§7. Moment-Equation Methods for Calculating Neoclassical Transport Coefficients in General Toroidal Plasmas

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A detailed comparison is made between moment-equation methods presented by Sugama and Nishimura [1] and by Taguchi [2] for calculating neoclassical transport coefficients in general toroidal plasmas including nonsymmetric systems [3]. It is shown that these methods can be derived from the drift kinetic equation with the same collision model used for correctly taking account of collisional momentum conservation. In both methods, the Laguerre polynomials of the energy variable are employed to expand the guiding-center distribution function and to obtain the moment equations, by which the radial neoclassical transport fluxes and the parallel flows are related to the thermodynamic forces. The methods are given here in the forms applicable for an arbitrary truncation number of the Laguerre-polynomial expansion so that their accuracies can be improved by increasing the truncation number. Differences between results from the two methods appear when the Laguerre-polynomial expansion is truncated up to a finite order because different weight functions are used in them to derive the moment equations. At each order of the truncation, the neoclassical transport coefficients obtained from the Sugama-Nishimura method show the Onsager symmetry and satisfy the ambipolar-diffusion condition intrinsically for symmetric systems. Also, numerical examples are given to show how the transport coefficients converge with the truncation number increased for the two methods.

In order to elucidate the differences between results from the Sugama-Nishimura and Taguchi's methods, we consider the axisymmetric case. In this case, the Sugama-Nishimura method is equivalent to the conventional moment approach [4]. The neoclassical transport of a single species of ions in the banana regime are given by

$$\begin{aligned} \left[\begin{array}{c} u_\theta \\ \frac{2}{5p_i} q_\theta \end{array} \right] &= -\frac{cIX_2}{e\chi' \langle B^2 \rangle} \left[\begin{array}{c} C_{0\theta} \\ C_{1\theta} \end{array} \right], \\ q^{\rm b} &= C_q \frac{f_t}{f_c} \frac{n_i m_i T_i c^2 I^2}{e^2 (\chi')^2 \langle B^2 \rangle \tau_{ii}} X_2, \end{aligned} \qquad (1)$$

where the poloidal flow velocity u_θ, the poloidal heat flow q_θ, and the radial heat flux $q^{\rm b}$ for ions are driven by the radial ion temperature gradient X_2 [see Ref. 3 for detailed definitions of variables in Eq. (1)]. The numerical values of the dimensionless coefficients $C_{0\theta}$, $C_{1\theta}$, and C_q obtained from the Sugama-Nishimura method and those from Taguchi's method are shown in Fig. 1 for the cases using the different truncation numbers $j_{\max} = 1, 2$ and 3 (referred to as the 13M, 21M and 29M approximations, respectively).

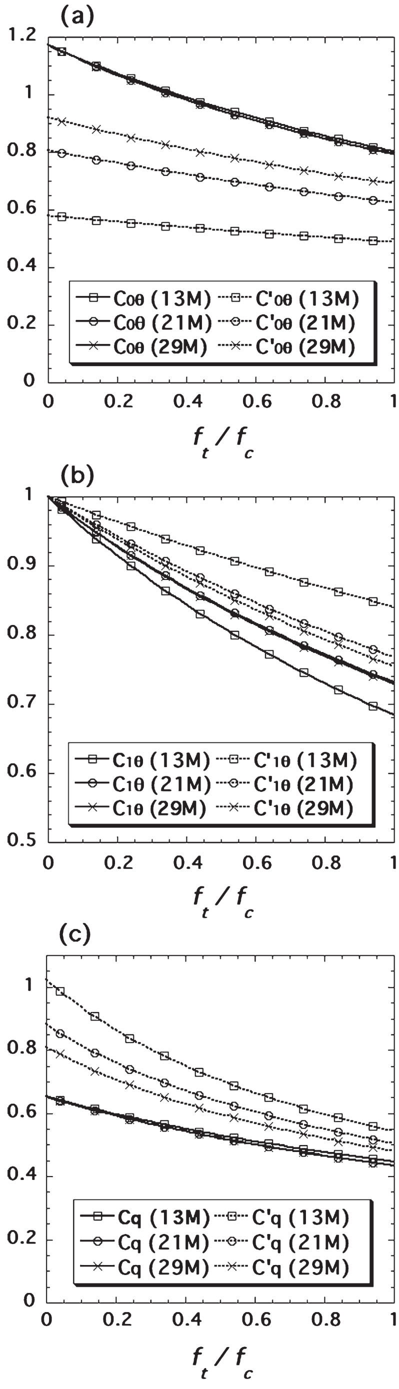

Fig.1. Dimensionless neoclassical coefficients calculated as functions of f_t/f_c for $j_{\max} = 1$ (13M), 2 (21M), and 3 (29M). Here, f_t denotes the fraction of trapped particles and $f_c \equiv 1 - f_t$. The coefficients $C_{0\theta}$, $C_{1\theta}$, and C_q in Eq. (1) obtained from the Sugama-Nishimura method are plotted by solid curves in (a), (b), and (c), respectively. For comparison, also plotted by dotted curves are $C'_{0\theta}$, $C'_{1\theta}$, and C'_q obtained from Taguchi's method.

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4) S. P. Hirshman and D. J. Sigmar, Nucl. Fusion **21**, 1079 (1981).