§12. Flows in a Precessing Sphere - Structure and Stability

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Last year we investigated the stability characteristics of steady flows in a precessing spherical cavity. In terms of the sphere radius a, the kinematic viscosity ν of fluid, the spin rotation angular velocity Ω_s and the precession rotation angular velocity Ω_p we introduce the Reynolds number $Re = a\Omega_s/\nu$ and the Poincaré number $\Gamma = \Omega_p/\Omega_s$. This system is then characterised by these two control parameters [1, 2, 3]. We found numerically the instability boundary over a wide range of parameters, i.e. $0 < Re < 5000$ and $0.04 < \Gamma < 1.8$ (see Fig. 1 in Ref. 1). The critical Reynolds number is about 1102 at $\Gamma = 0.18$.

The double limit of large Re and small Γ is of a particular importance with relation to the geodynamo due to the fluid motion in the interior of the Earth which is precessing with the spin period of one day and the precession period of 25,800 years. Knowledge of the spatial structure of steady flows is prerequisite to investigate the stability characteristics in this limit. Therefore we have performed direct numerical simulations at large Re and small Γ. In Fig. 1, we show the flow structure at $Re = 10^5$ and $\Gamma = 2 \times 10^{-4}$, where the contours of the velocity magnitude are drawn in the plane including the spin (x) axis and normal to the precession (z) axis. The pattern of slope $\pm 1/\sqrt{3}$ is prominent, which are cross-sections of circular cones having their apexes at the poles of the spin rotation axis.

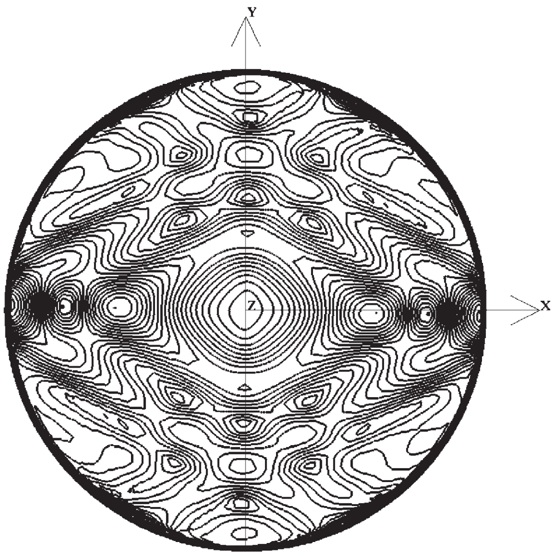

Fig. 1: The pattern of kinetic energy density on the plane including the spin rotation (x) axis and perpendicular to the precession rotation (z) axis. $Re = 10^5$, $\Gamma = 2 \times 10^{-4}$.

In parallel to the direct numerical simulation we have performed an asymptotic analysis of the flow in the limit $Re \gg 1$ and $\Gamma \ll 1$. In this limit there appears a thin boundary layer of thickness $\delta = 1/\sqrt{Re}$ over almost all the spherical boundary. As is well-known, this boundary layer has singularities at latitude 30^o in both the northern and southern hemispheres, which are called critical circles. Near these critical circles stronger fluid flows are induced over the thickness of $O(\delta^{4/5})$ in the radial direction and of $O(\delta^{2/5})$ in the latitudinal direction.

These singularities flow into the interior of the sphere along the characteristics of inertial waves to make circular cones observed in Fig. 1. The interior flow is seen in Fig.2, where we plot the velocity field on plane $x = 1/4$ with arrows, the precession rotation axis being vertical. The outermost circle indicates the spherical boundary on this cross-section. The two other smaller circles correspond to the circular cones shown in Fig.1. Abrupt variation in the velocity field across the circles is clearly visible.

The stability of this kind of steady flows is the next target of our research.

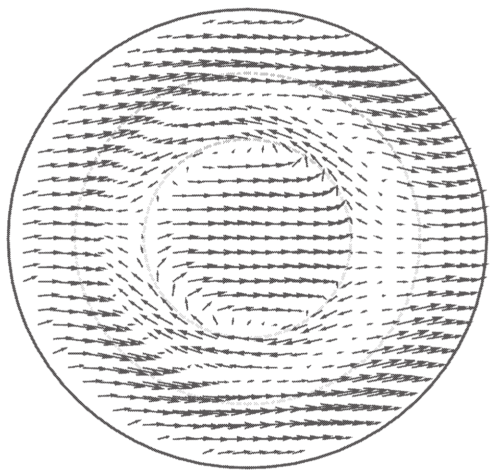

Fig. 2: The velocity field on a cross-plane $x = \frac{1}{4}$ obtained by an asymptotic analysis for $\Gamma \ll 1$. The largest circle is a cross-section of the spherical boundary. The other two circles indicate cross-sections of two circular cones emanating from critical circles. $Re = 10^5$.

1) Goto, S. *et al.*: Phys. Fluids **19** (2007) 061705.

2) Kida, S. *et al.*: Fluid Dyn. Res. **41** (2009) 011401.

3) Kida, S. and Nakayama, K.: J. Phys. Soc. Japan **77** (2008) 054401.

4) Kida, S: Ann. Rep. NIFS (2007 - 2008).