§36. Statistical Laws and Field Structure of Fluctuations in NS, MHD and Super Fluid Turbulence

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It is well known that there exist quantum vortices in quantum turbulence of liquid helium below critial temperature. Circulation of the vortex is quantized and its diameter is of order of a few angstrom. When temperature of the liquid helium is very low but non zero, interaction between super and normal fluid components exerts force on the filaments of the quantized vortices[1]. In order to examine the statistical law and singular nature of the turbulent fluctuations of the super fluid, we have simulated an ensemble of the vortex rings in the super fluid in terms of the vortex element method. Our aim here is to examine the spatial distribution law, energy spectrum and energy dissipation of the ensemble. A vortex ring is expressed as a ring consisting of small segments, and each element is assumed to move with the total velocity which are the sum of the induced velocity accoding to Biot-Savart's law and the velocity due to the normal fluid at the finite temperature.

Let an vortex element position be $\boldsymbol{s}(\xi,t)$. Then the velocity of the element is given by

$$
\begin{aligned}
\frac{d\boldsymbol{s}(\xi,t)}{dt} &= \frac{1}{4\pi}\sum_j \int_{C_j} \frac{\boldsymbol{s}'(\xi,t)(\boldsymbol{r}-\boldsymbol{s}(\xi',t))}{|\boldsymbol{r}-\boldsymbol{s}(\xi',t)|^3} d\xi' + \boldsymbol{v}_T, \\
\boldsymbol{v}_T(\boldsymbol{r},t) &= \alpha \boldsymbol{s}' \times (\boldsymbol{w} - \beta \boldsymbol{s}' \times \boldsymbol{s}") \\
&\quad -\alpha' \boldsymbol{s}' \times (\boldsymbol{s}' \times (\boldsymbol{w} - \beta \boldsymbol{s}' \times \boldsymbol{s}")), \\
&\quad \beta = \frac{\kappa}{4\pi}\log\left(\frac{cR}{a}\right),
\end{aligned}
$$

where ξ is the arc length of the vortex filament, $\boldsymbol{s}'$ and $\boldsymbol{s}''$ are tangential and normal vectors of the filament, respectively, $\boldsymbol{v}_n$ is the velocity of the normal fluid, $\boldsymbol{w} = \boldsymbol{v}_n - d\boldsymbol{s}/dt$, κ is the circulation, R is the curvature radius of the vortex ring, a is the radius of the vortex core and c is a constant of $O(1)$. α and α' are constants which increases with temperature. When length of an element of the filament is longer than a prescribed value, a new element of the filament is introduced. Also when the curvature of the filalment becomes larger than a given value, the node is removed, representing the dissipation of the vortex. The vortex reconnection is introduced when distance between two vortex filaments becomes shorter than 1.5 times the length of the elements according to Schwarts[2].

We have done numerical computation for the cases of one, two, three and ensemble of qunatized vortex rings. When the number of the vortex rings was small, the observed translational velocity, way of the vortex reconnection, stretching, and dissipation of the vortex rings were found to be consistent with the previous results. When a vortex ring was stretched enough, reconnection between the elements on the same vortex filament occured. As for the case of ensemble of 18 vortex rings, an initial configuration of vortex rings was assumed to be uniformly distributed in space and direction[3]. It was found that the rate of decay of the total arc length of all vortex rings in time increased with temperature, meaning the increase of the dissipation of the vortex rings (Figs.1, 2, and 3). The energy spectrum and the probability density functions of the velocity and its spatial derivatives will be examined and compared to the Navier-Stokes turbulence.

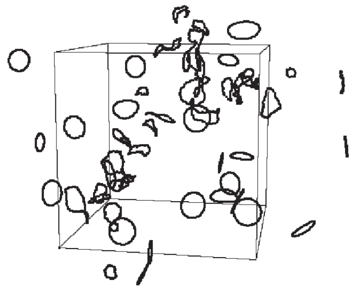

FIG. 1: Distribution of the vortex rings at $T = 1.0K$

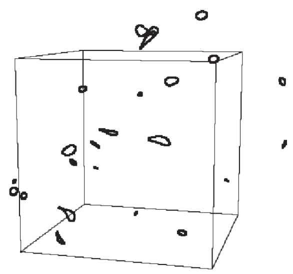

FIG. 2: Distribution of the vortex rings at $T = 1.6K$

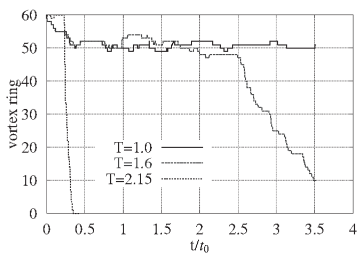

FIG. 3: Variation of the total number of the vortex rings.

1) K. Yamada and T. Ohmi, "Superfluid", Baifukan (in Japanese) (1995).
2) K. W. Schwarz, Phys. Rev. B, **31**, 5782 (1985), Phys. Rev. **38** 2398 (1988).
3) M. Kayahara, "Numerical study on the vortex rings in superfluid", Master thesis, Nagoya Institute of Technology, (200).