§3. Effective Helical Ripple and Zonal Flow Response as Functions of Outermost Flux Surface in L=2 Heliotron

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In order to investigate the neoclassical and anomalous transport quickly, we estimate the effective helical ripple $\varepsilon_{\rm eff}$ and zonal flow response K_L, in $L=2$ heliotron with various shapes of outermost flux surface. For this purpose, the $L=2$ heliotron is characterized by three small parameters δ as [1]

$$\left\{\begin{array}{cc} R_{0,0} & Z_{0,0} \\ R_{1,0} & Z_{1,0} \\ R_{1,M} & Z_{1,M} \\ R_{0,-M} & Z_{0,-M} \end{array}\right\} = R_0 \left\{\begin{array}{c} 1 \quad 0 \\ \delta_t \left\{\begin{array}{c} 1 \quad 1 \\ \delta_h \left\{\begin{array}{cc} -1 & 1 \\ \delta_b & \delta_b \end{array}\right\} \end{array}\right\} \end{array}\right\},$$

where the toroidicity δ_t and the helicity δ_h are positive while the helical twisting parameter of the surface δ_b can change the sign. In this report, $M=10$ is fixed and R_0 is nominal parameter.

The effective helical ripple is obtained as

$$\varepsilon_{\rm eff}^{3/2} = \frac{\pi 2^{7/2}}{256}\frac{r^2}{\varepsilon_t^2}\left(\frac{dr}{ds}\right)^2 H, \qquad (1)$$

where

$$H = \frac{1}{B_0(dV/d\chi)}\int_{\alpha_0}^{\alpha_0+2\pi/M} d\alpha \int_{h_{\min}}^{h_{\max}} d\Lambda \sum_l \frac{(H_1^{(l)})^2}{H_2^{(l)}},$$

$$H_1^{(l)}(\Lambda,\alpha) = \frac{B_\theta}{\chi'}\int_{\theta_1^{(l)}}^{\theta_2^{(l)}} d\theta h^2 \kappa_g \frac{|v_\parallel|}{v}\left(3+\frac{|v_\parallel|^2}{v^2}\right),$$

$$H_2^{(l)}(\Lambda,\alpha) = \frac{B_\theta}{B_0}\int_{\theta_1^{(l)}}^{\theta_2^{(l)}} d\theta h^3 \frac{v_\perp^2}{v^2}\frac{|v_\parallel|}{v}, \qquad (2)$$

$\kappa_g = \boldsymbol{b}\cdot\nabla\boldsymbol{b}\cdot\sqrt{g}\nabla s\times\nabla\theta$, $\sqrt{g} = (\nabla s\cdot\nabla\theta\times\nabla\alpha)^{-1}$, $dV/d\chi = (B_\theta/B_0^2)\int_{\alpha_0}^{\alpha_0+2\pi/M} d\alpha \int_{\theta_0}^{\theta_0+2\pi} d\theta h^2$, $B_\theta = I + qG$, $2\pi(I,G)/\mu_0$ are toroidal and poloidal currents respectively, and $2\pi\chi$ are poloidal flux. The zonal flow response is obtained as $\mathcal{K}_L(t) = I'(t)/(D_< + \mathcal{E}(t))$ with $I'(t) = \langle I(t)\rangle/(e_i\langle\phi_{k_r}(0)\rangle/T_i)$, where

$$I'(t) = \int_{\rm tt,c} d^3v_* F_{0i}\frac{\tau_h'}{v'}\langle\overline{J_{0i}e^{-ik_r\Delta_r}}\rangle_{\rm po}\langle e^{ik_r\Delta_r}\overline{(1-\Gamma_{0i})}\rangle_{\rm po}$$
$$+ \int_{\rm ht} d^3v_* F_{0i}\frac{\tau_h'}{v'}\langle\overline{J_{0i}}e^{-ik_r\overline{v_{di}^r}t}\overline{(1-\Gamma_{0i})}\rangle_{\rm po},$$

$$D_< = \int_{\rm tt,c} d^3v_* F_{0i}\frac{\tau_h'}{v'}(1-\langle\overline{J_{0i}e^{-ik_r\Delta_r}}\rangle_{\rm po}\langle\overline{J_{0i}}e^{ik_r\Delta_r}\rangle_{\rm po}),$$

$$\mathcal{E}(t) = \sum_{j=i,e}\frac{T_i}{T_j}\int_{\rm ht} d^3v_* F_{0j}\frac{\tau_h'}{v'}(1-\langle(\overline{J_{0j}})^2 e^{-ik_r\overline{v_{dj}^r}t}\rangle_{\rm po}), \qquad (3)$$

F_{0j} is the Maxwellian for j species, $J_{0j} = J_0(k_r\rho_j)$ is a Bessel function, $\Gamma_{0j} = \Gamma_0(b_j) = I_0(b_j)e^{-b_j}$ with I_0 being a modified Bessel function, $b_j = (k_r a_j)^2$, and $a_j = \sqrt{T_j/m_j}/\Omega_j$. These $\varepsilon_{\rm eff}$ and K_L are originally obtained in [2] and [3] respectively, and are rewritten to be suitable in the Boozer coordinates.

The (radially averaged) effective helical ripple $\varepsilon_{\rm eff}$ (triangles) and the (radially and time averaged) damped zonal flow, defined by $|1-K_L|^2$ (circles), are shown as a function of δ_b. Here $\delta_t = 1/6$ and $\delta_h = 1/3$ are fixed. It can be seen that both the damped zonal flow and effective helical ripple have the minimum points, which are nearly coincident on $\delta_b \sim 0.3$. The corresponding δ_b values of the realistic LHD configurations are pointed out by arrows, and the inward-shifted configuration with $R_0 = 3.53$m is confirmed to be nearly optimum. This shows the importance of the parameter δ_b, and gives a clearer visualization for the discussion in [3] that the neoclassical and anomalous transport are both optimized in the inward shifted configuration in the $L=2$ heliotron. The more detailed analysis like how the radial drift inwardly and outwardly is canceled out by the bounce average in each helical ripple can be found in [1]. This consideration is also the simple case in the low collisional regime, and more detailed analysis will be needed to investigate the transport in the realistic situation with other factors such as collisionality and instabilities in the various plasma profiles.

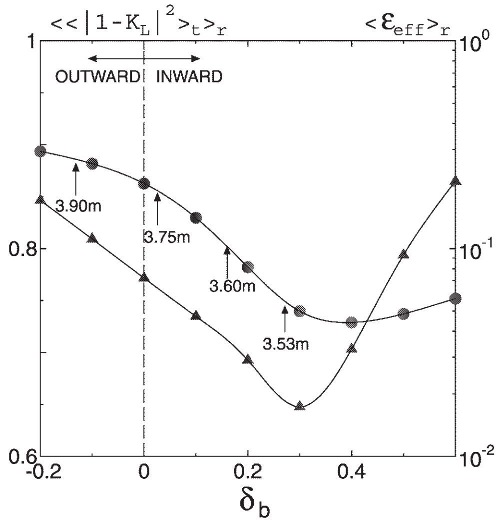

FIG. 1: Damped zonal flow (cirlces, left axis) and effective helical ripple (triangles, right axis) as a function of δ_b.

1) O.Yamagishi and S.Murakami Nucl. Fusion **49**, 045001 (2009)
2) V.V.Nemov et al., Phys. Plasmas **6**, 4622 (1999)
3) H.Sugama and T-H.Watanabe, Phys. Plasmas **13**, 012501 (2006)