§31. Numerical Renormalization Group in NS and MHD Turbulence by Using Parallel Massive Direct Numerical Simulation

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It is common in direct numerical simulations (DNSs) of NS, MHD, and passive scalar turbulence that $K_{max}\overline{\eta} > 1$ is a necessary condition for the spatial accuracy, where $\overline{\eta}$ is the average Kolmogorov length and K_{max} is the maximum wavenumber retained in the Fourier spectral method. However, in many cases of turbulence simulation at high Reynolds numbers, there is an actual cut off wavenumber k_c such that $k_c\overline{\eta} < K_{max}\overline{\eta} < 1$ because of limitation of the computational resources. In this case, effects of degrees of freedom (subgrid scale, SGS) in the band $k_c < k < K_{max}$ on grid scale (GS) are to be examined. We have examined those effects from the view points of dynamics, statistics, and renormalization by taking as $k_c = k_{max}/\beta, \beta = 1, 2, 4$ and comparing with the data which was computed by the full resolution DNS.

For this purpose, we have introduced a sharp cut off filter $\mathcal{F}$ at k_c and decomposed the velocity field as $\boldsymbol{u}(\boldsymbol{k}, t) = \boldsymbol{u}^<(\boldsymbol{k}, t) + \boldsymbol{u}^>(\boldsymbol{k}, t)$. The equation of motion of the GS velocity $\boldsymbol{u}^<(\boldsymbol{k}, t)$ is given by $\left(\partial_t + \nu k^2\right) \boldsymbol{u}^<(\boldsymbol{k}) = \boldsymbol{N}^<(\boldsymbol{k}) + \boldsymbol{R}^<(\boldsymbol{k})$, $\boldsymbol{N}^<(\boldsymbol{k}) = \mathcal{F}\boldsymbol{M}(\boldsymbol{k}) \sum^{\Delta}_{\boldsymbol{p},\boldsymbol{q}} \boldsymbol{u}^<(\boldsymbol{p})\boldsymbol{u}^<(\boldsymbol{q})$, where $\boldsymbol{N}^<$ is the nonlinear term consisting of GS components alone, and $\boldsymbol{R}^<$ represents contributions from the SGS components. We have done two series of DNSs, the first one is runs L1($N = 256, K_{max}\overline{\eta} = 1.0$), L2($N = 512, K_{max}\overline{\eta} = 2.0$), and L3($N = 1024, K_{max}\overline{\eta} = 3.8$) at $R_\lambda = 180$, and the second is run H1($N = 1024, K_{max}\overline{\eta} = 1.06$) at $R_\lambda = 420$ [1].

Figure 1 shows the compensated energy spectrum $k^{5/3}E(k)$ and $k^2E(k)$ in steady turbulence. Curves for $k^{5/3}E(k)$ collapse well and the one for $R_\lambda = 420$ has a horizontal part with finite width , showing the existence of the inertial range. For this turbulence, we have analysed the effects of the SGS components. Figure 2 shows $E_N(k|k_c) = 4\pi k^2\langle|\boldsymbol{N}^<(\boldsymbol{k})|^2\rangle$ and the square root of $E_R(k|k_c) = 4\pi k^2\langle|\boldsymbol{R}^<(\boldsymbol{k})|^2\rangle$. Except wavenumbers near k_c, there is no change in $E_N(k|k_c)$. This is because $E_R(k|k_c)$ is very small except $k = k_c$ and k_c lies in the dissipation range. $E_R(k|k_c)$ rises as $E_R(k|k_c) \propto (k/k_c)^{3.2}$ when $k \to k_c$ and the curve becomes cusp when $k \approx k_c$. If we write as $E_R(k|k_c) = 2\nu_e(k|k_c)k^2E_N(k|k)$, the term $\nu_e(k|k_c)$ is the eddy viscosity. As seen in Fig. 3, the eddy viscosity becomes large near $k = k_c$ and decays quickly when k becomes smaller. The contributions from SGS components may be written in the form of Langevin type equation as $\boldsymbol{R}^<(\boldsymbol{k}) = -\nu_e(k|k_c)k^2\boldsymbol{u}^<(\boldsymbol{k}) + \tilde{\boldsymbol{R}}(\boldsymbol{k})$. It is important to note that the random force $\tilde{\boldsymbol{R}}(\boldsymbol{k})$ is not white noise in time and its probability density function is not Gaussian at all, which is confirmed by using the above DNS data set [2, 3]. It is interesting and important to examine the possibility of nonlinear regression model.

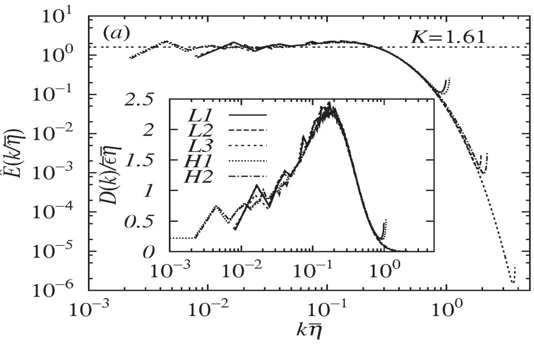

FIG. 1: Compensated kinetic energy spectrum $\hat{E}(x)$ at $R_\lambda = 420$. The dotted horizontal line is $K = 1.61$. The inset figure shows the normalised dissipation spectra.

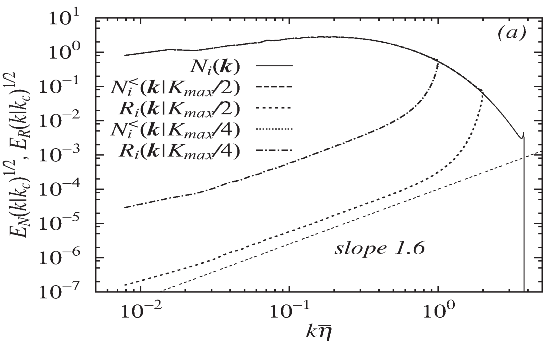

FIG. 2: Square root of the spectra for the nonlinear and residual terms $\boldsymbol{N}^<(\boldsymbol{k})$ and $\boldsymbol{R}^<(\boldsymbol{k})$.

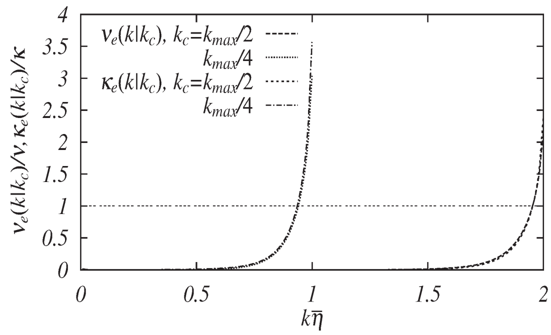

FIG. 3: The normalised eddy viscosity and eddy diffusivity extracted from the residual terms $\boldsymbol{R}(\boldsymbol{k})$ and $\boldsymbol{R}_\theta(\boldsymbol{k})$ obtained from run L3. $k_c = k_{max}/2$ and $k_c = k_{max}/4$.

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[3] H. Touli, M. Y. Hussaini, T. Gotoh, R. Rubinstein, and S. L. Woodruff, New J. Phys. **9** 215 (2007).