§28. Quasi-bound States in Continuum in a Two-channel Quantum Wire with an Adatom

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We report the prediction of quasi-bound states (resonant states with very long lifetimes) that occur in the eigenvalue continuum of propagating states for certain systems in which the continuum is formed by two overlapping energy bands[1]. Our Hamiltonian is the tight-binding model on a ladder with the adatom, or the dot (Fig. 1(a)):

$$\begin{aligned}\mathcal{H} &= -\frac{t_\mathrm{h}}{2}\sum_{y=1,2}\sum_{x=-\infty}^{\infty}\left(c_{x+1,y}^\dagger c_{x,y}+c_{x,y}^\dagger c_{x+1,y}\right)\\ &\quad -t_\mathrm{h}'\sum_{x=-\infty}^{\infty}\left(c_{x,2}^\dagger c_{x,1}+c_{x,1}^\dagger c_{x,2}\right)\\ &\quad +g\left(d^\dagger c_{0,1}+c_{0,1}^\dagger d\right)+E_\mathrm{d}d^\dagger d. \qquad (1)\end{aligned}$$

Here, $c_{x,y}$ are the annihilation operators of a spinless fermion at the site (x, y) with integer x ($-\infty < x < \infty$) and $y = 1, 2$, whereas $d^\dagger$ and d represent an adatom attached to the $(0, 1)$ site of the ladder. The first line of Eq. (1) gives the hopping matrix elements along the ladder, the second line the hopping elements across the ladder and the third line gives the hopping elements to and from the one-particle level of the adatom.

The ladder has two eigenmodes $c_{x,\pm} \equiv (c_{x,1} \pm c_{x,2})/\sqrt{2}$, which transform the Hamiltonian (1) as Fig. 1(b). The Fourier transform $c_{k_\pm,\pm}^\dagger = (2\pi)^{-1/2}\sum_{x=-\infty}^{\infty} e^{ik_\pm x}c_{x,\pm}^\dagger$ reveals that the system has two conduction channels with $-\pi \le k_\pm \le \pi$, each of which forms an energy band (a continuum) $\varepsilon_\pm(k_\pm) = -t_\mathrm{h}\cos k_\pm \mp t_\mathrm{h}'$.

When the energy bands of the two channels overlap, a would-be bound state that lays just below the upper energy band is slightly destabilized by the lower energy band and thereby becomes a resonant state with a very long lifetime (a second such state lays above the lower energy band). Unlike the bound states in continuum predicted by von Neumann and Wigner[2, 3], these states occur for a wide region of parameter space.

The quasi-bound states are resonant states because they diverge in space due to the negative imaginary part of only one of the wave numbers. Some of them, however, diverge in space very slowly and decay in time very slowly. One of the quasi-bound state, for example, *appears* to be a localized state around the x-axis origin and exponentially diverges only far away (Fig. 2).

Analytically solving the dispersion relation by perturbation expansion in g, we find that the imaginary part of the energy of state is proportional to g^6 (extremely small for $g \ll 1$) due to the interaction between the divergent van Hove singularity in the upper energy band and the continuum of the lower band[4]. Since the real part of the energy lays within the lower energy band, we refer to this state as a quasi-bound state in continuum (QBIC).

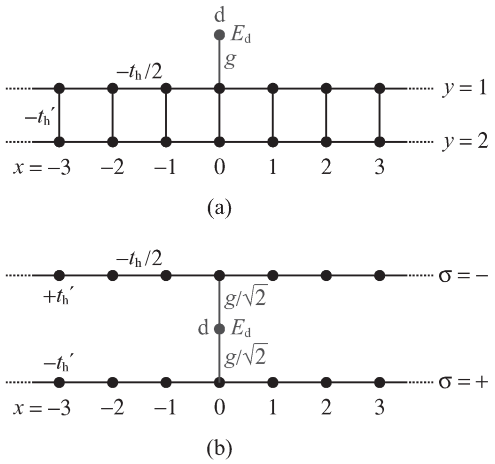

Figure 1: (a) An adatom attached to a ladder. (b) After diagonalizing the ladder, the system is composed of an adatom coupled to two independent channels.

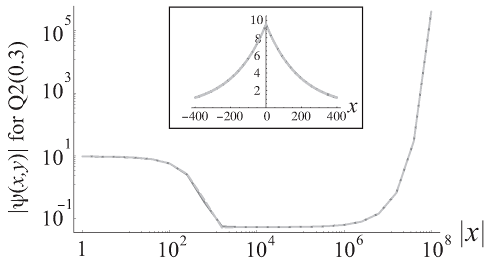

Figure 2: The wave function modulus $|\psi(x, y)|$ of the quasi-bound state around the origin on the linear scale (inset) and away from the origin on the logarithmic scale. The plots for $y = 1$ (the upper leg) and $y = 2$ (the lower leg) are almost indiscernible. The parameters are set to $t_\mathrm{h}' = 0.345t_\mathrm{h}$, $g = 0.1t_\mathrm{h}$ and $E_\mathrm{d} = 0.3t_\mathrm{h}$.

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[2] J. von Neumann and E. Wigner, Phys. Z. **30**, 465 (1929).

[3] T. Petrosky, C.-O. Ting, and S. Garmon, Phys. Rev. Lett **94**, 043601 (2005).

[4] N. Hatano, H. Nakamura, K. Sasada and T. Petrosky, Prog. Theor. Phys. **119**, 187(2008).