§4. Energy Cascade in Homogeneous Turbulence

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One of the most important characteristics of turbulence is the small-scale university: that is, statistics at small scales are independent of larger-scale structures which depend on the boundary conditions and/or external forcing. It is the *energy cascade* that is the basis of this small-scale universality. More precisely, the energy supplied to turbulence at a large scale (the integral length L) transfers, *scale by scale*, to smaller scales until it is dissipated by the molecular viscosity at the smallest scale (the Kolmogorov length η). Then, the information of the large scales may well be forgotten through this cascade process, and therefore the small-scale universality is established. Although the Fourier analysis of turbulence has been shown to support this energy cascade picture, it is unknown what happens in the real space.

In order to investigate the physical mechanism of the energy cascade, we have carried out the direct numerical simulations of homogeneous isotropic turbulence (in a periodic cube) of an incompressible fluid at relatively high Reynolds numbers (the Taylor-length based Reynolds number is about 200). Then, it is clearly observed that such turbulence consists of multi-scale coherent vortical structures with tubular shapes (Fig. 1).

This fact implies that the energy cascade is the process of the creation of smaller-scale (i.e. thinner) vortex tubes due to the vortex stretching in the strain around the pairs of vortex tubes at larger scales (i.e. fatter vortex tubes). More precisely, the cascade process can be described as follows[1)]: The energy supplied to the large scale, L, is possessed by vortex tubes with large radii of $O(L)$. When a pair (especially, an anti-parallel pair) of these vortex tubes encounter, a strongly straining region is created around the pair. In this straining region, smaller-scale (i.e. thinner) vortices are stretched and created. In other words, the energy possessed by the fatter, $O(L)$, vortex tubes transfers to thinner scales (L', say) by the process of the vortex stretching. Thus created vortex tubes with radii of the intermediate length scale, L', confines the energy inside them; and when a pair of them encounter, a strongly straining region at the scale, L', is created around the pair, and further smaller-scale (i.e. further thinner, L'') vortex tubes are stretched and created. Thus energy transfers from L' to L''. Such processes continue until η-scale vortex tubes are created, and the energy is finally dissipated by the molecular viscosity in strongly straining regions around those smallest-scale vortices.

Indeed, examples supporting the above scenario are easily found in the numerically simulated homogeneous isotropic turbulence. That is, one can frequently observe that anti-parallel pairs of fatter (i.e. larger-scale) vortex tubes stretch and create thinner (i.e. smaller-scale) vortex tubes, and that the energy transfers from larger to smaller scales.

More quantitative studies to support the proposed scenario of the energy cascade are now under investigation by the simulations, on the Plasma Simulator, of turbulence at higher Reynolds numbers.

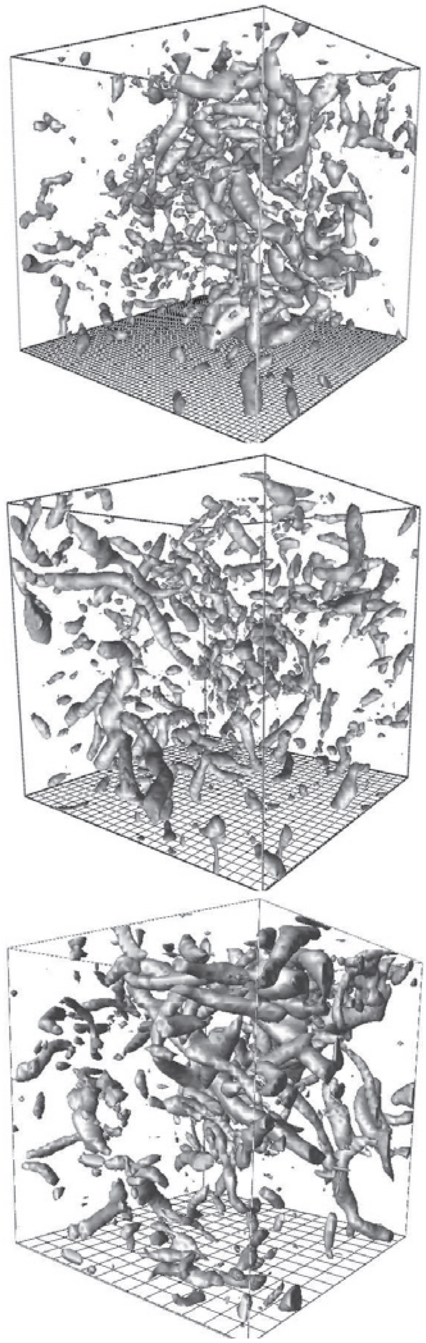

Fig. 1 Multi-scale coherent vortices in homogeneous isotropic turbulence. Iso-surfaces of squared coarse-grained vorticity are plotted for three different coarse-graining scales: (top) 74η, (middle) 36η and (bottom) 18η. Note that the bottom grid width is common. Coherent structures look statistically self-similar and have tubular shapes at these scales in the inertial subrange.

1) S. Goto: J. Fluid Mech. **605** (2008) 355.