§43. An Alternative Coarse Projective Integration Method for Multi- Scale Plasma Simulation

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Recently, a novel simulation framework called the Equation-Free Coarse Projective Integration (EFPI) was introduced to a variety of multi-scale problems, in which the macro-scale (coarse) behavior can be obtained through short-time simulations within the fine-micro-scale models (microscopic, stochastic, etc.) [1]. The first application to plasmas has been developed by M. Shay et al., [2] to study propagation and steepening of a 1D ion sound (IS) wave, based on a PIC code as a microscopic simulator. To start PIC, macro variables (three moments) are "lifted" to a fine micro- space. The PIC code is stepped forward for a short time, and the results are "restricted" or smoothed back to macro space. By extrapolation, time derivative is estimated and projected with a large step; the process is repeated. The EFPI scheme could reproduce full PIC results with limited success, attributed mainly to idealized assumption about the Maxwellian ion distribution and adiabatic electrons.

As a simple alternative, we have proposed and implemented, an original, so-called, *primal* EFPI method to simulate nonlinear IS paradigm including kinetic effects. The micro-simulator is the standard 1D ES PIC code. Ions have been assumed inherently coarse grained as compared to electron-scale dynamics [3]. Ion orbits are tracked and extrapolated in time to project. The potential is averaged over the electron plasma period to extrapolate and project. No adiabatic approximation for electrons has been used, instead, we self-consistently track and project non-uniform electron distribution found from the Poisson equation and ion density. Our preliminary and motivating results have been already discussed with some basic limitations of EFPI methods in large multi-scale simulations pointed out [3].

Furthermore, the coarse projective integration method [4] for multi-scale plasma simulation based on PIC micro- physics solver is extended to involve 2D cumulative distribution function (CDF) of the particle phase space and applied to simulate wave steepening and kinetic effects in the nonlinear ion- sound wave paradigm. Study of particle distribution using inverse-CDF (ICDF) is a convenient tool for one- dimensional problems. Still, an extension to more than 1D is non-trivial; the CDF itself, not being a bijective mapping, does not possess an inverse. To overcome such difficulties with multi-dimensional CDFs an alternative approach is used which represents a multi-D CDF in terms of its marginal and 1D conditional CDFs [4]. To study the EFPI feasibility in kinetic plasmas, we have attempted to incorporate the above procedure. Briefly, restricted macro-quantities are coefficients of the marginal CDFs and several conditional CDF-s projected on the Legendre polynomial basis. The macro-time projection involves the polynomial coefficients by adopting the linear regression method. Lifting to micro-space is done from the CDFs (ICDFs), previously reconstructed from the corresponding time- projected polynomial coefficients.

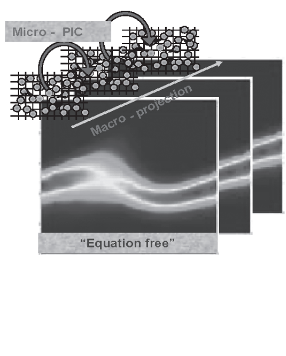

Fig 1. The schematic illustration of the EPFI method

We perform a number of simulation runs within the same plasma variables range, as [2], also by varying the scheme parameters. Our initial results seem encouraging. The macro- coarse variables, like ion density, PDF and even the ion phase space snapshots compare well. To show good conservation property of the scheme, in Fig. 2 we plot comparative time evolution of electron and ion kinetic, field and total energy. Also, phase mismatch, time-lag in the ion kinetics, typical for primal EFPI, is not observed. Although the projection was modest (10-20 steps) actual agreement with direct PIC is reasonable. However, an increasing noise level intrinsic to PIC, appears as a serious obstacle in repetitive particle reloading in the EFPI method. We plan to extend EFPI studies to other nonlinear plasma paradigm problems, as well as with a Vlasov micro-solver.

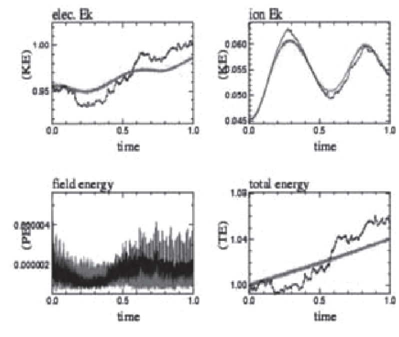

Fig.2. The kinetic energy of electrons and ions, field and total energy from PIC (gray) and EFPI.

References

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