solution of the large scale or not was not addressed. We analyze the nonlinear evolution of the zonal flow by the initial value problem in the one dimensional model, and study a selection rule for a radial wavelength of the zonal flow. As a first step in investigating a selection rule, we employ the model equation in the previous article. It is shown that, when the initial condition is at a small amplitude and random in space, the periodicity length of a finial state is dominated by a particular length.

We study the case that the zonal flow retains the coherent structure in a time much longer than the decorrelation time of drift wave fluctuations. This 'coherent regime' is one of the characteristic situation of the drift wave-zonal flow (DW-ZF) system. Such a time scale separation was given the basis from the direct numerical simulation (DNS) of the core plasmas for strongly unstable cases, and the relevance of the model was discussed. This corresponds to the case that the decorrelation rate of the drift wave fluctuations, is larger than the vorticity of zonal flow. In the derivation of analytic model, the nonlinear effects of zonal flows on wave kinetic equations for drift waves are expanded in a series of the parameter. Therefore, the nonlinear evolution equation for zonal flows was derived, by keeping the lowest order nonlinear term. The model equation used here is shown as

$$\frac{\partial u}{\partial t}+\frac{\partial^2}{\partial x^2}(u-u^3)+\frac{\partial^4 u}{\partial x^4}=0\,, \qquad (1)$$

where u is the normalized vorticity, x is the normalized axis, and t is the normalized time. We examine the competition between the zonal flows which have the short and long wavelengths by use of this model equation. We solve the model equation (1) numerically in the region $0<x<2\pi d$. As the constraints, we choose the periodic boundary condition and the zero net vorticity, $\int dxu = 0$. The effect of the finite size of the system is illuminated by observing the temporal evolution for the access to the stationary solution. The temporal evolution of the each values for the Fourier components is obtained. The Fourier components for k, I_k are defined as $I_k=\int_0^{16\pi} u(x)\sin(kx/8)dx/(16\pi)$, k = 1, . . . , 8, because we should study the eight Fourier components in the system $0<x<16\pi$. After the Fourier components grows with the linear rate (t<50), the growth rate of the mode coupling dominates the linear growth rate. Around the time t=400, the growth rates of the Fourier modes k=2, k=3 exceed those of the components k=5 and k=6 which are dominant between the times t=100 and t=400. After that, the u(x) profile is found to go to the steady state, where the modes k=2 and k=3 remain dominant. Which mode dominates in this system is determined by the competition of the growth rates of the model coupling. The energy in the system goes to the modes of the longer wavelength (k=2 and k=3) in the steady state. These observations are summarized by the following features. At first, the initial pump to the flow with small radial wavelength takes place in the order of the linear growth time. However, the excitation of longer wavelength modes occurs by the beat of fast growing modes. After a longer time which is characteristic to the large scale perturbations, the large scale modes have a finite amplitude. The induced long wavelength modes modify the nonlinear stability of the quasi-saturated mode. Thus, the energy is transferred from one mode to the adjacent mode. The final stationary state is achieved by involving the longest wavelength mode. Therefore, the time for realizing the stationary state is given by the characteristic time of the longest wavelength modes. The evolution time, in which the stationary equilibrium (of the model equation) is established by starting small random initial values, is proportional to the square of the system size.

We have studied the nonlinear dynamics of zonal flows, by solving a one dimensional model equation as an initial value problem with periodic boundary conditions. The accessibility was studied. tThe radial periodicity length chooses a value around a particular length. This result shows that, so long as an evolution from small noise source is studied, the final state is characterized by a characteristic radial length. Although the solution is aperiodic, the mean (periodical) scale length is given by a unique length. It should be noticed that the scale length of the final stationary state is achieved not in the linear growth time, but after the competition of modes through exciting longer wavelength modes. It is found that the energy is transferred to the mode, which has a long wavelength, when the system size is finite. The partition to the longer mode becomes smaller as the system size increases. Nevertheless, the excitation of longer wavelength modes governs the elapse time for establishing the final state. The time for reaching the stationary state is in proportion to the square of the system size.

Reference

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