§6. Axisymmetric Equilibria with Flow and Pressure Anisotropy in Single-fluid and Hall MHD

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The ellipticity criteria for the partial differential equations of axisymmetric single-fluid and Hall magnetohydrodynamic (MHD) equilibria with flow and pressure anisotropy are investigated. In the ideal MHD description of plasmas, axisymmetric toroidal equilibria with flow are obtained by solving the so-called generalized Grad-Shafranov (GS) equation and the Bernoulli law. When the flow is strong, the characteristics of this system of equations are quite different from for static case. The generalized GS partial differential equation (PDE) can be either elliptic or hyperbolic depending on the magnitude of the poloidal flow velocity relative to the velocities of MHD waves and, in particular, transonic flow profiles become hyperbolic [1]. Two-fluid effects resolve the Alfvén singularity and modify the conditions for ellipticity. In order to include further multisclale effects, one has to adopt proper fluid equations since the characteristics of flowing equilibria depend also on the closure models. As a simple example that brings the closure problem to the two-fluid theory, we consider Hall MHD in the presence of pressure anisotropy. The MHD systems are closed with cold ions and electron pressures derived from their parallel heat flux equations [2], a closure that reproduces the corresponding kinetic dispersion relation. This is different from the CGL double-adiabatic closure which, as is well known, is not consistent with kinetic theory. We have studied the conditions for ellipticity of the PDEs for single-fluid and Hall MHD axisymmetric equilibria with flow and pressure anisotropy [3].

Anisotropic single-fluid MHD equations are obtained by omitting the Hall term in the generalized Ohm's law. Equations for axisymmetric equilibrium with flow are the generalized GS equation and the Bernoulli law. In the absence of poloidal flow, the equilibrium is elliptic if the kinetic stability conditions for the firehose and mirror modes are satisfied. In the presence of poloidal flow, there are three elliptic regions for the poloidal flow velocity with the critical velocities corresponding to the velocities of the MHD waves modified by the pressure anisotropy. These critical velocities can be obtained from the kinetic dispersion relation in its single-fluid limit, and are different from those found with the double-adiabatic CGL model [4].

For Hall MHD, a set of anisotropic-pressure equilibrium equations has been derived. It consists of the coupled GS equations for the magnetic flux and the ion stream function and the Bernoulli law for ions. Unlike the isotropic case [5], in the presence of pressure anisotropy, the characteristic determinants for each GS equation are coupled and cannot be examined their ellipticity separately. One can find the conditions for ellipticity of such systems involving higher order derivatives by examining the existence of wave type solutions, a method also applicable to second order differential equations. If we consider a wave propagating in one-dimensional space and time and having discontinuity across the wave front, ellipticity of the coupled GS equations requires the non-existence of real values of the velocity of the wave front. If there is no poloidal flow, the condition for the ellipticity is satisfied by that of single-fluid MHD equilibria with purely toroidal flow. Provided this condition holds, we have examined the dependence of the ellipticity condition on the poloidal flow velocity in the presence of pressure anisotropy. We have obtained a sufficient condition for ellipticity corresponding to a poloidal flow velocity slightly smaller than the ion sound velocity.

The fluid moment equations used in this study include the Hall term and the pressure anisotropy for electrons. To include more small scale effects such as the gyro-viscosity and the Landau damping, a more advanced closure model applicable to finite ion pressures should be adopted.

References

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